Derivational order and ACGs

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ACG@10, Bordeaux, December 7 – 9, 2011
ACGs provide a way to understand many ideas from a uniform perspective

- Architectures for grammar (SP)
  - Syntactico-centrism vs Parallelism (SP, PdG, & CP)

- Grammar formalisms
  - CFHGs (MK)
  - CFG, LCFTG, MCFG (PdG, SP)
  - DTWT (SS)
  - MG (SS)
  - Distributional learning (RY)

- We want to add to the list:
  - Timing
The basic idea of using cosubstitution to handle scope is that we can build the nuclear scope of a quantifier before the quantifier enters the derivation.
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Thus on the cosubstitution approach, quantifier scope ambiguity is a matter of timing: quantifiers that enter later in the derivation take wider scope.

In other words, co-TAG has exactly the same weak and strong generative capacity as TAG.
[... ] ambiguity can arise even when there is no element of intensionality, simply because quantifying terms may be introduced in more than one order.

[... ] an expression may bind only those expressions that it c-commands when first merged
What’s behind this?

The problem:
Sometimes we have a semantic ambiguity without any obvious corresponding difference in the derivation tree.

The idea:
Enrich the derivation tree with information about order. (not only did I apply this rule, but I applied this rule before that one)
A concrete example

Example

The representation of a CFG derivation as a tree equates sequences of rewriting steps which rewrite nonterminal instances the same way. (Leftmost, rightmost, etc)

\[ \langle S \Rightarrow AB \Rightarrow aB \Rightarrow ab \rangle \equiv \langle S \Rightarrow AB \Rightarrow Ab \Rightarrow ab \rangle \]
What’s to come

The goals:
- Make sense of the idea of ‘timing’ in terms of derivations
- Understand (in particular) co-substitution in TAGs in these terms
- If SGC/WGC unchanged, what is going on?

Our main claims are that:
- Third order ACGs provide an elegant description of the derivations of context-free formalisms.
- “Timing” is not derivational order
1. Everyone’s favourite example (Montague)
2. 3rd order ACGs and context-free derivations
3. Cosubstitution in TAGs
Frameworks which make use of ‘late’ operations

Montague Grammar
- Quantifying In

Tree Adjoining Grammars
- co-substitution
- flexible composition

Minimalist Grammars
- Late adjunction
- ‘hypothetical reasoning’
Montague Grammar (MG)

- MG has order sensitive rules in the form of Quantifying In (QI), namely, S14:

  **Rules of quantification**

  S14. If $\alpha \in P_T$ and $\phi \in P_t$, then $F_{10,n}(\alpha, \phi) \in P_t$, where either (i) $\alpha$ does not have the form $he_k$, and $F_{10,n}(\alpha, \phi)$ comes from $\phi$ by replacing the first occurrence of $he_n$ or $him_n$ by $\alpha$ and all other occurrences of $he_n$ or $him_n$ by $\{he, she\}$ or $\{him, her, it\}$ respectively, according as the gender of the first $B_{CN}$ or $B_T$ in $\alpha$ is $\{masc., fem., neuter\}$, or

  (ii) $\alpha = he_k$, and $F_{10,n}(\alpha, \phi)$ comes from $\phi$ by replacing all occurrences of $he_n$ or $him_n$ by $he_k$ or $him_k$ respectively.

  S15. If $\alpha \in P_T$ and $\zeta \in P_{CN}$, then $F_{10,n}(\alpha, \zeta) \in P_{CN}$.

  S16. If $\alpha \in P_T$ and $\delta \in P_{IV}$, then $F_{10,n}(\alpha, \delta) \in P_{IV}$.
Montague Grammar (MG)

- MG has order sensitive rules in the form of Quantifying In (QI), namely, S14:

- QI has some complications we do not need to worry about right now: multiple occurrences of $\text{he}_n$ can be affected by rule $F_{10,n}$

- QI is order sensitive in the sense that if we have a derivation $d = C[F_{10,i}(\alpha, D[F_{10,j}(\beta, \phi)])]$, then $\alpha$ takes semantic scope over $\beta$
An Example - *De Re* vs *De Dicto* Readings

- John seeks a unicorn.

<table>
<thead>
<tr>
<th>De Re</th>
<th>De Dicto</th>
</tr>
</thead>
</table>
| There is a particular unicorn that John is looking for.  
\[ \exists y [\text{UNICORN}(y) \land \text{TRY}(\text{FIND}(y))(J)] \] | John will be satisfied with any unicorn.  
\[ \text{TRY}(\exists y [\text{UNICORN}(y) \land \text{FIND}(y)])(J) \] |

- Can be represented in terms of operator scope:

\[ x \text{ seeks } y \]
\[ \text{TRY}(\text{FIND}(y))(x) \]
Derivations for *De Re vs De Dicto* Readings

```
John seeks a unicorn, 4
  
  John

  seek a unicorn, 5
    
    seek

    a unicorn, 2
      
      unicorn

John seeks a unicorn, 10, 0

  a unicorn, 2
    
    unicorn

  John seeks him_{0}, 4
    
    John

    seek him_{0}, 5
      
      seek

      he_{0}
```
An ACG implementation (following de Groote 2001)

- A faithful ACG implementation of the derivation structure of MG might look as follows:
  - to each $n$-ary predicate (seek, love, ...) we associate a function symbol of type $np^n \rightarrow s$
  - to each common noun (boy, unicorn, ...) we associate a function symbol of type $n$
  - to each determiner (some, every, ...) we associate a function symbol of type $n \rightarrow d$
  - we have a function symbol $\text{QI}$ of type $(np \rightarrow s) \rightarrow (d \rightarrow s)$
- This allows us to have abstract terms like the following:
  - $\text{QI}(\lambda y^{np}.\text{QI}(\lambda x^{np}.\text{seek}(x)(y))(\text{some}(\text{unicorn}))))(\text{every}(\text{boy}))$
The lexica are as follows:

- on the concrete phonology side, the symbol $QI$ is interpreted as the (forwards application) combinator $\lambda xy. x(y)$ (and everything else is interpreted in the obvious way, given that all abstract types $n$, $d$, $np$ and $s$ are interpreted as the object type $o \to o$, the type of strings).
- on the concrete semantic side, the symbol $QI$ is interpreted as the (backwards application) combinator $\lambda xy. y(x)$. 
An alternative implementation substitutes for the type \( d \) the type \((np \to s) \to s\), and eliminates the symbol \( QI \). Then we have abstract terms like:

- \( \text{every}(\text{boy})(\lambda y^{np}.\text{some}(\text{unicorn})(\lambda x^{np}.\text{seek}(x)(y))) \)

Under this alternative, the lexica are as follows:

- The concrete phonological interpretation of a determiner \( \text{every} \) is as the function \( \lambda x.f.f(\text{every} \cdot x) \)
- The concrete semantic interpretation of a determiner \( \text{every} \) is as the function \( \lambda x.f.\forall(x)(f) \)
The ACG implementation does currently not deal with the ignored complexities of QI.

