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Mathematical Methods of Modern Physics - Problem Set 11

Summer Semester 2024

Due: The problem set will be discussed in the seminars on 27.06. and 28.06.

Internet: The problem sets can be downloaded from
https://home.uni-leipzig.de/stp/Mathematical_methods_2_ss24.html

37. Laurent series *2+3 Points*

Expand the function $f(z) = [(z-1)(z+1)]^{-1}$ into a Laurent series around $z_0 = 1$

a) in the circular ring $\left\{z \in \mathbb{C} \middle| \, 0 < |z-1| < 2\right\}$

b) in the circular ring $\left\{z \in \mathbb{C} \middle| \, |z-1| > 2\right\}$.

38. Residues *2+2+2 Points*

Calculate the residue of

a) $\dfrac{1}{\sin(z)}$ at $z_0 = 0$

b) $\dfrac{az^2 - 2}{z^2 - 1}$ at $z_0 = 1$

c) $(1 - \cos^2(z))^{-1}$ at $z_0 = 0$.

39. Function with multiple branch points *1+3 Points*

Consider the function

$$f(z) = (z^2 - 1)^{\frac{1}{2}} = (z+1)^{\frac{1}{2}}(z-1)^{\frac{1}{2}}$$

which is multivalued due to the complex root. We can make the function single-valued by introducing one or multiple branch cuts, i.e., we can make $\arg(f(z)) \bmod 2\pi$ well-defined and continuous for all z that are not on the branch cut(s). In this exercise, we want to explore two different choices for the branch cut(s).

a) One possible choice is to make branch cuts on the real axis from 1 to ∞ and from -1 to $-\infty$. Show that in this case $f(z)$ is a single-valued function.

b) Another possibility is to put a single branch cut between -1 and 1. Show that $f(z)$ is also a single-valued function in this case [although it is different from the function in a)].
Hint: Consider $\arg(f(z))$ along the contour shown in Fig. 1. Establish that $\arg(f(z)) \bmod 2\pi$ is continuous on the entire contour. Form this you can conclude that f is single-valued.

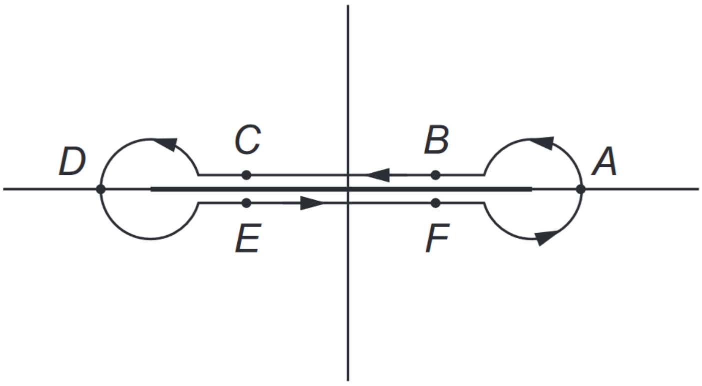

Figure 1: Branch cut as in 39 b). The aim is to show that $\arg(f(z))$ is continuous along the entire contour, i.e. in particular that it is continuous in D and A. You might want to calculate an approximate value of $\arg(f(z))$ at each of the marked points.