Climate Dynamics
Summer Semester 2019

UNIVERSITÄT LEIPZIG

Homework 4
Due 22 May 2019

Problem 1 Saturation water vapor pressure

Consider the Clausius–Clapeyron equation, (2.65):

$$\frac{d\ln e_s}{dT} = \frac{L}{R_v T^2} \tag{1}$$

(a) Show that e_s increases by approximately 7% K^{-1} for typical values of the near-surface air temperature.

(b) Approximately how large of a temperature increase is required for e_s to double?

Problem 2 Diabatic heating

Starting with the thermodynamic energy equation (2.46),

$$c_p \frac{dT}{dt} = T\frac{ds}{dt} + \alpha \frac{dp}{dt} = Q + \alpha\omega, \tag{2}$$

show that an equivalent formulation (involving the static stability $\partial\theta/\partial p$) is

$$c_{pd}\frac{T}{\theta}\left(\frac{\partial\theta}{\partial t} + \vec{v}\cdot\nabla_p\theta + \omega\frac{\partial\theta}{\partial p}\right) = Q \tag{3}$$

Problem 3 Thermodynamic structure of the atmosphere

ERA Interim reanalysis air temperature data is available in the file
`/home_local/quaas/data/ERA__Interim__T_GDS0_ISBL_123__1.5x1.5xL37__198901-200712.nc`.
Calculate zonal and temporal means of the following quantities and plot them as contour lines in the ϕ–p plane:

(a) T

(b) θ

(c) q_s; you may use the empirical approximation

$$e_s(T) = 6.1094\ \text{hPa} \times \exp\left(\frac{17.625T}{T + 243.04}\right) \tag{4}$$

for T *in Celsius*.

(d) θ_e

Problem 4 Averaging operators

The temporal mean of an arbitrary field A over a time period τ is defined as

$$\overline{A} = \overline{A}(\lambda, \phi, p) = \frac{1}{\tau} \int_{-\tau/2}^{\tau/2} A(\lambda, \phi, p, t)\, dt \tag{5}$$

The zonal mean (over all longitudes) is defined as

$$[A] = [A](\phi, p, t) = \frac{1}{2\pi} \int_{0}^{2\pi} A(\lambda, \phi, p, t)\, d\lambda \tag{6}$$

The instantaneous value of A is given by

$$A = \overline{A} + A' \tag{7}$$

where A' is called the *fluctuating* component of A. Likewise

$$A = [A] + A^* \tag{8}$$

where A^* is the departure from the zonal mean.
Show that for arbitrary fields A and B

(a) $A = \left[\overline{A}\right] + \left[A'\right] + \overline{A}^* + A'^*$

(b) $\left[\overline{AB}\right] = \left[\overline{A}\right]\left[\overline{B}\right] + \left[\overline{A}^*\overline{B}^*\right] + \left[\overline{A'B'}\right]$