O5e "Index of Refraction of Liquids (Refractometry)"

Aufgaben

1. Determine the index of refraction n_D of three organic liquids at room temperature using an Abbe refractometer.

2. Calculate the molar refraction and the molecular polarizability of water and the test liquids from *task 1.* Compare the experimental values and the theoretically calculated values of the molar refraction.

3. Measure the temperature dependence of the index of refraction n_D of one liquid from *task 1* at eight different temperatures between room temperature and 60°C using an Abbe refractometer.

4. Determine the slope dn/dT from the graph n vs. T and calculate the volume expansion coefficient χ_V of the liquid at 20°C.

5. Measure the dispersion curve of water using a Pulfrich refractometer. Plot the data; determine the average dispersion n_F-n_C by inter/extrapolation.

Literature

University Physics, H. Benson, Chap. 35.3, 35.4, 35.5
Tipler, Physics, 3rd Edition, Vol. 2, Chap. 30-4
http://www.ub.es/javaoptics/index-en.html (JAVA Optics, Light dispersion)
Physikalisches Praktikum, 13. Auflage, Hrsg. W. Schenk, F. Kremer, Optik, 3.0.1, 3.1

Accessories

Abbe refractometer, thermostat, test liquids, projection lamp, Pulfrich refractometer

Keywords for preparation

- Refraction, Snell's law, refractive index, total internal reflection, critical angle
- Normal and anomalous dispersion
- Polarizability, molar refraction
- Operating principles of the Abbe refractometer and the Pulfrich refractometer
- Volume expansion, volume expansion coefficient
- Rainbow
- Bohr model and emission of light

Remarks

In a first control measurement check whether the measured refractive index n_D of water at room temperature coincides with the given value (see table) in the range of uncertainty $u(n) = 2\text{x}10^{-4}$.

The necessary values for the calculation of the molar refraction are available in the laboratory.

Two additional Abbe refractometers connected to a thermostat are available in order to measure the temperature dependence of n_D parallel to the room temperature measurements. The index of refraction is determined as the average value of three measurements at each temperature when thermal equilibrium has been reached in the measuring prism.

Before starting measurements with the Pulfrich refractometer note the further information available in the laboratory. The supervisor will introduce you to the operation of the instrument.

The equation of Clausius and Mossotti *resp.* the Lorentz-Lorenz formula describes the dependence of the refractive index n on the density ρ, the molar mass M and the molecular polarizability α_P of the medium. N_A denotes Avogadro's number and ε_0 the vacuum permittivity.

$$\frac{n^2-1}{n^2+2} = \frac{N_A\,\alpha_P}{3\,\varepsilon_0\,M}\rho = a\,\rho \ . \tag{1}$$

In a first approximation the quantity a is a constant factor, since the polarizability does not depend on temperature, unlike the density. Note that $N_A\rho/M$ gives the number of particles per cubic meter (particle density). With increasing temperature the density decreases because of the thermal expansion, consequently, the index of refraction decreases as well. For small changes of ρ and n one obtains:

$$\frac{\Delta\rho}{\Delta n} \approx \frac{\mathrm{d}\rho}{\mathrm{d}n} = \frac{6\,n}{(n^2+2)^2}\frac{1}{a} = \frac{6\,n}{(n^2+2)\,(n^2-1)}\rho \ . \tag{2}$$

Furthermore one has for the thermal expansion coefficient γ_V:

$$\gamma_\mathrm{V} = \frac{1}{V}\frac{\mathrm{d}V}{\mathrm{d}T} = -\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}T} = -\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}n}\frac{\mathrm{d}n}{\mathrm{d}T} \ \text{ with } \ \frac{\mathrm{d}V}{\mathrm{d}T} = \frac{\mathrm{d}\left(\dfrac{m}{\rho}\right)}{\mathrm{d}T} = -\frac{m}{\rho^2}\frac{\mathrm{d}\rho}{\mathrm{d}T} \ . \tag{3}$$

Finally one obtains

$$\gamma_\mathrm{V} = \frac{6\,n}{(n^2+2)\,(n^2-1)}\left(-\frac{\mathrm{d}n}{\mathrm{d}T}\right) . \tag{4}$$

