Nonlinear ac conductivity of interacting one-dimensional electron systems

Bernd Rosenow and Thomas Nattermann
Institut für Theoretische Physik, Universität zu Köln, D-50923 Köln, Germany

(Received 7 July 2005; revised manuscript received 28 September 2005; published 8 February 2006)

We consider low-energy charge transport in one-dimensional electron systems with short-range interactions under the influence of a random potential. Combining renormalization group and instanton methods, we calculate the nonlinear ac conductivity and discuss the crossover between the nonanalytic field dependence of the electric current at zero frequency and the linear ac conductivity at small electric fields and finite frequency.

DOI: 10.1103/PhysRevB.73.085103 PACS number(s): 72.15.Nj, 71.10.Pm, 72.15.Rn

I. INTRODUCTION

In one-dimensional (1D) electron systems, the effects of both interactions and random potentials are very pronounced, and a variety of unusual phenomena can be observed.[1,2] The linear dc conductivity shows a power law dependence on temperature T at higher temperatures,[3,4] but is exponentially small at low temperatures and vanishes at zero temperature.[5,6] The ac conductivity vanishes like[7] $\sim\omega^2[\ln(1/\omega)]^2$ and shows several crossovers to other power laws at higher frequencies.[8]

Much less is known about the nonlinear conductivity. At zero temperature and frequency charge transport is only possible by tunneling of charge carriers, which can be described by instanton formation. The nonlinear dc conductivity is characterized by[9–12] $I\sim\exp(-\sqrt{E_0/E})$ provided the system is coupled to a dissipative bath. Without such a coupling, the current was recently suggested to vanish below a critical temperature.[13,14]

In this work, we calculate the low-energy nonlinear ac conductivity for systems with random pinning potentials and discuss the crossover between linear ac response at small fields and nonlinear dc response at large fields. To be specific, we consider a charge density wave (CDW) or spinless Luttinger liquid (LL) pinned by a random lattice potential which can be described by the quantum sine-Gordon model with random phases. We first scale the system to its correlation length, where the influence of the potential is strong and a semiclassical instanton calculation becomes possible. The response is dominated by energetically low-lying two-level systems (TLSs), whose dynamics is described by a Bloch equation.

Microfabrication of quantum wires or 1D CDW systems[15] should allow us to test our predictions experimentally. Indeed there are a number of recent experiments on carbon nanotubes[16–18] and polydiacetylene[19] which seem to confirm the variable-range hopping prediction for the dc conductivity made in Refs. 10 and 11.

II. ac CONDUCTIVITY OF 1D DISORDERED SYSTEMS

In the following, we present a heuristic derivation of the Mott-Halperin result[7] for the ac conductivity of a one-dimensional disordered electron system without interactions. In the end of the section, we indicate how this result can be generalized to interacting electrons.

In one spatial dimension, all electron states are localized and wave function envelopes decay on the scale of the localization length $\xi_{\rm loc}$. We divide the system into segments with size $\xi_{\rm loc}$. The typical energy separation of states within one segment is the mean level spacing $\Delta=1/(\rho_F\xi_{\rm loc})$, where ρ_F is the density of states at the Fermi level per unit length. Levels in neighboring segments are coupled by the Thouless energy $t(\xi_{\rm loc})=\Delta$. When we consider the coupling between more distant segments of separation L the coupling is reduced to $t(L)=\Delta\exp(-L/\xi_{\rm loc})$. The coupling splits (almost) degenerate energy levels in different segments by an amount $\Delta E=2t(L)$. An external perturbation with frequency ω causes transitions between levels with a separation $\Delta E=\hbar\omega$, hence we demand $2t(L)=\hbar\omega$ and therefore

$$L_x(\omega)=\xi_{\rm loc}\ln(2\Delta/\hbar\omega). \tag{2.1}$$

According to Fermi's golden rule, the transition rate for tunneling from a given state in one segment to states in a segment $L_x(\omega)$ away is given by $1/\tau=(2\pi/\hbar)|t(L_x(\omega))|^2\rho_F\xi_{\rm loc}$. By applying a voltage $V=EL_x(\omega)$ between the two segments, one couples $e_0V\rho_F\xi_{\rm loc}$ levels in the first segment to states in the second segment. The total current through a given site is $L_x(\omega)/\xi_{\rm loc}$ times the current between two individual segments separated by a distance $L_x(\omega)$, and the linear conductivity is given by

$$\sigma_{\rm ac}(\omega)\approx\sigma_0L_x^2(\omega)(\hbar\omega\rho_F)^2,\quad \sigma_0=\frac{e_0^2}{\hbar}\xi_{\rm loc}. \tag{2.2}$$

This result can be generalized to an interacting electron system[25] by remembering the basic idea of bosonization: the charge density is defined as the derivative of a displacement field, and a localized electronic state corresponds to a localized kink in the displacement field. In addition, the density of states ρ_F at the Fermi level has to be replaced by the compressibility $\kappa=\partial\rho/\partial\mu$. With these modifications, the above derivation can be repeated and one obtains a result analogous to Eq. (2.2).

