M5e  “Density of Solids and Liquids (Pycnometer, Law of Archimedes)”

Tasks

1. Determine the density of a solid with a pycnometer.

2. Determine the density of a short metal cylinder by using a buoyancy technique as well as by measuring its mass and geometric dimensions.

3. Determine the density of a liquid using a pycnometer and a Mohr-Westphal balance.

Additional task: Measure the temperature dependence of the density of the liquid from the third task with a Mohr-Westphal balance.

Literature

Physics, P. A. Tipler, 3rd Edition, Vol. 1, Chaps. 11-1, 11-3, 11-4
Physikalisches Praktikum, 13. Auflage, Hrsg. W. Schenk, F. Kremer, Mechanik, 1.0, 1.1, 1.2, 1.3

Accessories

Electronic balance, pycnometer with thermometer, glass beaker, test objects, test liquid, cellulose, vernier caliper, micrometer screw gauge, distilled water, Mohr-Westphal balance

Keywords for preparation

- Density of solids, liquids and gases, temperature dependence of the density
- Pressure in liquids, Archimedes’ principle
- Buoyancy in liquids and gases
- Forces on swimming and floating objects, meta center
- Methods for density measurement, pycnometer, buoyancy method, Mohr-Westphal balance, areometer
Remarks

Weighing is carried out with an electronic balance (precision balance of class II, measurement uncertainty $u(m) = 5$ mg). At the start of the experiment you will be briefly instructed in the use of the balance. After a warming-up time of about 10 min the balance reaches its operating temperature. Then the uncertainty of the balance is to be controlled with precision weights of class F under consideration of the measurement accuracy.

Derive all equations necessary for the determination of the density. The terms accounting for the buoyancy in air are negligible in the derivation of the maximum error relations. All density values obtained have to be compared to literature values taking the estimated experimental uncertainties into account.

During the measurements with the pycnometer it is not necessary that the fluid level in the capillary is exactly adjusted to mark S. It is sufficient when the fluid in the capillary is near the upper end of the capillary in all measurements.

Measurements with a pycnometer

![Fig. 1 Pycnometer with thermometer (schematic)](image)

In order to obtain small measurements uncertainties in the density measurements the water volume displaced by the test sample should be sufficiently large.

Density of a solid $\rho$ (task 1)

When weighing a sample in air the buoyancy of air has to be taken into account. Accordingly, the balance readings $m^*$ are given by

$$m^* = m - V\rho_{\text{air}},$$

(1)

where $m$ and $V$ denote mass and volume of the test samples and $\rho_{\text{air}}$ denotes the density of air.
In the experiment three masses are measured:
1. \( m^* \) mass of the solid test sample
2. \( m_1^* \) mass of the pycnometer filled with distilled water
3. \( m_2^* \) mass of the pycnometer filled with distilled water and the test sample.

Under the assumption that the temperature of the water is constant in the measurements one has

\[
m_2^* - m_1^* = m_2 - m_1. 
\]

Further

\[
m_1 = m_p + m_w \\
\]
\[
m_2 = m_p + m_w - \rho_w V + \rho V
\]

where \( m_p \) is the mass of the pycnometer, \( m_w \) the mass of the water, \( \rho_w \) the water density and \( \rho \) the density of the test sample. With Eqs. (2) and (3) one obtains

\[
m_2^* - m_1^* = (\rho - \rho_w)V; 
\]

Eq. (1) is equivalent to

\[
m^* = (\rho - \rho_{\text{air}})V. 
\]

Derive an equation for the solid density \( \rho \) from Eqs. (4) and (5). Calculate the maximum error estimate. The density in air under the conditions of the experiment can be obtained from the ideal gas equation.

**Density of a liquid \( \rho_l \) (task 3)**

In this case the following three weighings are made:
1. \( m_1^* \) mass of the pycnometer filled with distilled water
2. \( m_3^* \) mass of the pycnometer filled with air
3. \( m_4^* \) mass of the pycnometer filled with the test liquid

Derive the equation for the calculation of \( \rho_l \). Calculate the maximum error estimate.