Adjustment table for the Abbe refractometer

Adjustment fluid: distilled water

ϑ / °C	n_D	ϑ / °C	n_D
18	1,3332	23	1,3327
19	1,3331	24	1,3326
20	1,3330	25	1,3325
21	1,3329	26	1,3324
22	1,3328	27	1,3323

Refractive index n_D at a wavelength λ=589 nm
Systematic error as specified by the manufacturer: $2x10^{-4}$
Range and scale division for the refractive index: 1,300...1,700 / 0,001
Range and scale division for sugar (dry substance): 0...85 % / 0,1 %
Temperature range: 0...75 °C (scale division 1 °C)

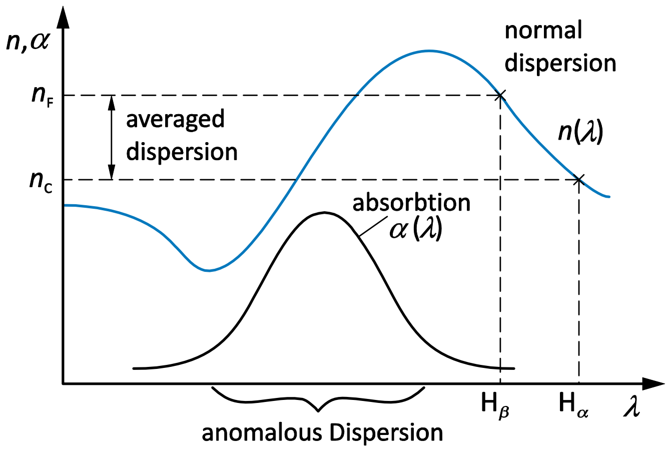
In *task 2* the molar refraction R_M can be calculated from

$$R_M = \frac{n^2 - 1}{n^2 + 2} \frac{M}{\rho}$$

where M is the molar mass and ρ the density of the sample at the measuring temperature.

In *task 5* the dispersion of the refractive index is measured. The velocity of light in general depends on the medium and the wavelength; this leads to dispersion, i.e. a dependence of the refractive index on the wave length, see Fig. 1.

Fig. 1. *Normal and anomalous dispersion.*

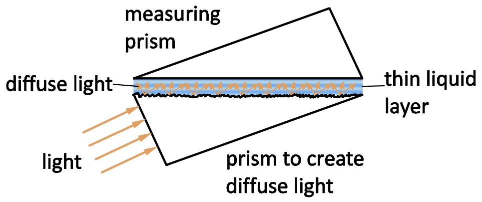


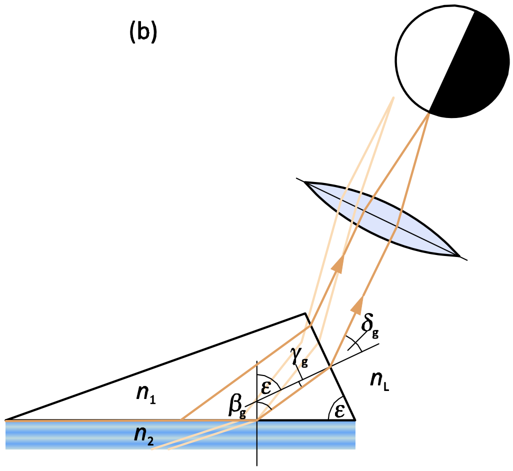
At large wavelengths the index of refraction decreases with increasing wavelength which is typical e.g. for normal glasses in the visible range of light. This regime is called normal dispersion; it can be characterized by the difference of the refractive indices corresponding to the H_α and H_β lines of hydrogen. The difference is also called averaged dispersion n_F - n_C (F and C are Fraunhofer's names of the H_α and H_β absorption lines in the sun spectrum). In Fig. 1 also the regime of anomalous dispersion is indicated in which the refractive index increases with increasing wavelength.

The operating principle of the Abbe refractometer is illustrated in Fig. 2. Fig. 2a shows two different prisms. The lower has a rough surface to create diffusely scattered light entering the liquid layer. The upper one is used to create a dark-bright boundary. Fig. 2b shows an incident ray at glancing angle to the upper prism that is refracted under the critical angle β_g.

