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Mathematical Methods of Modern Physics - Problem Set 6

Summer Semester 2024

Due: The problem set will be discussed in the seminars on 23.05. and 24.05.

Internet: The problem sets can be downloaded from
                 https://home.uni-leipzig.de/stp/Mathematical_methods_2_ss24.html

1. Contour integrals I   *4 Points*

Calculate $\oint_C |z|^2 dz$, where C is a positively (i.e. counterclockwise) oriented circle of
radius R and center $a \in \mathbb{C}$.

2. Contour integrals II   *3+2+2 Points*

Calculate $\int_C |z| dz$,

   a) where C is a straight line connecting $-i$ to i.

   b) where C is the left half of the unit circle connecting $-i$ to i.

   c) where C is the right half of the unit circle connecting $-i$ to i.

3. Integrating the exponential function   *2 Points*

Show by direct contour integration that for every path C from z_1 to z_2 it is

$$\int_C e^z dz = e^{z_2} - e^{z_1}$$

4. z^n on circular contour   *2 Points*

Calculate for all $n \in \mathbb{Z}$ by direct contour integration (i.e. not by using Cauchy's Integral Theorem) the integral

$$\int_C z^n dz,$$

for C being a positively oriented circle of radius r centered around zero.