The straightforward implementation takes the abstract terms to be pretty much isomorphic to the concrete semantic terms.

The big picture (that it admittedly seems is not very clear in this presentation) is that:

- higher order constants mark scope positions, and apply to lambda abstractions which make the variables in lower positions accessible.
- In other words, ‘timing’ requires of an expression that it simultaneously be connected to both an underlying (argument) position, and a ‘surface’ (timing) position.
Tree Adjoining Grammars

- Strongly equivalent to monadic linear CFTGs
  (Fujiyoshi & Kasai; Mönnich; Kepser & Rogers)
- Weakly equivalent to $2$-MCFL$^\text{wn}$
  (Vijay-Shanker et al.; Seki et al.; Kanazawa)
- A TAG consists of
  - initial trees: a finite set of trees with leaves labelled by either terminals or $X\downarrow$, where $X$ is a nonterminal symbol
  - auxiliary trees: as before, but with exactly one leaf node labelled by $X^\ast$, where $X$ is the label of the root
Substitute (1st Order Substitution)

\[
\begin{align*}
\text{DP} & \quad + \quad \text{S} \quad = \quad \text{S} \\
\text{John} & \quad \downarrow \quad \text{VP} \quad \text{left} \quad \downarrow \quad \text{John} \quad \text{left}
\end{align*}
\]
Not much happens in this transduction except that...

\[ S + \text{VP} = \begin{cases} \text{DP} & \text{VP} \\ \text{John} & \text{left} & \text{quietly} & \text{VP*} \\ \text{DP} & \text{VP} \\ \text{John} & \text{quietly} & \text{VP} & \text{left} \end{cases} \]
Barker’s Innovation: Unrestricted Derivation and Scope

Fact
Linear CFTGs derive the same tree languages under all (IO, OI, unrestricted) derivation modes.

Barker:
$\Gamma$ scopes over $\Delta$ iff $\Gamma$ was substituted in after a tree containing $\Delta$ was
Linear CFTGs

Definition (repeated)

\[ G = \langle N, T, P, s \rangle \text{ a linear CFTG} \]

- \( N \) the ranked alphabet of nonterminals
- \( T \) the ranked alphabet of terminals
- \( s \) the start symbol of rank 0 from \( N \)
- \( P \) the production rules each of which of the form

\[ a(x_1, \ldots, x_n) \rightarrow t \]

- \( a \in N \) such that \( \text{rank}(a) = n \)
- \( x_1, \ldots, x_n \) variables of rank 0
- \( t \) a linear tree over \( N \cup T \cup \{x_1, \ldots, x_n\} \)

- For expository reasons we assume that the start symbol \( s \in N \) does not appear on the righthand side of any rule
Linear CFTGs as ACGs

Composition as second-order substitution

(de Groote and Pogodalla 2004)

\[ G = \langle N, T, P, s \rangle \text{ a linear CFTG} \]
Linear CFTGs as ACGs
Composition as second-order substitution  
(de Groote and Pogodalla 2004)

$G = \langle N, T, P, s \rangle$ a linear CFTG

$\mathcal{G}_G = \langle \Sigma_G, \Sigma_T, \mathcal{L}_G, s \rangle$ associated 2nd order ACG

- $\Sigma_G = \langle N, \{ \bar{p} \mid p \in P \} , \tau \rangle$ higher-order linear signature
  
  $\tau(\bar{p}) = a_1 \rightarrow \cdots \rightarrow a_m \rightarrow a$

  $p \in P$ with skeleton $\langle a, a_1 \cdots a_m \rangle$
Linear CFTGs as ACGs

Composition as second-order substitution

\( G = \langle N, T, P, s \rangle \) a linear CFTG

\( \mathcal{G}_G = \langle \Sigma_G, \Sigma_T, \mathcal{L}_G, s \rangle \) associated 2nd order ACG

- \( \Sigma_G = \langle N, \{ p | p \in P \} \rangle \) higher-order linear signature
  \[ \tau(p) = a_1 \to \cdots \to a_m \to a \]
  \( p \in P \) with skeleton \( \langle a, a_1 \cdots a_m \rangle \)

- For \( p = a(x_1, \ldots, x_n) \to t \in P \) with skeleton \( \langle a, a_1 \cdots a_m \rangle \)
  by induction, linear \( \lambda \)-term
  \[ [p] := \lambda y_1 \ldots y_m . \lambda x_1 \ldots x_n . |t| \]
Linear CFTGs as ACGs

Composition as second-order substitution (de Groote and Pogodalla 2004)

\[ G = \langle N, T, P, s \rangle \] a linear CFTG

\[ G_G = \langle \Sigma_G, \Sigma_T, L_G, s \rangle \] associated 2nd order ACG

- \[ \Sigma_G = \langle N, \{ \overline{p} \mid p \in P \}, \tau \rangle \] higher-order linear signature
  \[ \tau(\overline{p}) = a_1 \to \cdots \to a_m \to a \]
  \[ p \in P \] with skeleton \( \langle a, a_1\cdots a_m \rangle \)

- For \( p = a(x_1, \ldots, x_n) \to t \in P \) with skeleton \( \langle a, a_1\cdots a_m \rangle \)
  by induction, linear \( \lambda \)-term
  \[ \llbracket p \rrbracket := \lambda y_1 \ldots y_m . \lambda x_1 \ldots x_n . \mid t \mid \]

\[
\begin{align*}
| x_i | &= x_i \\
| f() | &= \lambda x . (f x) \\
| f(t_1, \ldots, t_k) | &= |t_1| + \ldots + |t_k| \\
| a_j() | &= y_j \\
| a_j(t_1, \ldots, t_k) | &= y_j |t_1| \cdots |t_k|
\end{align*}
\]
Linear CFTGs as ACGs

Composition as second-order substitution (de Groote and Pogodalla 2004)

\[ G = \langle N, T, P, s \rangle \text{ a linear CFTG} \]

\[ \mathcal{G}_G = \langle \Sigma_G, \Sigma_T, L_G, s \rangle \text{ associated 2nd order ACG} \]

- \[ \Sigma_G = \langle N, \{ \bar{p} \mid p \in P \}, \tau \rangle \text{ higher-order linear signature} \]

\[ \tau(\bar{p}) = a_1 \to \cdots \to a_m \to a \]

\[ p \in P \text{ with skeleton } \langle a, a_1 \cdots a_m \rangle \]

- For \( p = a(x_1, \ldots, x_n) \to t \in P \) with skeleton \( \langle a, a_1 \cdots a_m \rangle \)

by induction, linear \( \lambda \)-term

\[ \lceil p \rceil := \lambda y_1 \ldots y_m . \lambda x_1 \ldots x_n . \lceil t \rceil \]