It is worthwhile to remark that the ac conductivity (2.2) can be rewritten as

$$\sigma_{\rm ac}(\omega)\sim\sigma_0e^{-2L_x(\omega)/\xi_{\rm loc}}\left(\frac{L_x(\omega)}{\xi_{\rm loc}}\right)^2. \tag{2.3}$$

This result resembles the form of the result for the nonlinear dc conductivity

$$\sigma_{\rm dc}(E) \sim \sigma_0 \frac{\hbar\omega_{\text{e-ph}}}{\Delta} e^{-2L_x(E)/\xi_{\rm loc}} \left(\frac{L_x(E)}{\xi_{\rm loc}}\right)^2. \tag{2.4}$$

Here $L_x(E)=\xi_{\rm loc}\sqrt{\Delta/e_0E_0\xi_{\rm loc}}$ denotes the spatial distance of the energy levels between which the tunneling events take place; it follows from a variational treatment.[9,10] The prefactor $\hbar\omega_{\text{e-ph}}/e_0E_0\xi_{\rm loc}$ takes into account that the dissipation rate is controlled by the typical frequency for electron-phonon coupling $\omega_{\text{e-ph}}$ and that the current is hence of the order[12] $\sim e_0\omega_{\text{e-ph}}e^{-2L(\omega)/\xi_{\rm loc}}$.

Finally, at finite temperatures, Mott variable range hopping gives a linear dc conductivity which follows from Eq. (2.4) by replacing $L_x(E)$ by $L_x(T)=\sqrt{\Delta/k_BT}$, i.e.,[5]

$$\sigma_{\rm dc}(T) \sim \sigma_0 \frac{\hbar\omega_{\rm ph}}{\Delta} e^{-2L_x(T)/\xi_{\rm loc}} \left(\frac{L_x(T)}{\xi_{\rm loc}}\right)^2. \tag{2.5}$$

The crossover between the three expressions (2.3)–(2.5) can most easily be understood by the dominance of a shortest tunneling distance, as will be discussed further below.

III. THE MODEL

The models we analyze are defined by the Euclidean action

$$\frac{S}{\hbar} = \frac{1}{2\pi K}\int dx \int_0^{v\hbar\beta} dy \left[\left(\frac{\partial\varphi}{\partial x}\right)^2 + \left(\frac{\partial\varphi}{\partial y}\right)^2 - 2u\cos[p\varphi + 2\pi\zeta(x)] + \frac{2Ke_0}{\pi v}\varphi E(y)\right] + \frac{S_{\rm diss}}{\hbar}, \tag{3.1}$$

where we have rescaled time according to $v\tau\to y$, and $\beta=1/k_BT$. The dissipative part of the action describes a weak coupling of the electron system to a dissipative bath, for example phonons. It is needed for energy relaxation in variable-range-hopping processes[11] and for equilibration in the presence of a strong ac field. We assume it to be so small that it does not influence the renormalization group (RG) equations for the other model parameters significantly. The smooth part of the density is given by $(1/\pi)\partial_x\varphi$, and $p=1,2$ for CDWs and LLs, respectively. For a disordered CDW or LL $\zeta(x)$ is equally distributed in the interval $[0,1]$ with correlation length equal to the lattice spacing a.

For $K>K_c(u)$ the potential is RG irrelevant and decays under the RG flow, while for $K<K_c(u)$ the potential is relevant and grows; here[3] $K_c(0)=6/p^2$. We assume $K<K_c(u)$ and scale the system to a length $\xi=ae^{l^*}$, on which the potential is strong. After the scaling process, the parameters K, v, and u in Eq. (3.1) are replaced by the effective, i.e., renormalized but not rescaled, parameters $K_{\rm eff}$, $v_{\rm eff}$, and $u_{\rm eff}$.

We note that the ratio K/v and hence the compressibility $\kappa=K/v\hbar\pi$ is not renormalized due to a statistical tilt symmetry.[20] The compressibility $\kappa=\partial\rho/\partial\mu$ is used as a generalized density of states for interacting systems. Our calculations are valid for energies below the generalized mean level spacing $\Delta_0=1/\kappa\xi$.

In this RG calculation, we do not attempt to treat a possible nonlinear dependence of coupling parameters on the external electric field. The full inclusion of the external field in an equilibrium theory is not possible as it renders the ground state of the system unstable. The quantum sine-Gordon model has an infinite number of ground states connected by a shift of the phase field by $2\pi/p$. Here, we concentrate on renormalizing each of these ground states separately and take into account the coupling between different ground states due to the external electric field in the framework of an instanton approach.

IV. INSTANTON CALCULATION

The wall width $1/\sqrt{p^2u_{\rm eff}}\approx\xi$ of an instanton solution to the action Eq. (3.1) is for weak external fields much smaller than the extension of the instanton. Hence, the instanton action can be expressed in terms of the domain wall position $X(y)$. The discussion of instantons in the case of random pinning is more involved than, e.g., for periodic pinning[21] and the calculation of closed-form instanton solutions is not possible. For this reason, we look for approximate instanton solutions with a rectangular shape and extensions L_x and L_y in x and y directions, respectively. As the disorder is correlated in time but not in space, instanton walls in the x and y directions contribute L_xs_y and $L_ys_x(x)$ to the action, respectively. While the surface tension $s_y=2\pi/p^2\xi K_{\rm eff}$ is essentially constant, the surface tension s_x has a strong and random position dependence. To calculate the statistical properties of s_x, we make use of the exact solution[22] of the classical ground state of a LL or CDW with random pinning in the following.

