O14e “Measurement of the Speed of Light”

Tasks

1. Determine the speed of light in air c_{air} from the time shift between the emitter and receiver signals using an electronic modulation technique.

2. Determine the speed of light in air c_{air} by an electronic modulation method using Lissajous figures.

3. Measure the speed of light in two fluids using the modulation technique analogous to *task 1*.
Calculate the refractive index of the fluids.

Literature

Physics, P. A. Tipler, Vol. 2, Chapt. 30-1, 30-2, 30-4, 34-2
Physics, Benson, Chapt. 35.8, 39.2
Physikalisches Praktikum, 13. Auflage, Hrsg. W. Schenk, F. Kremer, Elektrizitätslehre, 3.0, 3.1

Accessories

Two-channel oscilloscope, light emitter/detector setup, optical work bench, lens, digital counter

Keywords for preparation

- Methods to determine the speed of light (Roemer's method, Fizeau's method)
- Speed of light, role in the theory of special relativity
- Electronic signal modulation (addition and multiplication of two sinusoidal voltages)
- Measurements of phase difference using a two-channel oscilloscope
- Measurement of phase difference by Lissajous figures
- Refraction, index of refraction, normal dispersion

Basics

The way c_{air} is measured in this experiment is similar to Fizeau's experiment, who in 1849 interrupted a beam with a rotating cogwheel to produce a modulation at 10 kHz viewed over a distance of 10 miles. With our 'meter-experiment' the intensity of the light is modulated at 60 MHz and the transmitter and receiver signal are compared. The speed of light is calculated from the relationship between the changes in the light path and the time delay (time difference between the transmitter and receiver signals) or phase. The phase relationship between the transmitter and receiver signal is obtained from Lissajous figures using the X-Y mode of an oscilloscope.

An idealized set - up

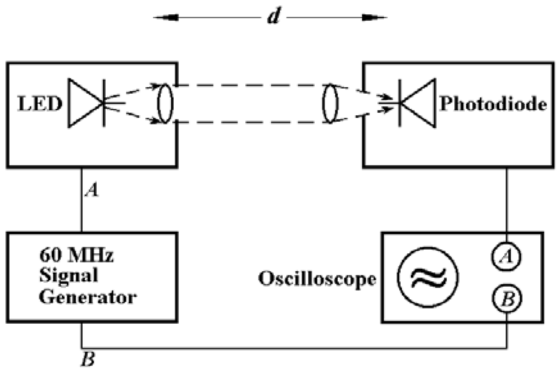


Fig. 1 Schematic setup.

Modulated light is emitted by an LED which is driven by a 60 MHz generator as shown in Fig. 1. The light travels a distance *d* through air to the photodiode, where it is detected, converted into a voltage signal and fed to the oscilloscope input A (CH. II). This signal is compared with the electric signal (reference signal) that is synchronous to the LED modulation and travels along a cable from the generator to input B of the oscilloscope (Input CH. I, triggering on channel II). The time difference in the two paths is: d/c + (unknown delay due to cables, position of the diode) resulting in a phase difference $\varphi_1 = 2\pi f\left(d / c_{air} + \tau\right)$, where f = 60 MHz, and τ is the unknown, but constant delay. If the distance between LED and photodiode is varied by Δd, then

$$\Delta\varphi = 2\pi f\left[\Delta d / c_{air}\right] \tag{1}$$

hence

$$c_{air} = 2\pi f\left[\Delta d / \Delta\varphi\right] \, . \tag{2}$$

The measurement of the phase difference would require an expensive oscilloscope which can sample at rates much higher than 60 MHz. One solution is to down convert the signal to obtain a signal with the relative phase information preserved, but with a signal frequency in a more easily measurable range.