- \( L_G : \Sigma_G \to \Sigma_T \)

\[ L_G(a) = \text{string}^{\text{rank}(a)} \to \text{string} \quad \text{for } a \in N \]

\[ L_G(\bar{p}) = \lceil p \rceil \quad \text{for } p \in P \]
Linear CFTGs as ACGs

Composition as second-order substitution

(de Groote and Pogodalla 2004)

$G = \langle N, T, P, s \rangle$ a linear CFTG

$\mathcal{G}_G = \langle \Sigma_G, \Sigma_T, \mathcal{L}_G, s \rangle$ associated 2nd order ACG

- $\Sigma_G = \langle N, \{ \bar{p} \mid p \in P \}, \tau \rangle$ higher-order linear signature
  
  $\tau(\bar{p}) = a_1 \to \cdots \to a_m \to a$
  
  $p \in P$ with skeleton $\langle a, a_1 \cdots a_m \rangle$

- For $p = a(x_1, \ldots, x_n) \to t \in P$ with skeleton $\langle a, a_1 \cdots a_m \rangle$
  
  by induction, linear $\lambda$-term $\lfloor p \rfloor := \lambda y_1 \ldots y_m \cdot \lambda x_1 \ldots x_n \cdot t$

- $\mathcal{L}_G : \Sigma_G \to \Sigma_T$
  
  $\mathcal{L}_G(a) = string^{rank(a)} \to string$ for $a \in N$
  
  $\mathcal{L}_G(\bar{p}) = \lfloor p \rfloor$ for $p \in P$

- $\Sigma_T = \langle \{ o \}, T, \tilde{\tau} \rangle$ higher-order linear signature
  
  $\tilde{\tau}(f) = string = o \to o$ for $f \in T$
Linear CFTGs as ACGs

Composition as second-order substitution  
(de Groote and Pogodalla 2004)

\[ G = \langle N, T, P, s \rangle \text{ a linear CFTG} \]

\[ \mathcal{G}_G = \langle \Sigma_G, \Sigma_T, \mathcal{L}_G, s \rangle \text{ associated 2nd order ACG} \]

- \[ \Sigma_G = \langle N, \{ \overline{p} \mid p \in P \}, \tau \rangle \text{ higher-order linear signature} \]

\[ \tau(\overline{p}) = a_1 \rightarrow \cdots \rightarrow a_m \rightarrow a \]
Linear CFTGs as ACGs
Composition as third-order substitution (including derivation order)

\[ G = \langle N, T, P, s \rangle \] a linear CFTG

\[ \mathcal{G}_G = \langle \Sigma_G, \Sigma_T, \mathcal{L}_G, s \rangle \] associated 2nd order ACG

- \[ \Sigma_G = \langle N, \{ \overline{p} \mid p \in P \} , \tau \rangle \] higher-order linear signature

\[ \tau(\overline{p}) = a_1 \rightarrow \cdots \rightarrow a_m \rightarrow a \]

\[ \mathcal{G}_G' = \langle \Sigma'_G, \Sigma_T, \mathcal{L}'_G, s \rangle \] 3rd order ACG resulting from \( \mathcal{G}_G \)
Linear CFTGs as ACGs
Composition as third-order substitution (including derivation order)

\[ G = \langle N, T, P, s \rangle \text{ a linear CFTG} \]

\[ \mathcal{G}_G = \langle \Sigma_G, \Sigma_T, L_G, s \rangle \text{ associated 2nd order ACG} \]

- \[ \Sigma_G = \langle N, \{ \overline{p} \mid p \in P \} , \tau \rangle \text{ higher-order linear signature} \]
  \[ \tau(\overline{p}) = a_1 \rightarrow \cdots \rightarrow a_m \rightarrow a \]

\[ \mathcal{G}'_G = \langle \Sigma'_G, \Sigma_T, L'_G, s \rangle \text{ 3rd order ACG resulting from } \mathcal{G}_G \]

- \[ \Sigma'_G = \langle N, \{ \overline{p} \mid p \in P \} , \tau' \rangle \text{ higher-order linear signature} \]
  \[ \tau'(\overline{p}) = \text{trans}(\tau(\overline{p})) = \text{trans}(a_1 \rightarrow \cdots \rightarrow a_m \rightarrow a) \]

\[ \text{trans}(s) = s \quad s \text{ the start symbol} \]

\[ \text{trans}(a) = (a \rightarrow s) \rightarrow s \quad a \in N - \{s\} \]

\[ \text{trans}(\alpha \rightarrow \beta) = \alpha \rightarrow \text{trans}(\beta) \]
Example grammar

### CFTG

\[ p_1 = S \rightarrow f(T(c), G(c)) \]

\[ p_2 = T(x) \rightarrow f(T(x), G(c)) \]

\[ p_3 = G(x) \rightarrow h(x) \]

\[ p_4 = T(x) \rightarrow G(x) \]

### 2nd order ACG

\[ \bar{p}_1 : T \rightarrow G \rightarrow S \]

\[ \bar{p}_2 : T \rightarrow G \rightarrow T \]

\[ \bar{p}_3 : G \]

\[ \bar{p}_4 : G \rightarrow T \]
Example grammar

CFTG

\[ p_1 = S \rightarrow \mathcal{f}(T(c), G(c)) \]
\[ p_2 = T(x) \rightarrow \mathcal{f}(T(x), G(c)) \]
\[ p_3 = G(x) \rightarrow h(x) \]
\[ p_4 = T(x) \rightarrow G(x) \]

2nd order ACG

\[ \overline{p}_1 : T \rightarrow G \rightarrow S \]
\[ \overline{p}_2 : T \rightarrow G \rightarrow T \]
\[ \overline{p}_3 : G \]
\[ \overline{p}_4 : G \rightarrow T \]

3rd order ACG

\[ \overline{p}_1 : T \rightarrow G \rightarrow S \]
\[ \overline{p}_2 : T \rightarrow G \rightarrow (T \rightarrow S) \rightarrow S \]
\[ \overline{p}_3 : (G \rightarrow S) \rightarrow S \]
\[ \overline{p}_4 : G \rightarrow (T \rightarrow S) \rightarrow S \]
Example derivation — \( p_1 p_4 p_3 p_3 \)

```
\[
\begin{align*}
\vdash \bar{p}_4 & : G \to (T \to S) \to S & \text{(var, app)} \\
\vdash \bar{p}_4 \bar{p}_2 & : (T \to S) \to S & \text{(var, app)} \\
\vdash \bar{p}_3 & : (G \to S) \to S & \text{(var, app)} \\
\dashv & \text{left-to-right}
\end{align*}
\]