In the limit $K_{\rm eff}\ll 1$, quantum fluctuations are strongly suppressed and the (classical) ground state of the model Eq. (3.1) can be determined exactly.[10] After renormalization to the scale ξ, the effective action can be rewritten as a discrete model on a lattice with grid size ξ, and the integration over x can be replaced by a summation over discrete lattice sites $i=x/\xi$. In the classical ground state, the solution $\varphi(x,y)$ does not depend on y any more and the y integral in Eq. (3.1) simply yields an overall factor $v_{\rm eff}\hbar\beta$. Dividing the action by $\hbar\beta$, one obtains the classical Hamiltonian[10]

$$H_{\rm class} = \frac{\Delta_0}{2\pi^2}\sum_{i=1}^{L_0/\xi}[(\phi_{i+1}-\phi_i)^2 - 2\xi^2u_{\rm eff}\cos(p\phi_i - 2\pi\zeta_i)]. \tag{4.1}$$

Here, $u_{\rm eff}$ is the disorder strength with $u_{\rm eff}\,\xi^2\gg 1$ and $\zeta_i\in[0,1]$ is a random phase. In the effective Hamiltonian Eq. (4.1), the disorder term dominates the kinetic term and the classical ground state of the system can be explicitly constructed.[22] One minimizes the cosine potential for each lattice site by letting $p\phi_i=2\pi(\zeta_i+n_i^0)$ with integer n_i^0. The set of integers $\{n_i^0\}$ is chosen in such a way that the elastic term in Eq. (4.1) is minimized,

$$n_i^0 = m + \sum_{i<j}[\zeta_{i+1}-\zeta_i]_G. \tag{4.2}$$

Here, $[\zeta]_G$ denotes the closest integer to ζ, and m is an integer parametrizing the infinitely many equivalent ground states. Excitations of the ground state change $n_i^0\to n_i^0\pm 1$ for sites with $i_0 < i < i_0 + L_x/\xi$, they bifurcate from one ground state characterized by $m = m_0$ to another ground state with $m = m_0 \pm 1$. The potential energy necessary for a bifurcation at position i is according to Eq. (4.1)

$$\Delta H(i) = \frac{\Delta_0}{2\pi^2}(2\pi/p)^2(1 \pm 2\{(\zeta_i - \zeta_{i-1}) - [\zeta_i - \zeta_{i-1}]_G\}) \equiv \hbar v_{\mathrm{eff}} s_y g_i \tag{4.3}$$

with a random $g_i \in [0,2]$. Defining the localization length $\xi_{\mathrm{loc}} = p^2 K_{\mathrm{eff}} \xi / 2\pi$, one has $s_y = 1/\xi_{\mathrm{loc}}$.

Quantum effects are due to the time derivative in the action Eq. (3.1) and give rise to tunneling between the ground state and excited states in the presence of an external electric field. Similar to the case of periodic pinning,[21] these tunneling events are described by instantons. In the following, we describe how the action of a quadratic instanton can be calculated.

The action of a bifurcation with extension L_y is just $H_{\mathrm{kink}}(i_0) L_y / v_{\mathrm{eff}}$. The action of a wall with constant y and length L_x can be calculated by an analogous consideration if one introduces a lattice of grid size ξ in the y direction. As the disorder is correlated in the time direction, one need not consider random phases ζ_i and find an action $\hbar s_y L_x$. Adding up the contributions from all four walls of an instanton, one finds the action of a rectangular $L_x \times L_y$ instanton[10]

$$\frac{\Delta S}{\hbar} = [s_x(i_0) + s_x(i_0 + L_x)] L_y + 2 s_y L_x, \quad s_x(i) = s_y g_i. \tag{4.4}$$

Pairs of sites with $g_i \ll 1$ correspond to energetically low-lying excitations and form TLSs, which dominate the response to an external electric field. Typically, the two lowest g_i in an interval of length L_x are of the order $1/L_x$, and the boundaries of a typical instanton will be at positions with a small surface tension $s_x \approx s_y \xi / L_x$. Taking into account the contribution of a dc external electric field, the total action of a typical instanton is

$$\frac{S(L_x, L_y)}{\hbar} = 2 s_y \frac{\xi}{L_x} L_y + 2 s_y L_x - \frac{2 e_0 E_0}{p \pi v_{\mathrm{eff}} \hbar} L_x L_y. \tag{4.5}$$

Extremizing the action with respect to L_x, L_y one finds[10]

$$L_x(E) = \sqrt{\frac{2\pi}{p \kappa e_0 E_0}}, \quad L_y = \frac{\pi}{p \kappa \xi e_0 E_0}. \tag{4.6}$$

The creation rate of these instantons is

$$P_{\mathrm{random}} \sim e^{-2 L_x(E_0)/\xi_{\mathrm{loc}}}. \tag{4.7}$$

Next we consider an ac field $E(t)$, which upon analytical continuation $it \to \tau$ turns into a field $E(\tau)$. In imaginary time, the electric field has to obey the same periodic boundary condition $E(\tau + \beta) = E(\tau)$ as other bosonic fields, e.g., the displacement field $\varphi(\tau)$. This boundary condition is respected by a discrete Fourier representation[23]

$$E(\tau) = T \sum_{\omega_n} E(\omega_n) e^{-i\omega_n \tau}, \quad \omega_n = \frac{n 2\pi k_B T}{\hbar} \tag{4.8}$$

with Matsubara frequencies ω_n. A monochromatic external field is hence described by $E(y) = E_0 \cos \tilde{\omega}_n y$, where time is rescaled as $y = v_{\mathrm{eff}} \tau$ and frequency as $\tilde{\omega}_n = \omega / v_{\mathrm{eff}}$. In the end of our calculation, we analytically continue Matsubara frequencies to retarded real frequencies $i \tilde{\omega}_n v_{\mathrm{eff}} \to \omega + i\eta$.

