Mathematical Methods of Modern Physics - Problem Set 5

Summer Semester 2024

Due: The problem set will be discussed in the seminars on 16.05. and 17.05.

Internet: The problem sets can be downloaded from
https://home.uni-leipzig.de/stp/Mathematical_methods_2_ss24.html

1. Parametrization of paths   *1+1+1+2+4 Points*

For each of the following curves give a parametrization that is consistent with the indicated direction.

a) A straight line from $z = 1 + i$ to $z = -2 + -3i$.

b) The circle $|z - 2i| = 4$ transversed once in the clockwise direction starting from the point $z = 4 + 2i$.

c) The segment of the parabola $y = x^2$ from point $(1, 1)$ to the point $(3, 9)$.

d) The ellipse $x^2/a^2 + y^2/b^2 = 1$ transversed once in the counterclockwise direction starting from the point $(a^2, 0)$.

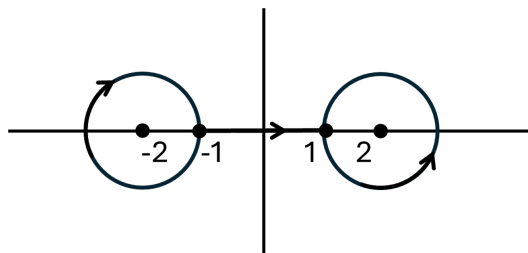e) The curve shown in Figure 1, where the start point is $z = -1$ and the end point is $z = 1$.

Figure 1:

2. Length of curve   *2+2+2 Points*

Use the formula for the length of a curve from the lecture to...

a) verify that the length of a straight line from z_1 to z_2 is indeed $|z_2 - z_1|$.

b) verify that the length of the circle $|z - z_0| = R$ is indeed $2\pi R$.

c) determine the length of the curve parametrized by $z(t) = 5e^{3it}$, $0 \le t \le \pi$.