```

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\[
\begin{align*}
\vdash \bar{p}_1 & : T \to G \to S & \text{(var, app)} \\
\vdash \bar{p}_1 t_1 & : G \to S & \text{(var, app)} \\
\vdash \bar{p}_1 t_1 g_1 & : S & \text{(abs)} \\
\vdash \lambda t_1 \bar{p}_1 t_1 g_1 & : T \to S & \text{(app)} \\
\vdash \lambda t_1 \bar{p}_1 t_1 g_1 \bar{p}_4 \bar{p}_2 & : T \to S & \text{(app)} \\
\vdash \lambda g_2 \bar{p}_3(\lambda g_1 \bar{p}_4 \bar{p}_2(\lambda t_1 \bar{p}_1 t_1 g_1)) & : G \to S & \text{(app)} \\
\vdash \lambda g_2 \bar{p}_3(\lambda g_1 \bar{p}_4 \bar{p}_2(\lambda t_1 \bar{p}_1 t_1 g_1)) & : G \to S & \text{(app)} \\
\vdash \bar{p}_3(\lambda g_2 \bar{p}_3(\lambda g_1 \bar{p}_4 \bar{p}_2(\lambda t_1 \bar{p}_1 t_1 g_1))) & : S & \text{(app)} \\
\vdash \bar{p}_3(\lambda g_2 \bar{p}_3(\lambda g_1 \bar{p}_4 \bar{p}_2(\lambda t_1 \bar{p}_1 t_1 g_1))) & : S & \text{(app)} \\
\end{align*}
\]

```
Example derivation — $p_1 p_4 p_3 p_3$ left-to-right

\[
\begin{array}{c}
\vdash \overline{p}_4 : G \rightarrow (T \rightarrow S) \rightarrow S \\
g_2 : G \vdash \overline{p}_4 g_2 : (T \rightarrow S) \rightarrow S \\
g_1 : G \vdash \lambda g_1 \cdot \overline{p}_4 g_2 (\lambda t_1 \cdot \overline{p}_1 t_1 g_1) : S \\
g_2 : G \vdash \overline{p}_3 (\lambda g_1 \cdot \overline{p}_4 g_2 (\lambda t_1 \cdot \overline{p}_1 t_1 g_1)) : G \rightarrow S \\
\vdash \overline{p}_3 (\lambda g_2 \cdot \overline{p}_3 (\lambda g_1 \cdot \overline{p}_4 g_2 (\lambda t_1 \cdot \overline{p}_1 t_1 g_1))) : S
\end{array}
\]
Example derivation — $p_1 \; p_3 \; p_4 \; p_3$

\begin{align*}
\vdash \overline{p}_1 : T \rightarrow G \rightarrow S \\
\vdash \overline{p}_3 : (G \rightarrow S) \rightarrow S \\
\vdash \overline{p}_4 : G \rightarrow (T \rightarrow S) \rightarrow S \\
\vdash \overline{p}_3 : (G \rightarrow S) \rightarrow S \\
\vdash \lambda t_1 . \overline{p}_3 (\lambda g_1 . \overline{p}_1 t_1 g_1) : T \rightarrow S \\
\vdash \lambda g_2 . \overline{p}_4 g_2 (\lambda t_1 . \overline{p}_3 (\lambda g_1 . \overline{p}_1 t_1 g_1)) : G \rightarrow S \quad \text{(app)} \end{align*}
Example derivation — $p_1 p_3 p_4 p_3$

right-to-left

\[
\begin{align*}
& \vdash \overline{p}_1 : T \to G \to S \\
& t_1 : T \vdash \overline{p}_1 \overline{t}_1 : G \to S \\
& g_1 : G, t_1 : T \vdash \overline{p}_1 \overline{t}_1 \overline{g}_1 : S \\
& t_1 : T \vdash \lambda g_1 \cdot \overline{p}_1 \overline{t}_1 \overline{g}_1 : G \to S \\
\end{align*}
\]

\[
\begin{align*}
& \vdash \overline{p}_3 : (G \to S) \to S \\
& \vdash \overline{p}_4 : G \vdash \overline{p}_4 \overline{g}_2 : (T \to S) \to S \\
& g_2 : G \vdash \overline{p}_4 \overline{g}_2 : (T \to S) \to S \\
& \vdash \overline{p}_3 : (G \to S) \to S \\
& \vdash \overline{p}_3 : (G \to S) \to S \\
& \vdash \lambda g_2 \cdot \overline{p}_4 \overline{g}_2 (\lambda t_1 \cdot \overline{p}_3 (\lambda g_1 \cdot \overline{p}_1 \overline{t}_1 \overline{g}_1)) : S \\
& \vdash \overline{g}_2 (\lambda t_1 \cdot \overline{p}_3 (\lambda g_1 \cdot \overline{p}_1 \overline{t}_1 \overline{g}_1)) : S \\
& \vdash \overline{p}_3 (\lambda g_2 \cdot \overline{p}_4 \overline{g}_2 (\lambda t_1 \cdot \overline{p}_3 (\lambda g_1 \cdot \overline{p}_1 \overline{t}_1 \overline{g}_1))) : S \\
\end{align*}
\]
Back towards the object language

The transformation $g$ which identifies ‘equivalent’ derivations:

$$
\overline{p} \quad \begin{cases} 
\overline{p} & \text{if } p \in P \text{ is a non-lifted rule} \\
\lambda y_1 \cdots y_m f \cdot f (\overline{p}y_1 \cdots y_m) & \text{otherwise}
\end{cases}
$$

- This actually maps a derivation with order information (qua third order lambda term) to a derivation tree, i.e.

  a representation which keeps track only of which rule was used to rewrite which nonterminal, and not the relative order in which the rules were used.
**Example**

The transformation $g$ which identifies ‘equivalent’ derivations:

\[ \overline{p} \rightarrow \begin{cases} \overline{p} & \text{if } p \in P \text{ is a non-lifted rule} \\ \lambda y_1 \cdots y_m f \cdot f (\overline{p} y_1 \cdots y_m) & \text{otherwise} \end{cases} \]

- \[ \overline{p}_3 (\lambda g_2 \cdot \overline{p}_3 (\lambda g_1 \cdot \overline{p}_4 g_2 (\lambda t_1 \cdot \overline{p}_1 t_1 g_1))) \]

left-to-right

and

- \[ \overline{p}_3 (\lambda g_2 \cdot \overline{p}_4 g_2 (\lambda t_1 \cdot \overline{p}_3 (\lambda g_1 \cdot \overline{p}_1 t_1 g_1))) \]

right-to-left

both are identified as

\[ \overline{p}_1 (\overline{p}_4 \overline{p}_3) \overline{p}_3 \]

transformation under $g$
The object language of the new ACG is gotten by ‘composing’ the map $g$ which turns a new abstract term into the old ‘IO’ one, and the original lexicon. (Note that this simply reuses the original ACG’s lexicon — the object language of the new ACG is obtained by first translating it back into the old ACG, and then interpreting the old ACG as usual.)
The object language