Which type of instantons determines the current in the presence of an ac field with period $L_\omega = 2\pi / \tilde{\omega}_n$? In the limit of a very small external electric field, the only external length scale in the problem is the ac period L_ω, and the contribution of typical instantons to the linear response can be estimated by assuming that L_y has to be of order L_ω. A typical instanton obtained from minimizing Eq. (4.5) with respect to L_x for a fixed $L_y = L_\omega$ and for vanishing E_0 has $L_x = \sqrt{\xi L_\omega}$. For this solution, the current would be proportional to $\exp[-(8\pi/p^2)(1+i)\sqrt{\Delta_0/\hbar\omega}]$ and vanish nonanalytically for small frequencies. However, according to the Mott-Halperin law, the true frequency dependence should be proportional to[7] $\omega^2 \ln^2(1/\omega)$. We conclude that typical instantons do not yield the leading contribution to the current and that a discussion of rare instantons is needed.

Indeed, besides typical instantons with $s_x \approx s_y \xi / L_x$, there are rare instantons with an exceptionally low $s_x(i) + s_x(i + L_x)$. Such a pair of sites i and $i + L_x$ allows for the hopping of a kink without changing the kink's potential energy much. The potential energy difference between two sites can become arbitrarily small in sufficiently large samples. For the following considerations, we will set it to zero in the sense that it is much smaller than any other energy scale in the system. In the discussion of the dc electric field, quantum fluctuations, i.e., spontaneous creation of typical instantons in the absence of an external field, were unimportant. For pairs of sites with exceptionally low surface tensions, quantum fluctuations are important and have to be taken into account. Here, the spontaneous formation of instantons describes the physics of level repulsion.[23] In our approximation of vanishing s_x, the instanton action does not depend on the extension L_y in time direction any more, hence the occurrence of single domain walls of length L_x with constant y is possible. Such a domain wall describes the hopping of a kink across the distance L_x, and its action is

$$S_{\mathrm{single}}/\hbar = s_y L_x. \tag{4.9}$$

To obtain the partition function for this tunneling degree of freedom, we must sum over all possible domain wall configurations in the interval L_ω. A configuration with three hopping events is displayed in Fig. 1. Summation over all possible configurations yields

$$Z(L_\omega) = \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \left(\int_0^{L_\omega} \frac{dy_i}{\xi K_{\mathrm{eff}}} \right) e^{-n s_y L_x} = \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{L_\omega}{\xi K_{\mathrm{eff}}} e^{-s_y L_x} \right)^n = \exp\left(\frac{L_\omega}{\xi K_{\mathrm{eff}}} e^{-s_y L_x} \right). \tag{4.10}$$

The integration measures in the y_i integrals are normalized

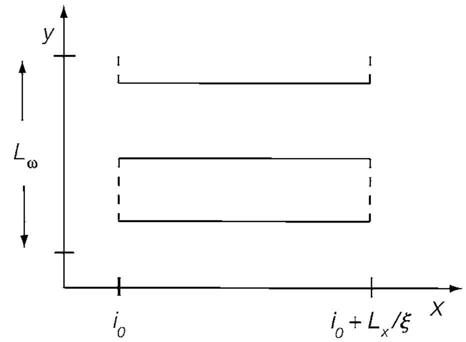

FIG. 1. Hopping of kinks from position i_0 to i_0+L_x/ξ and back (full lines) in a time interval of length L_y. The lines parallel to the y axis do not contribute to the action if the line tensions s_x are neglected.

by $\xi K_{\rm eff}$ rather than by ξ because the short-time cutoff $\hbar/\Delta_0$ is determined by the high-energy cutoff Δ_0 and not by the short-distance cutoff ξ. We note that the exponent of the outer exponential function is positive, indicating a lowering of the ground-state energy due to frequent tunneling between the degenerate states. The summation over all possible instanton configurations describes the quantum mechanical effect of level repulsion. Coupled energy levels repel each other and are separated at least by

$$t_0(L_x) = -\frac{\hbar v_{\rm eff}}{L_\omega}\ln Z(L_\omega) = \Delta_0 \exp(-L_x/\xi_{\rm loc}). \quad (4.11)$$

The probability of having exactly one instanton (hopping of a kink forth and back) within the time interval L_τ is given by

$$p_1(L_x) = e^{-2s_yL_x}/Z(L_\omega) = \exp\left(-\frac{L_\omega}{\xi K_{\rm eff}}e^{-s_yL_x} - 2s_yL_x\right). \quad (4.12)$$

The optimal length L_x for such an instanton is found by minimizing the exponent in Eq. (4.12) with respect to the tunneling length. We find

$$L_x = \frac{1}{s_y}\ln\frac{L_\omega}{2\xi K_{\rm eff}}. \quad (4.13)$$

Using this expression for L_x, we find the probability for having exactly one instanton of length L_ω

$$p_1 = e^{-2}\left(\frac{2\xi K_{\rm eff}}{L_\omega}\right)^2 = \left(\frac{\tilde{\omega}_n \xi K_{\rm eff}}{e\pi}\right)^2. \quad (4.14)$$

This proportionality of the tunneling probability to frequency squared is the essence of the Mott-Halperin conductivity Eq. (2.2).