Fig. 2 *Ray diagram to illustrate the operating principle of an Abbe refractometer.*

(a) (b)





Let n_1 and n_2 denote the refractive indices of glass and of the liquid under study, respectively; ε is the refraction angle of the prism, δ_g is the deflection angle. From Fig. 2b the following relationships can be derived:

$$\sin\beta_g = n_2 / n_1\,, \quad \sin\delta_g = \frac{n_1}{n_L}\sin\gamma_g \approx n_1 \sin\gamma_g\,, \quad \beta_g + \gamma_g = \varepsilon\,,$$

where the refractive index of air is approximated by n_L=1.

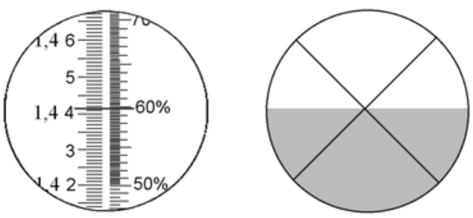
From this the refractive angle of the liquid can be related to the deflection angle δ_g:

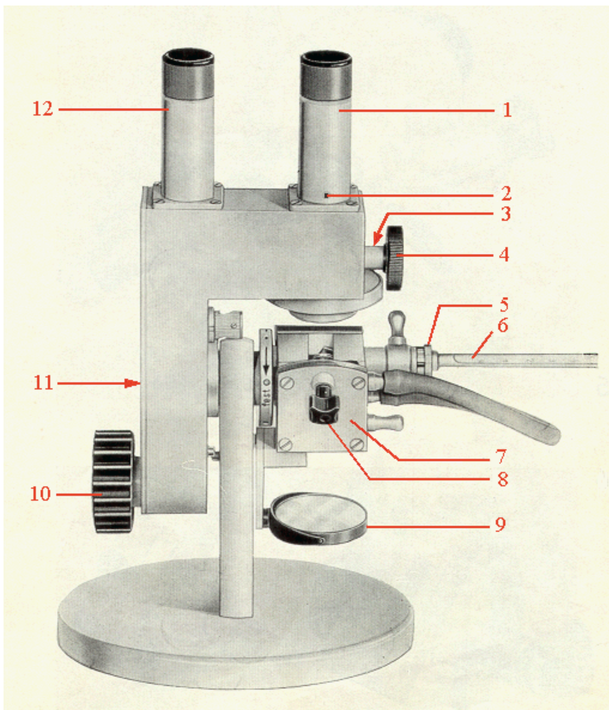
$$n_2 = n_1 \sin\beta_g = n_1 \sin(\varepsilon - \gamma_g) = \sin\varepsilon\sqrt{n_1^2 - \sin^2\delta_g} - \cos\varepsilon\sin\delta_g\,.$$

The commercial Abbe refractometer available in the Undergraduate Physics Laboratory, see Fig. 3, works with a calibrated scale allowing for a direct reading of the refractive index n_D (sodium D line, λ = 589 nm). The refractometer is equipped with two telescopes (Fig. 3, 1 and 12), the right one for adjusting the dark-bright boundary created by the total refraction, the left one for direct reading of the refractive index.

Fig. 3 *Abbe refractometer.*

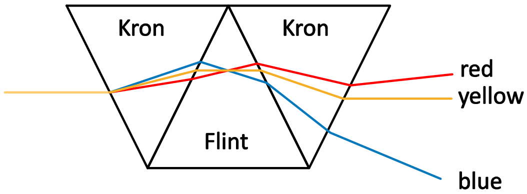
scale bright-dark boundary





1 telescope (bright-dark boundary)
4 dispersion compensation by Amici prisms
5/6 thermometer
7 prisms
9 mirror for illumination of prisms
10 knob (bright-dark boundary)
12 telescope (calibrated scale, n_D)

Fig. 4 *Amici prism and ray diagram inside* the *Abbe refractometer.*



A pair of Amici prisms (Fig. 4a) is integrated into the measuring telescope to eliminate the dispersion of white light between the dark and the bright area in Fig. 2b. Due to the construction of the Amici prisms the index of refraction n_D corresponding to the wavelength λ = 589 nm is determined.