We need to prove that the translation $\text{trans}$ over types engenders an ACG whose abstract language exactly corresponds to the derivations of the original linear context-free tree grammar.

- One problem: the notion of derivation is not rich enough. We need to enrich it with information about which nonterminal is being rewritten. One way to do this is to redefine the notion of derivation:

$$ \Rightarrow \subseteq (T_\Sigma \times P \times \mathbb{N}) \times T_\Sigma $$

That is, the one step derivation relation is between a tree (a sentential form), a rule, a natural number, and another tree:

$$ C[A(t_1, \ldots, t_k)] \Rightarrow^p_n C[t[t_1, \ldots, t_k]] $$

if there are exactly $n - 1$ nonterminal symbols to the left of the bullet in $C[\bullet]$, and if there is some rule $p = A(x_1, \ldots, x_k) \rightarrow t$. 
The object language

Now we want to say that each type derivation of an abstract term (a term in the abstract language) can be put into correspondence with a unique derivation in the CFTG, and what properties this correspondence should have.

- We will be interested in canonical ($\beta$-reduced, but $\eta$-long) terms. That is, in terms where all argument positions are explicitly filled, but where all applications have been computed.
- Note that the abstract language is a subset of the following:

$$L := \delta Y_1 \ldots Y_m \mid \rho Y_1 \ldots Y_m \lambda Y.L$$

where $\delta$ is a rule rewriting the start symbol, and $\rho$ is not.
The object language

- To define a mapping $\langle \cdot \rangle$ from such terms to derivations in the underlying CFTG, we first need to be able to turn a term into a sentential form. We do this in the following way:
- For each nonterminal $T$ in the CFTG, there is a type $T$ in the ACG. We introduce a new zero-place constant $\overline{T}$ of type $T$ for each nonterminal $T$. We define the lexicon $L''$ to be the extension of $L'$ which maps each $\overline{T}$ to

\[
L''(\overline{T}) = \lambda x_1 \ldots x_{\text{rank}(T)} \cdot T \ x_1 \ldots x_{\text{rank}(T)}
\]

For $M \in \Lambda(\Sigma'_G)$ of atomic type with $\text{FV}(M) = \{y_1, \ldots, y_m\}$ we define

\[
[M] := L''((\lambda y_1, \ldots, y_m \cdot M) \overline{T_1} \ldots \overline{T_m}),
\]

where $y_i : T_i$. Now $[M]$ is simply the sentential form which $M$ represents.
The object language

- We can define the translation from terms to derivations after introducing the following notation: Let $|\lambda x . M|$ denote the number of free variables which occur to the left of the first occurrence of $x$ in $M$.

- Note, this definition treats the $\lambda$-term as a syntactic object — it distinguishes between terms which are $\beta, \eta$-equivalent. This is why we are working with the unique representative of a $\beta, \eta$-equivalence class which is $\beta$-reduced and $\eta$-long.

- Formally, define for any set $B$, any $a$ being a constant or variable, and any lambda terms $M$ and $N$:

  $\text{str}(B, a) = \begin{cases} 
  \varepsilon & \text{if } a \in B \\
  a & \text{otherwise}
  \end{cases}$

  $\text{str}(B, \lambda x . M) = \text{str}(B \cup \{x\}, M)$

  $\text{str}(B, (MN)) = \text{str}(B, M) \cup \text{str}(B, N)$

  Then set $|\lambda x . M| := |u|$, where $\text{str}(C, M[x \rightarrow \bullet]) = u \cdot w$
The object language

The translation can be given as

\[ \begin{align*}
\llangle M \rrangle &= \begin{cases} 
S \Rightarrow_1^\delta [M] & \text{if } M = \delta y_1 \cdots y_m \\
\llangle N \rrangle \Rightarrow_{n+1}^\rho [M] & \text{if } M = \rho y_1 \cdots y_m \lambda y.N \\
\end{cases} \\
\text{and if } n = |\lambda y.N| \\
\end{align*} \]

Claim

For every term \( M \) of the abstract language \( A(G'_G) \), \( \llangle M \rrangle \) is a derivation in \( G \) of \( [M] \).
Translating back to ACGs

- $\langle S \Rightarrow D' \rangle^{-1} = [D'] (\lambda y. \delta \bar{y})$

- $[\phi \Rightarrow D'] (M) = [D'] (\lambda x \bar{u} \bar{v} . \rho \bar{x} \bar{z} . (M \bar{u} \bar{z} \bar{v}))$
  
  such that $n = |\lambda z . (M \bar{u} \bar{z} \bar{v})|$

- $[\phi] (M) = M$

Claim

For every complete derivation $D$ of $t$, $[\langle D \rangle^{-1}] = t$.
Moreover, $\langle \cdot \rangle^{-1}$ is the inverse of $\langle \cdot \rangle$. 
Third order lambda terms really are nicer

- dequotienting of ‘equivalent’ derivations implicit in the use of derivation trees
- but easy to use because substitution is managed by $\beta$-reduction
- canonical terms correspond exactly to derivations
Barker’s Analysis (qua CFTG)

- Some person left.

\[ S \rightarrow s(DP,v) \]
\[ DP \rightarrow d(det,NP) \]
\[ NP \rightarrow n \]

- Some person from Jamaica left.

\[ DP \rightarrow d(det,NP'(NP)) \]
\[ NP'(x) \rightarrow NP'(np(x,pp(p,DP))) \]
\[ NP'(x) \rightarrow x \]
\[ DP \rightarrow name \]
Some person from every city left.

\[ S \Rightarrow s(DP,v) \]
\[ \Rightarrow s(d(det, NP'(NP)),v) \]