With the knowledge of the probability Eq. (4.14) for an instanton in resonance with the external field, we can now set up a calculation of the ac current. It is calculated as a derivative $I(x,\omega_n)=-\hbar[\delta/\delta a(x,-\omega_n)]\ln Z$ of the partition function with respect to the vector potential $a(x,\omega_n)=E(x,\omega_n)/\omega_n$. The field ϕ couples to the vector potential via

$$S_E/\hbar = \frac{e_0}{\pi\hbar}\int dx \frac{1}{\beta v_{\rm eff}}\sum_{\tilde{\omega}_n} a(x,-\tilde{\omega}_n)(-\tilde{\omega}_n)\phi(x,\tilde{\omega}_n), \quad (4.15)$$

where β denotes the inverse temperature. Hence, the current is given by

$$I(x,\tilde{\omega}_n) = -e_0\tilde{\omega}_n v_{\rm eff}\frac{\int D[\phi]\phi(\tilde{\omega}_n)e^{-S[\phi]/\hbar}}{\int D[\phi]e^{-S[\phi]}}. \quad (4.16)$$

In the low-energy regime, we do not perform the full functional integral over ϕ in order to evaluate the partition function. Instead, we sum over the relevant tunneling degrees of freedom. We label such a degree of freedom by the position i_0 of the first weak link and by the distance L_x between the first and the second weak links. The field $\phi(i_0) = \phi(i_0+1) = \cdots = \phi(i_0+L_x)$ takes the values ϕ_0 and $\phi_0 + 2\pi/p$, respectively. Furthermore, we make use of the self-averaging properties of the current. Instead of averaging it over the position x in the system and letting all different types of TLSs contribute to it, we calculate the contribution of one TLS and average over the parameters L_x, $s_x(i_0)$, and $s_x(i_0+L_x)$. So far we considered instantons with vanishing surface tension $s_x=0$. We now extend these considerations to instantons with surface energies smaller than $\hbar\omega$, i.e., we are concentrating on sites with $g_i < K_{\rm eff}\xi\tilde{\omega}_n$, where $s_x(i_0) = s_y g_1, s_x(i_0+L_x) = s_y g_2$. The probability that the position x, for which we want to evaluate the current, is inside an active instanton is L_x/ξ times the probability for finding a weak tunneling link at a given site. In this way, we obtain for the average current

$$\langle I\rangle(\tilde{\omega}_n) = -e_0\tilde{\omega}_n v_{\rm eff}\int dg_1 dg_2 \frac{dL_x}{\xi}\frac{L_x}{\xi}p_1(L_x) \times \int \frac{dy_1}{\xi K_{\rm eff}}\frac{dy_2}{\xi K_{\rm eff}}\phi_0(\tilde{\omega}_n;y_1,y_2)e^{-S_E[\phi_0]}. \quad (4.17)$$

Here, we evaluate the current under the approximation that there is exactly one instanton (two tunneling events at times y_1 and y_2) in the interval of length L_ω. The two g_i integrals contribute a factor $(K_{\rm eff}\xi\tilde{\omega}_n)^2$ as we neglect the dependence of S_E on the g_i. The integral over L_x is evaluated by the saddle point method just taking into account instantons of the optimal length $L_x = (1/s_y)\ln(L_\omega/2\xi K_{\rm eff})$. As p_1 is a function of $L_x/\xi_{\rm loc}$, we have to use the same variable in the integration measure to perform a saddle point approximation and obtain a factor $\xi_{\rm loc}/\xi$ from this transformation. If y_1 is the time of the first tunneling event and y_2 the time of the second tunneling event, we define the new variables $\tilde{y}=(y_1+y_2)/2$ and $L_y = y_2 - y_1$. The Fourier transform of the displacement field ϕ for such an instanton is given by

$$\phi(\tilde{\omega}_n;L_y,\tilde{y}) = \frac{1}{\pi}e^{i\tilde{\omega}_n\tilde{y}}\sin\frac{\tilde{\omega}_n L_y}{2}. \tag{4.18}$$

For this field configuration, the coupling to the external electric field contributes the action

$$S_E(L_y,\tilde{y})/\hbar = \frac{e_0E_0L_x}{\pi\hbar v_{\rm eff}}\frac{2}{\tilde{\omega}_n}\sin(\tilde{\omega}_n\tilde{y})\sin(\tilde{\omega}_nL_y/2). \tag{4.19}$$