In *task 5* a Pulfrich refractometer is used to determine the refractive index of a fluid for the following wavelengths:

Linie		λ / nm	V-prism Vo F3 n_1	V-prism Vo F2 n_1
Hg	e (green)	546.1	1.746428	1.748219
Hg	g (dark blue)	435.9	1.775521	1.777484
Hg	h (violet)	404.7	1.790168	1.792221
He	d (yellow)	587.6	1.740102	1.741855
Cd	C' (red)	643.8	1.733584	1.735296
Cd	(green-blue)	508.6	1.753785	1.755625
Cd	F' (blue-green)	480.0	1.760848	1.762724
Cd	(blue)	467.8	1.764442	1.766312
H_2	C (red)	656.3	1.732378	1.734089
H_2	F (blue-green)	486.1	1.759280	1.761020

The refractive indices are those of the V-glass-prisms.

With the V-prisms used in this experiment not the critical angle for total reflection, but the deflection of the light is measured. The light beam passing through the collimator is incident on the V-prism and is refracted at the oblique surfaces that hold the fluid; finally the light beam leaves the V-prism under an angle γ. This angle γ is measured with a telescope observing the collimator slit. γ is given by

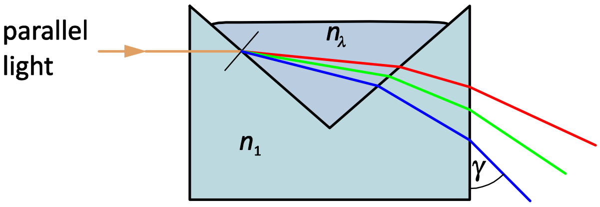
$$\sin(90°-\gamma)=\frac{1}{\sqrt{2}}\left(\sqrt{1{,}5\,n_1^2-n_\lambda^2}-\sqrt{n_\lambda^2-0{,}5\,n_1^2}\right)\;;\quad (5)$$

alternatively, the wavelength dependent refractive index n_λ is given by

$$n_\lambda=\sqrt{n_1^2-\sin(90°-\gamma)\cdot\sqrt{n_1^2-\sin^2(90°-\gamma)}}\quad. \qquad (6)$$

See the appendix for a derivation of Eq. (6).

Fig. 5 *Light-beam diagram in the V-prism of a Pulfrich refractometer*.



The angle γ is measured relative to the exit face of the V-prism. The refractive indices n_1 are related to measurements in air (not vacuum) at 20 °C and normal pressure. Turning the knob 36, see Fig. 6, the collimator image is brought onto the collimation mark, see Fig. 6.

Fig. 6 *Pulfrich refractometer*.