\[ \Rightarrow s(d(det, NP'(n)),v) \]
\[ \Rightarrow s(d(det, NP'(np(n, pp(p, DP)))),v) \]
\[ \Rightarrow s(d(det, np(n, pp(p, DP))),v) \]
\[ \Rightarrow s(d(det, np(n, pp(p, d(det, NP'))))),v) \]
\[ \Rightarrow s(d(det, np(n, pp(p, d(det, NP'))))),v) \]
\[ \Rightarrow s(d(det, np(n, pp(p, d(det, n)))),v) \]
Some person from every city left.

\[ S \Rightarrow s(DP, v) \]
\[ \Rightarrow s(d(det, NP'(NP)), v) \]
\[ \Rightarrow s(d(det, NP'(n)), v) \]
\[ \Rightarrow s(d(det, NP'(np(n, pp(p, DP)))), v) \]
\[ \Rightarrow s(d(det, np(n, pp(p, d(det, NP')))), v) \]
\[ \Rightarrow s(d(det, np(n, pp(p, d(det, NP')))), v) \]
\[ \Rightarrow s(d(det, np(n, pp(p, d(det, n)))), v) \]
Some person from every city left.

\[ S \Rightarrow s(DP, v) \]
\[ \Rightarrow s(d(det, NP'(NP)), v) \]
\[ \Rightarrow s(d(det, NP'(n)), v) \]
\[ \Rightarrow s(d(det, NP'(np(n, pp(p, DP)))), v) \]
\[ \Rightarrow s(d(det, np(n, pp(p, d(det, NP')))), v) \]
\[ \Rightarrow s(d(det, np(n, pp(p, d(det, NP')))), v) \]
\[ \Rightarrow s(d(det, np(n, pp(p, d(det, n)))), v) \]
Some person from every city left.

\[ S \Rightarrow s(DP, v) \]
\[ \Rightarrow s(d(det, NP')(NP)), v) \]
\[ \Rightarrow s(d(det, NP')(n)), v) \]
\[ \Rightarrow s(d(det, NP')(np(n, pp(p, DP)))), v) \]
\[ \Rightarrow s(d(det, np(n, pp(p, d(det, NP')))), v) \]
\[ \Rightarrow s(d(det, np(n, pp(p, d(det, NP')))), v) \]
\[ \Rightarrow s(d(det, np(n, pp(p, d(det, n)))), v) \]
Some person from every city left.

\[ S \Rightarrow s(DP, v) \]
\[ \Rightarrow s(d(det, N P'(NP)), v) \]
\[ \Rightarrow s(d(det, N P'(n)), v) \]
\[ \Rightarrow s(d(det, N P'(n, pp(p, DP)))), v) \]
\[ \Rightarrow s(d(det, np(n, pp(p, DP))), v) \]
\[ \Rightarrow s(d(det, np(n, pp(p, d(det, N P'(NP)))), v) \]
\[ \Rightarrow s(d(det, np(n, pp(p, d(det, N P'))), v) \]
\[ \Rightarrow s(d(det, np(n, pp(p, d(det, N P'))), v) \]
\[ \Rightarrow s(d(det, np(n, pp(p, d(det, N P'))), v) \]**
Some person from every city left.

\[ S \Rightarrow s(DP, v) \]
\[ \Rightarrow s(d(det, NP'(NP)), v) \]
\[ \Rightarrow s(d(det, NP'(n)), v) \]
\[ \Rightarrow s(d(det, NP'(np(n, pp(p, DP)))), v) \]
\[ \Rightarrow s(d(det, np(n, pp(p, d(det, NP'))))), v) \]
\[ \Rightarrow s(d(det, np(n, pp(p, d(det, NP'))))), v) \]
\[ \Rightarrow s(d(det, np(n, pp(p, d(det, n)))), v) \]
Some person from every city left.

\[ S \Rightarrow s(DP, v) \]
\[ \Rightarrow s(d(det, NP'(NP)), v) \]
\[ \Rightarrow s(d(det, NP'(n)), v) \]
\[ \Rightarrow s(d(det, NP'(np(n, pp(p, DP)))), v) \]
\[ \Rightarrow s(d(det, np(n, pp(p, d(det, NP'))))), v) \]
\[ \Rightarrow s(d(det, np(n, pp(p, d(det, NP'))))), v) \]
\[ \Rightarrow s(d(det, np(n, pp(p, d(det, n))))), v) \]
Some person from every city left.

\[
S \Rightarrow s(DP, v) \\
\Rightarrow s(d(det, NP'(NP)), v) \\
\Rightarrow s(d(det, NP'(n)), v) \\
\Rightarrow s(d(det, NP'(np(n, pp(p, DP)))), v) \\
\Rightarrow s(d(det, np(n, pp(p, d(det, NP'))))), v) \\
\Rightarrow s(d(det, np(n, pp(p, d(det, NP'))))), v) \\
\Rightarrow s(d(det, np(n, pp(p, d(det, n)))), v)
\]
Here are the rules, and their second-order types:

\[
\begin{align*}
S & \rightarrow s(DP, v) \\
DP & \rightarrow d(det, NP'(NP)) \\
NP & \rightarrow n \\
NP'(x) & \rightarrow NP'(np(x, pp(p, DP))) \\
NP'(x) & \rightarrow x
\end{align*}
\]
Here are the rules, and their second-order types:

- \( S \rightarrow s(DP, v) \)
- \( DP \rightarrow d(det, NP'(NP)) \)
- \( NP \rightarrow n \)
- \( NP'(x) \rightarrow NP'(np(x, pp(p, DP))) \)
- \( NP'(x) \rightarrow x \)

- \( DP \rightarrow S \) \( (\rho_S) \)
- \( DP \rightarrow S \) \( (\rho_{DP}) \)
- \( NP \rightarrow n \) \( (\rho_N) \)
- \( NP'(x) \rightarrow NP'(np(x, pp(p, DP))) \) \( (\rho_{NP_a}') \)
- \( NP'(x) \rightarrow x \) \( (\rho_{NP_b}') \)
Here are the rules, and their second-order types:

\[
\begin{align*}
S & \rightarrow s(DP, v) & \quad (\rho_S) \\
DP & \rightarrow d(det, NP'(NP)) & \quad (\rho_{DP}) \\
NP & \rightarrow n & \quad (\rho_N) \\
NP'(x) & \rightarrow NP'(np(x, pp(p, DP))) & \quad (\rho_{NP_a'}) \\
NP'(x) & \rightarrow x & \quad (\rho_{NP_b'}) \\
\end{align*}
\]
Here are the rules, and their second-order types:

\[ S \to s(DP,v) \]
\[ DP \to d(det, NP'(NP)) \]
\[ NP \to n \]
\[ NP'(x) \to NP'(np(x, pp(p, DP))) \]
\[ NP'(x) \to x \]
\[ DP \to S \]
\[ NP' \to NP \to DP \]
\[ NP \]
\[ NP'(x) \to NP'(np(x, pp(p, DP))) \]
\[ NP'(x) \to x \]
The abstract language of Barker’s TAG

Here are the rules, and their second-order types:

\[
\begin{align*}
S & \rightarrow s(DP, v) \\
DP & \rightarrow d(det, NP'(NP)) \\
NP & \rightarrow n \\
NP'(x) & \rightarrow NP'(np(x, pp(p, DP))) \\
NP'(x) & \rightarrow x \\
DP & \rightarrow S \quad (\rho_S) \\
NP' & \rightarrow NP \rightarrow DP \quad (\rho_{DP}) \\
NP & \quad (\rho_N) \\
NP'(x) & \rightarrow DP \rightarrow NP' \quad (\rho_{NP_a'}) \\
NP'(x) & \rightarrow x \quad (\rho_{NP_b'})
\end{align*}
\]
The abstract language of Barker’s TAG