The barrier size is not fixed by the electric field as in the dc limit, and one has to consider both forward and backward jumps as for the standard thermally assisted flux flow argument.[24] Hence, the probability for the instanton to be in phase with the external field is given by $2\sinh[S_E(L_\omega,L_x)/\hbar]$. Expanding to linear order in S_E and integrating over L_Y and $\tilde{y}$, we find for the current

$$\begin{aligned}\langle I\rangle(\tilde{\omega}_n) &= -e_0v_{\rm eff}\tilde{\omega}_n\left(\frac{\tilde{\omega}_n\xi K_{\rm eff}}{e\pi}\right)^2(\tilde{\omega}_n\xi K_{\rm eff})^2\frac{L_x}{\xi}\frac{\xi_{\rm loc}}{\xi}\\ &\quad\times\int_0^{L_\omega/2}\frac{d\tilde{y}}{\xi K_{\rm eff}}\int_0^{L_\omega}\frac{dL_y}{\xi K_{\rm eff}}\phi(\tilde{\omega}_n;L_y,\tilde{y})\frac{2S_E(L_y,\tilde{y})}{\hbar}\\ &= -\frac{8}{e^2}\frac{e_0^2}{\hbar}\xi_{\rm loc}L_x^2\tilde{\omega}_n^2K_{\rm eff}^2E_0.\end{aligned} \tag{4.20}$$

After the analytical continuation $\tilde{\omega}_nv_{\rm eff}\to -i\omega$ the real part of the conductivity agrees with formula Eq. (2.2) up to a numerical factor. As both $\xi_{\rm loc}$ and L_x are proportional to $K_{\rm eff}$, the conductivity contains a factor of $K_{\rm eff}^5$ in agreement with the result.[8,25]

V. DESCRIPTION BY BLOCH EQUATION

The instanton calculation presented in the last section needs to be improved upon in two respects: first, the classical level separation parametrized by the g_i was not fully taken into account, and second, nonlinear corrections in the strength of the external field are not considered yet. To achieve these goals, we use the concepts developed in the last section in a real-time quantum mechanical calculation. Instantons in the imaginary-time formalism correspond to the hopping of kinks from one level just below the chemical potential to another level just above the chemical potential. The localized states of a 1D disordered system can be modeled by an ensemble of these TLSs, and the average properties of the system can be calculated by averaging over the parameters of the TLSs.

We consider a TLS with spatial extension L_x and on site energies ϵ_1 and ϵ_2 with $\epsilon_i=g_i\Delta_0$. The two sites are coupled by a distance-dependent hopping integral $t_0(L_x)$ according to Eq. (4.11). Such a TLS is described by the Hamiltonian $\hat{H}=\hat{H}_0+\hat{H}_E$ with

$$\hat{H}_0 = \frac{1}{2}(\epsilon_1+\epsilon_2)\sigma_z - t_0(L_x)\sigma_x, \quad \hat{H}_E = \frac{1}{2}e_0L_xE_0\cos(\omega t)\sigma_z. \tag{5.1}$$

The position of the tunneling kink is measured by the operator $\hat{x}=\frac{1}{2}L_x\sigma_z$, and the current operator is given by $\hat{I}=\rho(\epsilon_1,\epsilon_2,L_x)e_0\dot{\hat{x}}$. Here,

$$\rho(\epsilon_1,\epsilon_2,L_x) = \frac{1}{\xi}\frac{d\epsilon_1}{\Delta_0}\frac{d\epsilon_2}{\Delta_0}\frac{dL_x}{\xi} \tag{5.2}$$

denotes the spatial density of TLSs with given parameter values. $\hat{H}_0$ is diagonalized by the unitary transformation

$$\hat{H}\to\exp(i\varphi\sigma_y/2)\hat{H}\exp(-i\varphi\sigma_y/2) \tag{5.3}$$

with $\varphi=\arctan[2t(L_x)/(\epsilon_1+\epsilon_2)]$. In the new basis, $\hat{H}_0$ corresponds to a static field in the z direction, and $\hat{H}_E$ to an oscillating field with x component proportional to $\sin\varphi$ and z component proportional to $\cos\varphi$.

In principle, the transformed Hamiltonian should now be solved in a nonequilibrium setup in a dissipative environment. In general, this type of problem is difficult to deal with in full generality.[26] However, the problem simplifies if one does not treat an individual quantum system but averages over a whole ensemble instead. Such an ensemble of TLSs or spins interacting with an oscillatory electric field and subject to relaxation processes can be described by Bloch equations.[27] We denote the ensemble polarization of the TLSs in the transformed basis by the pseudospin vector $\underline{p}$. The current I is then proportional to the the pseudospin component in the y direction,

$$I = -\frac{1}{2}p_y\rho(\epsilon_1,\epsilon_2,L_x)e_0L_x\frac{t_0(L_x)}{\hbar}. \tag{5.4}$$

The pseudospin polarization $\underline{p}$ of the TLSs follows the Bloch equation

$$\frac{d}{dt}\underline{p} = -\frac{1}{\tau_0}(\underline{p}-\underline{p}_0) + \alpha\underline{p}\times\underline{E}. \tag{5.5}$$

Here, $\underline{E}=[E_0\sin(\varphi)\cos(\omega t),0,2\Delta/eL_x+E_0\cos(\varphi)\cos(\omega t)]$, $\underline{p}_0=[0,0,-1]$, $\Delta=\sqrt{(\epsilon_1^2+\epsilon_2^2)/4+t_0^2(L_x)}$, and $\alpha=e_0L_x/\hbar$. Inelastic processes are described by the phenomenological damping constant τ_0.

