Climate Dynamics
Summer Semester 2019

UNIVERSITÄT LEIPZIG

Homework 9
Due 10 July 2019

Problem 1 Time evolution of the surface temperature

Recall from lecture that the response of the surface temperature to climate forcing depends on the strength of the various feedbacks, and that we can write the following linearized equation:

$$\Delta R = \lambda \Delta T_s + F. \tag{1}$$

The radiative flux imbalance ΔR increases the internal energy of the climate system (on short timescales, mostly the upper ocean):

$$c\frac{\partial \Delta T_s}{\partial t} = \lambda \Delta T_s + F(t) \tag{2}$$

where c is the heat capacity of the upper ocean, λ is the climate feedback parameter, F is the forcing, and ΔT_s is the ocean surface temperature perturbation. The base-state temperature is taken to be the temperature at $t = 0$, so that $\Delta T_s(t = 0) = 0$.

(a) Solve (2) for F a general function of t. *Hint: To solve an inhomogeneous linear differential equation such as (2), try writing the solution as the product of two functions,*

$$\Delta T_s(t) = \Delta T_{hom}(t) \cdot f(t) \tag{3}$$

$\Delta T_{hom}(t)$ solves the homogeneous differential equation

$$c\frac{\partial \Delta T_{hom}}{\partial t} = \lambda \Delta T_{hom} \tag{4}$$

Applying the product rule to (3), inserting the result into (2), and using (4), you can solve for $f(t)$. As a check that you got the right result, your solution should have this form:

$$\Delta T_s = \exp(At) \int_0^t \exp(-At')BF(t')\, dt' \tag{5}$$

(b) Find $\Delta T_s(t)$ when the perturbation is a sudden step at $t = 0$:

$$F(t) = \begin{cases} 0 & (t \leq 0) \\ Q_0 & (t > 0) \end{cases} \tag{6}$$

What is the initial behavior ($t \ll c/|\lambda|$)? What is the asymptotic behavior ($t \gg c/|\lambda|$)? (Note: we write $|\lambda|$ because $\lambda < 0$.)

(c) Find $\Delta T_s(t)$ when the perturbation is a linear increase,

$$F(t) = \begin{cases} 0 & (t \leq 0) \\ Q_t t & (t > 0) \end{cases} \tag{7}$$

What is the initial behavior ($t \ll c/|\lambda|$)? What is the asymptotic behavior ($t \gg c/|\lambda|$)?

Problem 2 A tale of two climate changes

The feedback parameter λ of the climate system is not well known. Let us explore the consequence of uncertainties on λ for the climate system. Consider two extreme cases: in one case the feedback parameter has a certain value $\lambda = \hat{\lambda}$, while in the other $\lambda = \hat{\lambda}/3$.

(a) Which climate system is more sensitive (greater equilibrium temperature change for the same forcing)?

(b) How does the temperature change in the early period of global warming ($t \ll c/|\lambda|$) depend on λ? You may use your solution from 1(b) or 1(c).

(c) How does the equilibrium temperature change ($t \gg c/|\lambda|$) depend on λ?

(d) What does this mean for using historical observations to predict equilibrium climate change?

(e) How would you modify Eq. (2) to account for heat storage in the deep ocean? How does this change your conclusion from part (d)?

Problem 3 Geoengineering and its termination

This week we will look at a modeling study of geoengineering. The scenario is the following. After some amount of warming has occurred (following the RCP4.5 scenario, which stabilizes at 4.5 W m^{-2} radiative forcing in 2100), society decides in the year 2020 that further rapid temperature increases need to be avoided. The method of geoengineering is stratospheric sulfate aerosol, which decreases the solar radiation absorbed by the climate system. The amount of sulfate aerosol is chosen to stabilize the radiative forcing at its 2020 value. (See Figure 1.)

In 2070, the sulfate-aerosol geoengineering is terminated. Due to the short lifetime of the stratospheric aerosol (compared to the CO_2 lifetime), the radiative forcing rapidly approaches the RCP4.5 value.

(a) What are the surface temperature increases in 2050–2069 (the geoengineering period) and 2070–2089 (the termination period), for RCP4.5 and G3 (the geoengineering program)? Denote the temperature in 2006–2035 as T_1, 2050–2069 as T_2, temperature in 2070–2089 as T_3. Denote the temperature under the RCP4.5 scenario as T^R, and under the G3 scenario as T^G. You may wish to arrange the plots on a 3×3 grid as follows:

$$\begin{array}{ccc} T_2^R - T_1^R & T_3^R - T_1^R & T_3^R - T_2^R \\ T_2^G - T_1^R & T_3^G - T_1^R & T_3^G - T_2^G \\ T_2^G - T_2^R & T_3^G - T_3^R & \end{array}$$

and divide the temperature differences by time elapsed between the midpoints of the periods (3.9 decades between T_1 and T_2, 2 decades between T_2 and T_3) to get temperature increases per decade. The files containing the 2D temperature fields are on `ora` in `/home_local/tgoren/ex8/tas_<scenario>_<time period>_mean.nc`.

(b) Plot the time series of the global annual-mean temperatures under RCP4.5 and G3 from 2006 to 2089. The files containing the time series are called `/home_local/tgoren/ex8/tas_<scenario>.nc`.

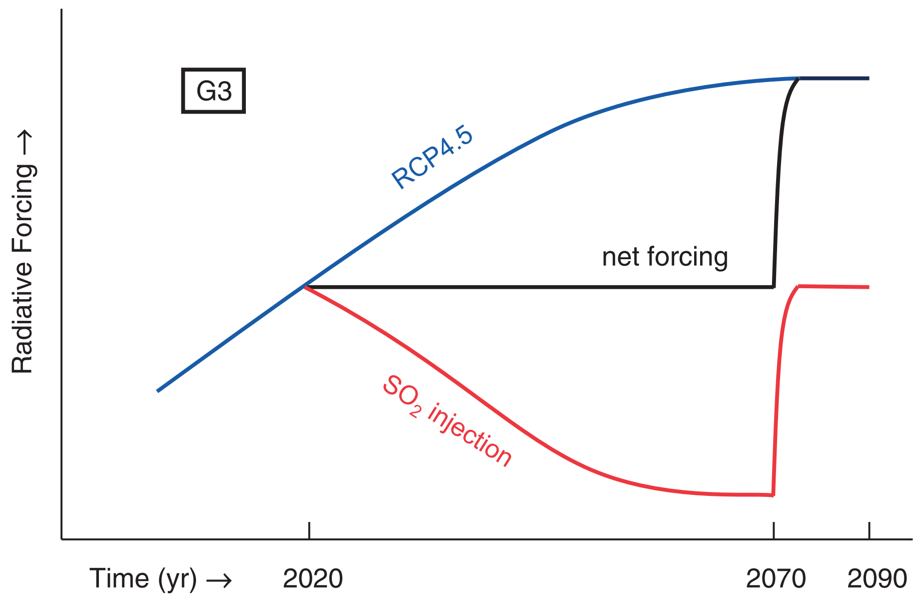

Figure 1: Time evolution of radiative forcing under RCP4.5 and under the G3 geoengineering scenario. From Kravitz et al., 2011.