**Measurements using a buoyancy technique**

Measure the mass of the test sample \( m^* \) in air.

For the buoyancy technique an electronic balance is used. Place the beaker filled with a liquid of density \( \rho_l \) on the balance. Using the Tare button zero the balance reading. Subsequently, the test sample (mass \( m_b \)) is completely immersed in the liquid. The value \( m_A \) read off from the balance is equal to the mass of the displaced fluid \( m_n = \rho_n V_k \). From this one obtains

\[
\rho_k = \frac{m_A}{V_k} - \frac{m_n}{m_A} \rho_n. 
\]

Distilled water is used as liquid. To keep the influence of evaporation negligible, the measurements should be quickly performed after pressing the Tare button. The buoyancy of the suspension wire can be estimated (diameter of the wire: \( d = 0.15 \) mm).
Fig. 2 Weighing with an electronic balance

mass $m_A$ (test sample immersed in liquid)

Mohr-Westphal balance

The principle of the balance is based on the known buoyancy of a reference glass body. The density of the test liquid is calculated according to

$$\rho_i = \frac{m_2}{m_1} \rho_w + \left(1 - \frac{m_2}{m_1}\right) \rho_{air},$$

where $m_1$ and $m_2$ are the masses of the glass cylinder immersed in water and in the test liquid, respectively.

Fig. 3 Mohr-Westphal balance

1 Foot with adjustment screws
2 Metal frame with adjustable height
3 Balance beam with notches
4 Plummets (glass cylinder with thin Platinum wire)
The beam of the balance is balanced with the plummet (glass cylinder hanging on a thin platinum wire attached to a hook on the beam) in air using the adjustable screws on the foot. When adjusted, the index pointer on the end of the beam lines up with the point on the frame. The plummet is then completely immersed in the unknown liquid, and the system is rebalanced, using a series of riders on the nine equally spaced notches on the beam, thus specifying the value of the added mass for each decimal place. This gives the buoyant force of the liquid relative to water, and hence the density, which may be obtained to three decimal places.

**Accessories**

Plummet (glass cylinder): 5 cm³ (20 °C)

Weights:  
2 riders 5.000 g  
1 rider 0.500 g  
1 rider 0.050 g

Example:  
1 rider 5.000 g on the hook (notch 10)  
1 rider 5.000 g notch 5  
1 rider 0.500 g notch 9  
1 rider 0.050 g notch 3.

This yields a density 1.593 g cm⁻³ = 1593 kg m⁻³.

Tab. 1 Nominal values of weights from the precision classes E₁, F₁ und M₁ and their respective uncertainties \( u(m) \)

<table>
<thead>
<tr>
<th>Nominal value</th>
<th>E₁ ( u(m)/mg )</th>
<th>F₁ ( u(m)/mg )</th>
<th>M₁ ( u(m)/mg )</th>
</tr>
</thead>
<tbody>
<tr>
<td>m / g</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>0.002</td>
<td>0.025</td>
<td>0.25</td>
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<tr>
<td>0.02</td>
<td>0.003</td>
<td>0.03</td>
<td>0.3</td>
</tr>
<tr>
<td>0.05</td>
<td>0.004</td>
<td>0.04</td>
<td>0.4</td>
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<td>0.05</td>
<td>0.5</td>
</tr>
<tr>
<td>0.2</td>
<td>0.006</td>
<td>0.06</td>
<td>0.6</td>
</tr>
<tr>
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<td>0.008</td>
<td>0.08</td>
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<tr>
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</tr>
<tr>
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<td>1.5</td>
</tr>
<tr>
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<td>0.20</td>
<td>2.0</td>
</tr>
<tr>
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<td>0.025</td>
<td>0.25</td>
<td>2.5</td>
</tr>
<tr>
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<td>3</td>
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<tr>
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<td>5</td>
</tr>
<tr>
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<td>1.0</td>
<td>10</td>
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