The solution of Eq. (5.5) is described in detail in Ref. 27 and we do not reproduce it here. We find that to order $O(E_0^3)$, one type of TLS contributes to the conductivity,

$$\begin{aligned}\sigma_{\rm TLS}(\omega;\epsilon_1,\epsilon_2,L_x) &= \\ &\rho(\epsilon_1,\epsilon_2,L_x)\frac{e_0^2L_x^2t^2(L_x)}{2\hbar^2\Delta}\frac{-i}{\omega-2\Delta/\hbar-i\hbar/\tau_0}\\ &\times\left(1-\frac{1}{\hbar^2\Delta^2}\frac{[e_0L_xE_0t_0(L_x)]^2}{(\omega-2\Delta/\hbar)^2+1/\tau_0^2}\right).\end{aligned} \tag{5.6}$$

The reduction of the linear conductivity becomes effective for strong ac fields, when both states of the TLS are occupied with comparable probability. In order to calculate the conductivity of the disordered sample, we integrate over all possible parameter values ϵ_1, ϵ_2, and L_x and obtain the final result

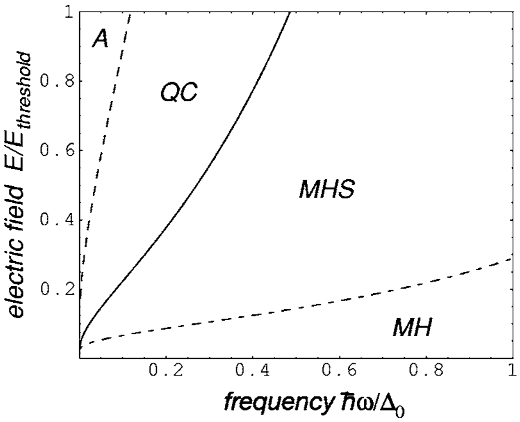

FIG. 2. Different transport regimes in a disordered CDW or LL with dissipation as a function of field strength in units of $E_{\text{threshold}} = \Delta_0 / p e_0 K_{\text{eff}} \xi_{\text{loc}}$ and frequency: adiabatic quantum creep in A, quantum creep with increased current noise in QC, ac conductivity following a Mott-Halperin law in MH, and Mott-Halperin law with saturation for large field strengths in MHS.

$$\text{Re}\,\sigma_{\text{ac}}(\omega) = \sigma_0 \frac{\pi}{4} L_x^2(\omega)(\hbar\omega\kappa)^2 \left(1 - 2\frac{e_0^2 L_x(\omega)^2 E_0^2}{\hbar^2/\tau_0^2}\right) \tag{5.7}$$

with the optimal tunneling length given by

$$L_x(\omega) = \xi_{\text{loc}} \ln \frac{2\Delta_0}{\hbar\omega}. \tag{5.8}$$

The linear part of Eq. (5.7) is proportional to K_{eff}^5 and agrees with the result of Fogler.[8] This linear conductivity describes the response of a disordered 1D system in region MH of Fig. 2. For an unscreened Coulomb interaction, in Eq. (5.7) one factor of $\hbar\omega$ has to be replaced by[28] $e_0^2/\epsilon L_x(\omega)$, where ϵ is the dielectric constant of the system.

When $e_0 E_0 L_x(\omega) \approx \hbar/\tau_0$, higher-order terms become important and the ac current will saturate as a function of E_0. The value E_s of the electric field where the current saturates can be estimated from Eq. (5.7) as

$$E_s = \frac{\hbar}{e_0 \tau_0 L_x(\omega)}. \tag{5.9}$$

As the nonlinear conductivity is defined as the ratio of current and electric field, in the saturation regime one obtains

$$\sigma_{\text{ac}}(\omega, E > E_s) = \sigma_0 \frac{p}{8} L_x(E)^2 \frac{L_x(\omega)}{\xi} \frac{\hbar}{\tau_0 \Delta_0} (\hbar\omega\kappa)^2. \tag{5.10}$$

The region in ω-E space where the nonlinear conductivity Eq. (5.10) can be observed is labeled MHS in Fig. 2.

VI. DISCUSSION

How does the linear conductivity Eq. (5.7) connect to the creep current in strong fields? The calculation of the nonlinear dc conductivity involves the optimal length scale $L_x(E)$ in Eq. (4.6) for tunneling processes. The crossover from ac to dc conductivity takes place when the two length scales $L_x(\omega)$ and $L_x(E)$ match, i.e., for a crossover frequency

$$\omega_\times = \frac{\Delta_0}{\hbar} e^{-L_x(E_0)/\xi_{\text{loc}}}. \tag{6.1}$$

For $\omega \approx \omega_\times$, the magnitude of the creep current $I_{\text{creep}} \sim \exp[-2L_x(E)/\xi_{\text{loc}}]$ agrees with the magnitude of the ac current described by the conductivity Eq. (5.7), as the ω^2 term in Eq. (5.7) matches the exponential dependence on field strength of I_{creep}. Then, the expression Eq. (5.10) turns into

$$\sigma(E) = \sigma_0 \frac{p}{8} \frac{\hbar}{\tau_0 \Delta_0} \frac{L_x(E)^3}{\xi^3} e^{-2L_x(E)/\xi_{\text{loc}}}, \tag{6.2}$$

providing us with an estimate of the prefactor of the exponential factor describing dc creep. Identifying $\hbar/\tau_0\Delta_0$ as a dimensionless measure for the dissipation strength, this estimate agrees with a more sophisticated calculation, in which a TLS is coupled to phonons with an Ohmic spectral function.[29]

In the crossover region $\omega \approx \omega_\times$ between regimes QC and MHS in Fig. 2, there are two different types of TLS contributing to the current. In the ground state of a typical TLS with bare energy difference $\epsilon_1 + \epsilon_2 \approx \Delta_0 \xi / L_x(E) \gg t(L_x)$, most of the charge is localized at one of the levels. Under the influence of an external field, the charge hops *irreversibly* from one level to the other, as the hop is generally accompanied by an inelastic process. On the other hand, the ground state of a TLS with exceptionally low bare energy separation $\epsilon_1 + \epsilon_2 \leqslant t(L_x)$ is the even-parity combination of wave functions centered around the individual levels, and absorption of a photon excites the TLS to the odd-parity state.