For the conversion of a high frequency signal into a signal with considerably lower frequency, but the same phase information one might use the method of multiplicative signal mixing. A specific electronic circuit multiplies the high frequency signal $u_A(t) = \hat{u}_A \cos(2\pi f_1 t + \varphi)$ with frequency f_1 with a signal $u_B(t) = \hat{u}_B \cos(2\pi f_2 t)$ with frequency f_2 that is slightly different from the original frequency. This creates a beat that – according to the addition theorem for trigonometric functions $2\cos x \cos y = \left[\cos(x+y) + \cos(x-y)\right]$ – might be decomposed into a low (f_1 - f_2) and a high (f_1 + f_2) frequency signal:

$$u_M(t) = u_A(t) u_B(t) = \hat{u}_A \cos(2\pi f_1 t + \varphi) \hat{u}_B \cos(2\pi f_2 t) \, , \tag{3}$$

$$u_M(t) = \frac{1}{2} \hat{u}_A \, \hat{u}_B \left\{\cos\left[2\pi\left(f_1 + f_2\right)t + \varphi\right] + \cos\left[2\pi\left(f_1 - f_2\right)t + \varphi\right]\right\} . \tag{4}$$

The high frequency contribution is removed from the signal by a low pass filter such that only the low frequency contribution appears at the output of the mixer

$$u_M(t) = \frac{1}{2} \hat{u}_A \, \hat{u}_B \cos\left[2\pi\left(f_1 - f_2\right)t + \varphi\right] \, , \tag{5}$$

with the same phase difference as the original signal. However, this phase difference appears on a different time scale compared to the original signal with frequency f_1, since the time scale was expanded by a factor $f_1/(f_1\text{-}f_2)$. Accordingly the time difference Δt is obtained from the time difference Δt_M of the mixer signal by

$$\Delta t = \frac{f_1 - f_2}{f_1} \Delta t_M \, . \tag{6}$$

Δt_M is measured with the oscilloscope as the time difference between transmitter and receiver signal. Both signals are mixed in the same way before being fed to the oscilloscope.

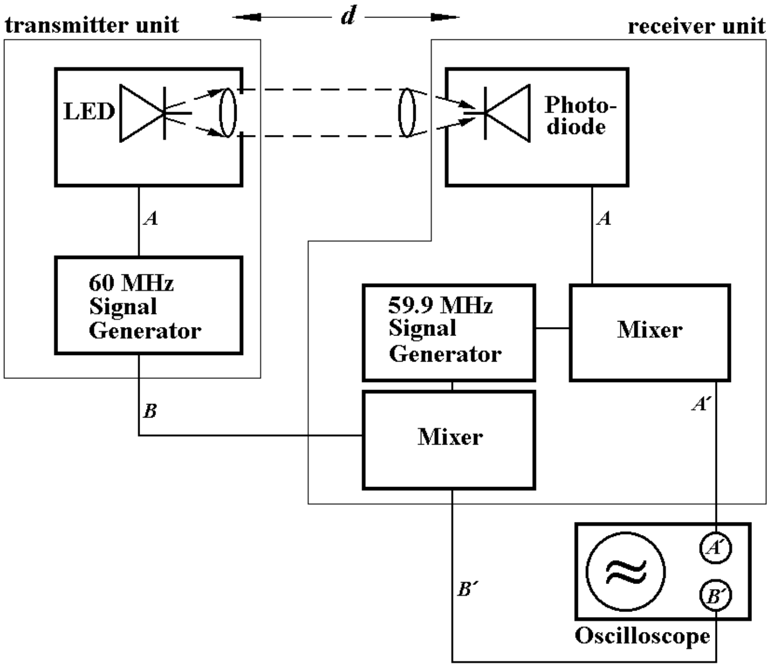
By multiplicative mixing of a 59.9 MHz signal (f_2) with a 60.0 MHz (f_1) of transmitter and receiver signal one obtains a mixer signal at a much lower frequency (difference frequency (f_1 - f_2) of about 100 kHz). For the exact measurement of the scaling factor the difference frequency should be measured with a precision counter.

Method using the Mixer (experimental setup)

Following the signal conversion procedure as explained above the experimental setup is realized as shown in Fig. 2.

Fig. 2

Actual setup.

