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Mathematical Methods of Modern Physics - Problem Set 7

Summer Semester 2024

Due: The problem set will be discussed in the seminars on 30.05. and 31.05.

Internet: The problem sets can be downloaded from
https://home.uni-leipzig.de/stp/Mathematical_methods_2_ss24.html

22. Partial fraction decomposition *2+2 Points*

We want to compute the integral,

$$\int_C \frac{1}{z^2+z} dz,$$

for C being a circle defined by $|z| = R$ and positively oriented.

a) Construct the partial fraction decomposition of $\frac{1}{z^2+z}$.
b) Use the partial fraction decomposition to evaluate the integral for $R < 1$ and $R > 1$.

Hint: You can use without further proof that from exercise 4 of Problem set 6 it follows that:

$$\int_C \frac{1}{z-z_0} dz = \begin{cases} 0, & \text{if } z_0 \text{ is outside the circle } C \\ 2\pi i, & \text{if } z_0 \text{ is inside the circle } C \end{cases}$$

23. Continious deformation of curves *1+1+1+1 Points*

Let D be the domain consisting of the complex plane with the three points $0,\ 2i,$ and 4 deleted. Let Γ be the (solid-line) contour shown in Figure 1. Decide which of the following contours are continuously deformable to Γ in D, i.e., you do not have to pass through one of the deleted points.

a) The dashed line contour Γ_0 in Fig. 1.

b) The circle $|z| = 3$ traversed once in the positive direction starting from the point $z = 3$.

c) The circle $|z| = 10^4$ traversed once in the positive direction starting from the point $z = 10^4$.

d) The circle $|z - 2| = 1$ traversed once in the positive direction starting from the point $z = 3$.

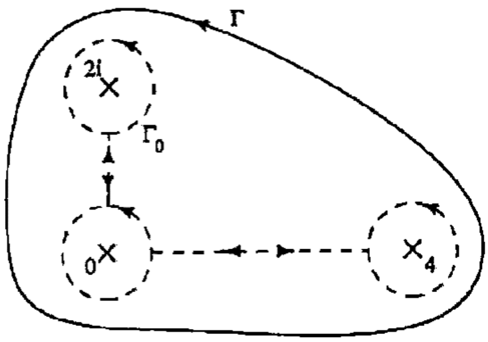

Figure 1:

24. Cauchy integral theorem *1+1+1 Points*

Determine the largest domain in which the following functions are holomorphic and explain why

$$\oint_{|z|=2} f(z)dz = 0$$

$$\text{a) } f(z) = \frac{z}{z^2+25} \qquad \text{b) } f(z) = e^{-z}(2z+1) \qquad \text{c) } f(z) = \frac{\cos(z)}{z^2-6z+10}$$

25. Fourier transformation of a Gaussian function *4 Points*

Let f be a real function defined on all of $\mathbb{R}$. We call

$$\tilde{f}(p) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)e^{ipx}dx, \; p \in \mathbb{R}$$

the Fourier transform of f.
Show by using the Cauchy integral theorem that the Fourier transform of a Gaussian function ($f(x) = be^{-ax^2}$; $a > 0$) is again a Gaussian function. Compare the width of the original Gaussian function with the width of its Fourier transform.