In the quantum creep regime, the time dependence of the current is calculated using the time-dependent field in the formula for the dc current I_{creep}. The adiabatic regime (region A in Fig. 2) is reached for frequencies smaller than the dc hopping rate I_{creep}/e_0. While the average current is stronger than the current noise in the adiabatic regime, the current noise is stronger than the current for $I_{\text{creep}}/e_0 < \omega < \omega_\times$.

In summary, we have discussed the crossover from a nonlinear creep current in a static electric field to the linear ac response in 1D disordered interacting electron systems. While the linear ac conductivity is described by a generalized Mott-Halperin law, for stronger fields one finds a reduction of this linear conductivity as both states of a TLS are occupied with comparable probability. The crossover between nonlinear ac conductivity and dc creep current occurs when the spatial extension of TLSs matches the length scale for tunneling of kinks.

ACKNOWLEDGMENTS

We thank T. Giamarchi, S. Malinin, D. Polyakov, and B. I. Shklovskii for useful discussions and the SFB 608 for support.

[1] T. Giamarchi, *Quantum Physics in One Dimension* (Oxford University Press, Oxford, 2003).
[2] G. Grüner, *Density Waves in Solids* (Addison-Wesley, New York, 1994).
[3] T. Giamarchi and H. J. Schulz, Phys. Rev. B **37**, 325 (1988).
[4] C. L. Kane and M. P. A. Fisher, Phys. Rev. B **46**, 15233 (1992).
[5] N. F. Mott and E. A. Davis, *Electronic Processes in Non-Crystalline Materials* (Clarendon Press, Oxford, 1971).
[6] B. I. Shklovskii and A. L. Efros, *Electronic Properties of Doped Semiconductors* (Springer, Berlin, 1984).
[7] N. F. Mott, Philos. Mag. **17**, 1259 (1968); B. I. Halperin, as cited by Mott.
[8] M. M. Fogler, Phys. Rev. Lett. **88**, 186402 (2002).
[9] B. I. Shklovskii, Fiz. Tekh. Poluprovodn. (S.-Peterburg) **6**, 2335 (1972) [Sov. Phys. Semicond. **6**, 1964 (1973)].
[10] T. Nattermann, T. Giamarchi, and P. Le Doussal, Phys. Rev. Lett. **91**, 056603 (2003).
[11] S. V. Malinin, T. Nattermann, and B. Rosenow, Phys. Rev. B **70**, 235120 (2004).
[12] M. M. Fogler and R. S. Kelley, Phys. Rev. Lett. **95**, 166604 (2005).
[13] I. V. Gornyi, A. D. Mirlin, and D. G. Polyakov, cond-mat/0506411 (unpublished).
[14] D. M. Basko, I. L. Aleiner, and B. L. Altshuler, cond-mat/0506617 (unpublished).
[15] E. Slot, H. S. J. van der Zant, K. O'Neill, and R. E. Thorne, Phys. Rev. B **69**, 073105 (2004).
[16] Z. K. Tang, D. D. Sun, and J. Wang, Physica B **279**, 200 (2000).
[17] J. Cumings and A. Zettl, Phys. Rev. Lett. **93**, 086801 (2004).
[18] M. Tzolov, B. Chang, A. Yin, D. Straus, and J. M. Xu, Phys. Rev. Lett. **92**, 075505 (2004).
[19] A. N. Aleshin, J. Y. Lee, S. W. Chu, S. W. Lee, B. Kim, S. J. Ahn, and Y. W. Park, Phys. Rev. B **69**, 214203 (2004).
[20] U. Schultz, J. Villain, E. Brezin, and H. Orland, J. Stat. Phys. **51**, 1 (1988).
[21] K. Maki, Phys. Rev. Lett. **39**, 46 (1977).
[22] A. Glatz and T. Nattermann, Phys. Rev. B **69**, 115118 (2004).
[23] J. W. Negele and H. Orland, *Quantum Many-Particle Systems* (Addison-Wesley, Reading, MA, 1988).
[24] P. W. Anderson and Y. B. Kim, Rev. Mod. Phys. **36**, 39 (1964).
[25] M. V. Feigelman and V. M. Vinokur, Phys. Lett. **87A**, 53 (1981).
[26] U. Weiss, *Quantum Dissipative Systems* (World Scientific, Singapore, 1999).
[27] C. Cohen-Tannoudji, B. Diu, and F. Laloë, *Quantum Mechanics* (John Wiley, New York, 1999), Vol. II.
[28] B. I. Shklovskii and L. Efros, Zh. Eksp. Teor. Fiz. **81**, 406 (1981) [Sov. Phys. JETP **54**, 218 (1981)].
[29] B. Rosenow (unpublished).