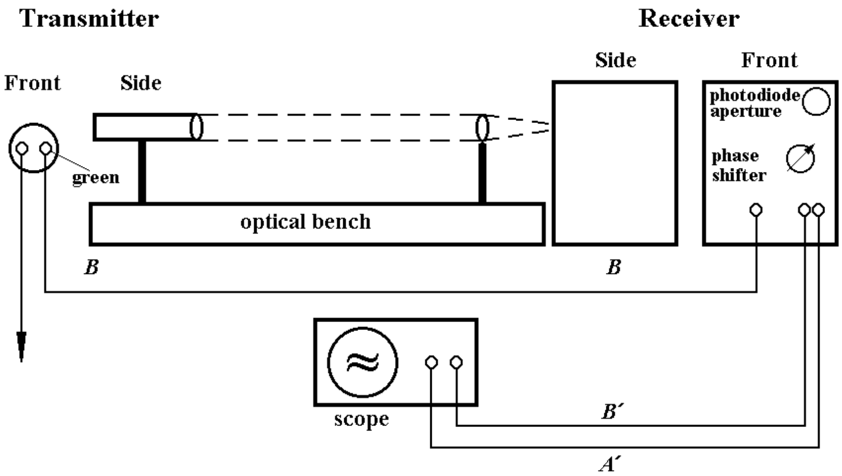


Hints to realize the experiment

Experimental Procedure - Adjustment

For adjustment you might follow the procedure as described below.

Fig. 3



1) The apparatus is wired up as shown in Fig 3.
2) Make sure all rails are horizontal and aligned. Make sure all lenses, LEDs and detectors are at the appropriate height.
3) Switch on, and adjust the LED unit to give a parallel beam using the condenser lens.
4) Focus the LED beam onto the diode inside the receiver box. The diode is 13 mm inside the box.
5) Check the beam alignment. This is time worth spending, since the experiment runs smoothly once good alignment is obtained.
6) Adjust oscilloscope parameters to display signals *A′* and *B′* on the scope screen with a peak-to-

peak voltage larger than 1 V.

7) Measure $\Delta\varphi$ of trace A' relative to B' by measuring the time delay.

8) Check the mixing frequency of about 100 kHz using a precision digital counter.

Using the zero crossings

1) Put the LED close to the lens, then adjust the phase shifter (see Fig. 3) on the front of the receiver box until the zero crossings of traces A' and B' coincide (A' and B' are now in phase). Inverting one of the traces might improve the accuracy.

2) Measure d.

3) Move the LED to a larger value $d+\Delta d$. Read off the time displacement Δt.

4) Plot distance Δd *vs*. time displacement Δt and determine the speed of light by linear regression from the slope.

Using Lissajous figures

1) Switch the scope into X-Y mode; you should see an ellipse. Vary the LED-photodiode distance and observe how the ellipse changes.

2) Bring the LED near to the lens and set the phase shifter of the receiver box so that you get a straight line. The phase between the two signals is zero.

3) Vary the LED-photodiode distance and determine the phase from the Lissajous figure using

$$\left(\frac{X}{A_0'}\right)^2+\left(\frac{Y}{B_0'}\right)^2-\frac{2XY\cos\Delta\varphi}{A_0'B_0'}-\sin^2\Delta\varphi=0\ . \tag{7}$$

Since the beam is not perfectly parallel to the optical bench, A_0' and B_0' vary with d; therefore the phase shift has to be determined for each d from Eq. (7). Plot distance Δd *vs*. phase shift $\Delta\phi$ and determine the speed of light by linear regression from the slope.

Calculating the refractive index

In the final task of this experiment two fluids in long tubes should be identified by measuring their refractive indices n_x. By inserting the tube (length $d_x + 2d_g$, where d_x = 994 mm is the length of the volume containing the liquid and d_g = 3 mm is the thickness of one glass window) into the path of the light, an extra optical path length is imposed. Derive the equation

$$n_\mathrm{x}-1=\frac{c_\mathrm{air}\,\Delta t}{d_\mathrm{x}}-2\frac{d_\mathrm{g}}{d_\mathrm{x}}(n_g-1) \tag{8}$$

for the calculation of the index of refraction n_x of the unknown fluid. Δt in Eq. (8) is the time difference related to the presence of the liquid filled tube, i.e. Δt = (signal with tube – signal without tube). Take the refractive index of glass n_g to be 1.5 and the speed of light c_air and the refractive index n_air in air to be $3\cdot10^8$ m/s and 1.0, respectively.