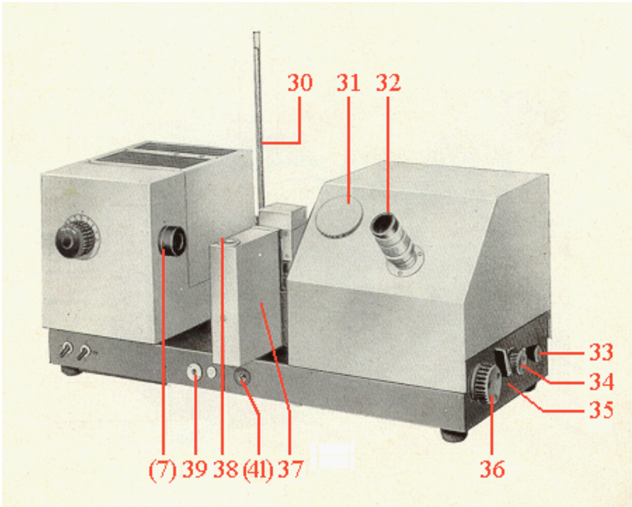
Top: instrument with

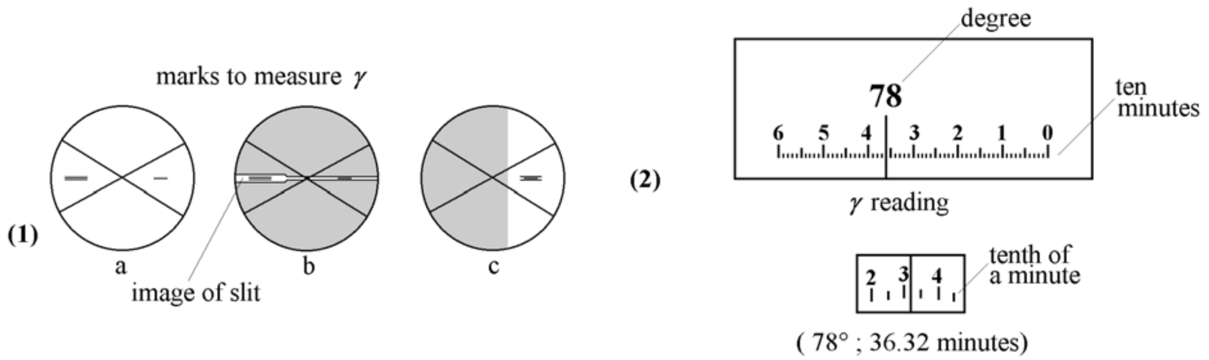
(34) knob for fine-adjustment of the measuring scale

(35) knob to switch angular scale and collimator image

(36) knob to turn the measuring telescope behind the V-prism

(38) collimator





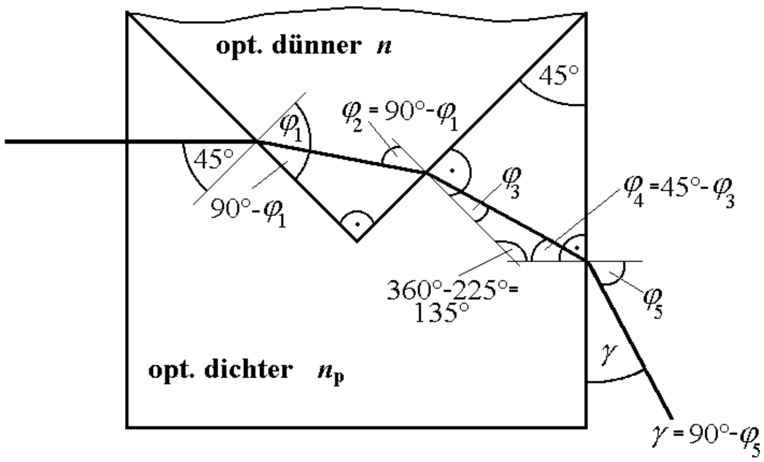
(1) collimator image with collimator mark in the telescope

(2) angle scale

Appendix: Derivation of the formula for the calculation of the refractive index using the Pulfrich refractometer with V-prism

Relation between the refractive index of a fluid (n) and the angle of deflection (γ)

n_p = refractive index of the V-prism (flint glass or crown glass)



$$\varphi_2 = 90° - (90° - 90° + \varphi_1) = 90° - \varphi_1$$

$$\frac{n}{n_p} = \frac{\sin 45°}{\sin\varphi_1} \quad \rightarrow \quad \sin\varphi_1 = \frac{1}{\sqrt{2}} \cdot \frac{n_p}{n}$$

$$\frac{n}{n_p} = \frac{\sin\varphi_3}{\sin\varphi_2} = \frac{\sin\varphi_3}{\cos\varphi_1} = \frac{\sin\varphi_3}{\sqrt{1-\sin^2\varphi_1}} \quad \rightarrow \quad \sin\varphi_3 = \frac{n}{n_p}\sqrt{1-\frac{n_p^2}{2n^2}}$$

$$\varphi_3 = 45° - \varphi_4$$

$$\frac{\sin\varphi_5}{\sin\varphi_4} = n_p \quad \rightarrow \quad \sin\varphi_4 = \frac{\sin\varphi_5}{n_p}$$

$$\varphi_5 = 90° - \gamma \quad \rightarrow \quad \sin\varphi_4 = \frac{\cos\gamma}{n_p}$$

Successive insertion yields the final equation for n:

$$\frac{n}{n_p}\sqrt{1-\frac{1}{2}\frac{n_p^2}{n^2}} = \frac{1}{2}\sqrt{2}\left(\sqrt{1-\frac{\cos^2\gamma}{n_p^2}} - \frac{\cos\gamma}{n_p}\right).$$

Rearranging and simplifying yields

$$n = \sqrt{n_p^2 - \cos\gamma\sqrt{n_p^2 - \cos^2\gamma}}$$

resp.

$$n = \sqrt{n_p^2 - \sin(90-\gamma)\cdot\sqrt{n_p^2 - \sin^2(90-\gamma)}} \; .$$