UNIVERSITÄT LEIPZIG
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Physikalisches Grundpraktikum

M13e "Moment of Inertia Tensor"

Introduction

The moment of inertia constitutes a tensor of second rank that relates the angular momentum of a rigid body to its angular velocity. The components of the moment of inertia tensor I_{ij} are obtained by integration over the volume of the rigid body according to

$$\overset{\leftrightarrow}{I} = \begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{pmatrix} = \iiint_V \begin{pmatrix} y^2+z^2 & -xy & -xz \\ -yx & z^2+x^2 & -yz \\ -zx & -zy & x^2+y^2 \end{pmatrix} \rho dV .$$

The moment of inertia tensor is symmetric, $I_{ij} = I_{ji}$. The eigenvalues of the moment of inertia tensor are the principal values of the moment of inertia; these are real and non-negative. The corresponding eigenvectors define the principal axes. When a rigid body is rotating around a fixed axis with a direction defined by the unit vector $\hat{n}$, the moment of inertia with respect to this axis is given by

$$I_n = \hat{n}^T \overset{\leftrightarrow}{I} \hat{n} .$$

The moment of inertia tensor plays a central role e.g. in the analysis of gyro problems.
The tensor of the radius of gyration

$$\overset{\leftrightarrow}{R}{}_G^2 = m^{-1} \overset{\leftrightarrow}{I}$$

is often used for the characterization of the moment of inertia tensor. In case of a homogeneous rigid body the mass density is independent of the position within the body and is given by $\rho = M/V$, such that the tensor of the radius of gyration is a purely geometric quantity independent of the mass density distribution.

The aim of this experiment is the investigation of the moment of inertia tensor of homogeneous rigid bodies with simple form. To this end the components of the moment of inertia tensor should be calculated and measured.

Tasks

0. During preparation time calculate the principal values of the moment of inertia tensor of a homogeneous cube with edge length *a* as well as of a cuboid with edge lengths *a*, *b* and *c*. Calculate further the radii of gyration for rotation around the body diagonal as well as around rotation axes through the center of mass parallel to the face diagonals. What are the values of the ratios I_{xx} / I_{zz}, I_{yy} / I_{zz} and I_d / I_{zz} in case of a cuboid with axis ratios a:b:c = 4:2:1? I_d denotes the moment of inertia with respect to the body diagonal.

1. Determine the principal moments of inertia of a cube and a cuboid as well as the moments of inertia for rotation around the body diagonal by measurements of the oscillation period of a turntable, on which the body is fixed.

Literature

Physikalisches Praktikum, 13. Auflage, Hrsg. W. Schenk, F. Kremer, Mechanik, 2.5
M. Alonso, E. J. Finn, Physics, 13.7
P. A. Tipler, Physics, Chap. 8

Accessories

Turntable with photoelectric relay, electronic counter, brass bodies

Keywords for preparation

- Rigid body, angular momentum, torque
- Tensor of the moment of inertia, principal axes, radius of gyration, calculation of these quantities
- Steiner's theorem (parallel axis theorem)
- Harmonic oscillation, eigenfrequency, period, spring moment of a spiral spring

Remarks

The turntable consists of a brass holder screwed onto a rotation axis that, after being deflected by 180°, performs harmonic oscillations under the action of the unknown spring moment of a spiral spring. The period of oscillation is measured with an electronic counter. The rigid body under investigation is screwed onto the brass holder in a distance s = -10…+10 cm from the rotation axis. The measurements should be made in such a way that for each orientation of the rigid body, the oscillation period is measured for distances from the rotation axis of at least s = -4…4 cm (in steps of 1 cm). Further the oscillation period T_0 of the turntable itself has to be measured.

The total moment of inertia is given by the moment of inertia of the turntable I_T and the moment of inertia of the rigid body under study, whereby the latter is the sum of the moment of inertia of the rigid body I_K for a rotation around an axis through the center of mass and the contribution Ms^2 from Steiner's theorem. M denotes the mass of the rigid body.

Devise a data analysis technique that allows for the determination of the spring moment D of the spiral spring as well as the moments of inertia of the turntable I_T and the rigid body I_K. Take into account that the geometrical distances s that are determined by the screw positions on the turntable, are probably not identical to the distances of the rigid body from the rotation axis, but that there might be some offset s_0. Determine s_0 from the data. Calculate the measurement uncertainty and compare the experimental and theoretical values of the moments of inertia along the various rotation directions.