Here are the rules, and their second-order types:

\[ S \rightarrow s(DP, v) \]
\[ DP \rightarrow d(det, NP' (NP)) \]
\[ NP \rightarrow n \]
\[ NP' (x) \rightarrow NP' (np(x, pp(p, DP))) \]
\[ NP' (x) \rightarrow x \]

\[ DP \rightarrow S \quad (\rho_S) \]
\[ NP' \rightarrow NP \rightarrow DP \quad (\rho_{DP}) \]
\[ NP \quad (\rho_N) \]
\[ NP' \rightarrow DP \rightarrow NP' \quad (\rho_{NP_a}) \]
\[ NP' \quad (\rho_{NP_b}) \]
Lifting Barker’s TAG

And here the types lifted over $S$:

$$\rho_S : DP \rightarrow S$$
$$\rho_{DP} : NP' \rightarrow NP \rightarrow DP$$
$$\rho_N : NP$$
$$\rho_{NP_a} : NP' \rightarrow DP \rightarrow NP'$$
$$\rho_{NP_b} : NP'$$
And here the types lifted over $S$:

\[
\begin{align*}
\rho_S : DP & \rightarrow S \\
\rho_{DP} : NP' & \rightarrow NP \rightarrow DP \\
\rho_N : NP & \\
\rho_{NP'} : NP' & \rightarrow DP \rightarrow NP' \\
\rho_{NP'_b} : NP' & 
\end{align*}
\]
And here the types lifted over $S$:

\[
\begin{align*}
\rho_S &: DP \to S \\
\rho_{DP} &: NP' \to NP \to DP \\
\rho_N &: NP \\
\rho_{NP'} &: NP' \to DP \to NP' \\
\rho_{NP'_a} &: NP' \to DP \to NP' \\
\rho_{NP'_b} &: NP'
\end{align*}
\]
And here the types lifted over $S$:

$$
\begin{align*}
\rho_S &: DP \rightarrow S \\
\rho_{DP} &: NP' \rightarrow NP \rightarrow (DP \rightarrow S) \rightarrow S \\
\rho_N &: NP \\
\rho_{NP_a} &: NP' \rightarrow DP \rightarrow NP' \\
\rho_{NP_b} &: NP'
\end{align*}
$$
And here the types lifted over $S$:

\[
\begin{align*}
\rho_S &: DP \rightarrow S \\
\rho_{DP} &: NP' \rightarrow NP \rightarrow (DP \rightarrow S) \rightarrow S \\
\rho_N &: NP \\
\rho_{NP_a} &: NP' \rightarrow DP \rightarrow NP' \\
\rho_{NP_b} &: NP'
\end{align*}
\]
And here the types lifted over $S$:

\[
\begin{align*}
\rho_S &: DP \rightarrow S \\
\rho_{DP} &: NP' \rightarrow NP \rightarrow (DP \rightarrow S) \rightarrow S \\
\rho_N &: (NP \rightarrow S) \rightarrow S \\
\rho_{NP'} &: NP' \rightarrow DP \rightarrow NP' \\
\rho_{NP'_a} &: NP' \rightarrow NP' \\
\rho_{NP'_b} &: NP'
\end{align*}
\]
And here the types lifted over $S$:

$$\rho_S : DP \to S$$

$$\rho_{DP} : NP' \to NP \to (DP \to S) \to S$$

$$\rho_N : (NP \to S) \to S$$

$$\rho_{NP_a} : NP' \to DP \to NP'$$

$$\rho_{NP_b} : NP'$$
And here the types lifted over $S$:

- $\rho_S : DP \to S$
- $\rho_{DP} : NP' \to NP \to (DP \to S) \to S$
- $\rho_N : (NP \to S) \to S$
- $\rho_{NP_a} : NP' \to DP \to (NP' \to S') \to S$
- $\rho_{NP_b} : NP'$
And here the types lifted over $S$:

\[
\begin{align*}
\rho_S &: DP \rightarrow S \\
\rho_{DP} &: NP' \rightarrow NP \rightarrow (DP \rightarrow S) \rightarrow S \\
\rho_N &: (NP \rightarrow S) \rightarrow S \\
\rho_{NP'_a} &: NP' \rightarrow DP \rightarrow (NP' \rightarrow S) \rightarrow S \\
\rho_{NP'_b} &: NP'
\end{align*}
\]
And here the types lifted over $S$:

\[
\begin{align*}
\rho_S &: DP \to S \\
\rho_{DP} &: NP' \to NP \to (DP \to S) \to S \\
\rho_N &: (NP \to S) \to S \\
\rho_{NP_a} &: NP' \to DP \to (NP' \to S) \to S \\
\rho_{NP_b} &: (NP' \to S) \to S
\end{align*}
\]
Derivations

- Some person left
  basic (2nd order):
  \[ \rho_S \left( \rho_{DP} \left( \rho_{NP_b} \rho_{NP} \right) \right) \]
  lifted (3rd order):
  \[ \rho_N \left( \lambda x_N . \rho_{NP_b} \left( \lambda x_{NP} . \rho_{DP} \left( x_{NP} x_N \lambda x_D . \rho_S x_D \right) \right) \right) \]

- Some person from every city left
  basic:
  \[ \rho_S \left( \rho_{DP} \left( \rho_{NP_a} \left( \rho_{NP_b} \rho_{DP} \left( \rho_{NP_b} \rho_{NP} \right) \right) \rho_{NP} \right) \right) \]
  lifted
  \[ \rho_{NP} \cdot \rho_{NP_b} \left( \lambda x_{np} . \rho_{DP} \left( x_{np} x_{np} \lambda x_d . \rho_{NP_b} \left( x_{np} x_{np} \lambda x_{np} . \rho_{NP}\left( \lambda x_{np} . \rho_{DP} \left( x_{np} x_{np} \lambda x_d . \rho_S x_d \right) \right) \right) \right) \right) \]
The Problem of Inverse Linking

Observation:

derivation order does not permit surface scope in:

- *every person from a city left.*
The Problem of Inverse Linking

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derivation order does not permit surface scope in:

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The problem:

the most deeply embedded quantifier cannot be introduced until *after* its ‘argument’ position is present, which is introduced only once its containing DP is present.
The Problem of Inverse Linking

Observation:

derivation order does not permit surface scope in:

- *every person from a city left.*

The problem:

the most deeply embedded quantifier cannot be introduced until after its ‘argument’ position is present, which is introduced only once its containing DP is present.

Example

Every derivation begins:

\[ \langle S \Rightarrow s(DP, v) \rangle^{-1} = \lambda x_D . \rho_S(x_D) \]
The Problem of Inverse Linking

Observation:

Derivation order does not permit surface scope in:

- every person from a city left.

The problem:

The most deeply embedded quantifier cannot be introduced until after its ‘argument’ position is present, which is introduced only once its containing DP is present.

Example

Every derivation begins:

\[ \langle S \Rightarrow s(DP, v) \Rightarrow s(d(det, NP'(NP)), v) \rangle^{-1} = \lambda x_{NP'} \ x_{N} \cdot \rho_{DP}(x_{NP'} \ x_{N} \lambda x_{D} \cdot \rho_{S}(x_{D})) \]
A timing based approach to quantifier scope simply cannot derive surface scope relations between embedded scope-takers: the scope relations predicted are always inverted.

\[
\begin{array}{c}
\rho_S \\
\mid \\
\rho_{DP} \\
\rho_{NP} \\
\rho_{NP_a}' \\
\rho_{NP_b} \\
\rho_{NP_b}' \\
\rho_{NP}' \\
\rho_{NP}' \\
\end{array}
\]

The possible derivations for \( t \) are given by \( h(t) \), where

\[
h(\sigma(t_1, \ldots, t_n) := \sigma \cdot \biguplus \{h(t_1), \ldots, h(t_n)\}
\]

\[
\rho_S \cdot (\rho_{DP} \cdot (\rho_{NP} \sqcup (\rho_{NP_a}' \sqcup (\rho_{NP_b}' \sqcup (\rho_{NP}' \sqcup (\rho_{NP}' \sqcup (\rho_{NP_b}')))))))
\]
Barker wants scope relations such that $\rho_{DP} < \rho_{DP}$.

There is no derivation which provides this.

No system which derives this can be expressed in terms of derivations.

Whatever people mean by “timing”

it is not describable in terms of rewriting order.
We simply lifted all of our types
  because we wanted to look at every possible derivation order
Barker only allows for cosubstitution of certain initial trees (the DPs)
  this corresponds to us only lifting types ending in DP
  the resulting signature has \textit{nothing} to do with derivations
Selectively Lifting Barker’s TAG

Only types ending in $DP$: 

$$
\rho_S : DP \rightarrow S \\
\rho_{DP} : NP' \rightarrow NP \rightarrow DP \\
\rho_N : NP \\
\rho_{NP'_a} : NP' \rightarrow DP \rightarrow NP' \\
\rho_{NP'_b} : NP'
$$
Selectively Lifting Barker’s TAG

Only types ending in $DP$:

$$
\rho_S : DP \rightarrow S
$$

$$
\rho_{DP} : NP' \rightarrow NP \rightarrow DP
$$

$$
\rho_N : NP
$$

$$
\rho_{NP'_a} : NP' \rightarrow DP \rightarrow NP'
$$

$$
\rho_{NP'_b} : NP'
$$
Selectively Lifting Barker’s TAG

Only types ending in $DP$:

\[ \rho_S : DP \to S \]
\[ \rho_{DP} : NP' \to NP \to DP \]
\[ \rho_N : NP \]
\[ \rho_{NP''_a} : NP' \to DP \to NP' \]
\[ \rho_{NP''_b} : NP' \]
Selectively Lifting Barker’s TAG

Only types ending in $DP$:

- $\rho_S : DP \rightarrow S$
- $\rho_{DP} : NP' \rightarrow NP \rightarrow (DP \rightarrow S) \rightarrow S$
- $\rho_N : NP$
- $\rho_{NP_\alpha} : NP' \rightarrow DP \rightarrow NP'$
- $\rho_{NP_\beta} : NP'$
Only types ending in \( DP \):

- \( \rho_S : DP \rightarrow S \)
- \( \rho_{DP} : NP' \rightarrow NP \rightarrow (DP \rightarrow S) \rightarrow S \)
- \( \rho_N : NP \)
- \( \rho_{NP'_a} : NP' \rightarrow DP \rightarrow NP' \)
- \( \rho_{NP'_b} : NP' \)
Selectively Lifting Barker’s TAG

Only types ending in $DP$:

\[ \rho_S : DP \to S \]
\[ \rho_{DP} : NP' \to NP \to (DP \to S) \to S \]
\[ \rho_N : NP \]
\[ \rho_{NP'_a} : NP' \to DP \to NP' \]
\[ \rho_{NP'_b} : NP' \]
Selectively Lifting Barker’s TAG

Only types ending in $DP$:

$$\rho_S : DP \to S$$
$$\rho_{DP} : NP' \to NP \to (DP \to S) \to S$$
$$\rho_N : NP$$
$$\rho_{NP'_a} : NP' \to DP \to NP'$$
$$\rho_{NP'_b} : NP'$$
Selectively Lifting Barker’s TAG

Only types ending in \( DP \):

\[
\begin{align*}
\rho_S &: DP \rightarrow S \\
\rho_{DP'} &: NP' \rightarrow NP \rightarrow (DP \rightarrow S) \rightarrow S \\
\rho_N &: NP \\
\rho_{NP'_a} &: NP' \rightarrow DP \rightarrow NP' \\
\rho_{NP'_b} &: NP'
\end{align*}
\]
Selectively Lifting Barker’s TAG

Only types ending in $DP$:

$$\rho_S : DP \rightarrow S$$

$$\rho_{DP} : NP' \rightarrow NP \rightarrow (DP \rightarrow S) \rightarrow S$$

$$\rho_N : NP$$

$$\rho_{NP_a} : NP' \rightarrow DP \rightarrow NP'$$

$$\rho_{NP_b} : NP'$$
Selectively Lifting Barker’s TAG

Only types ending in \( DP \):

\[
\begin{align*}
\rho_S &: DP \to S \\
\rho_{DP} &: NP' \to NP \to (DP \to S) \to S \\
\rho_N &: NP' \\
\rho_{NP'_a} &: NP' \to DP \to NP' \\
\rho_{NP'_b} &: NP'
\end{align*}
\]
We *still* don’t get the surface reading!

- but this time its easy to fix:

**Barker’s solution**

allow a version of rule $\rho_{NP_a}$ to directly select for a continuized $DP$:

$$\rho'_{NP_a} := NP' \to ((DP \to S) \to S) \to NP$$

**A lexical solution**

introduce new atomic type $XP$, and duplicate rules as appropriate:

$$XP \to d(det, NP'(NP)) \quad \rho_{XP}$$

$$NP'(x) \to NP'(np(x, pp(p, XP))) \quad \rho'_{NP_c}$$

lexically interpret $L(XP) = L((DP \to S) \to S)$
Conclusions

Cotags

- leave the realm of context-free grammar formalisms
- WGC and SGC preservation
  - NOT because IO and OI coincide on linear CFTGs
  - instead, because spellout involves going back through the 2\textsuperscript{nd} order derivation term
- Complexity heuristics in ACG differ from those of TAG:
  - ‘simpler’ to deal with in-situ readings by making lexicon higher order than the abstract language . . . (??)
Conclusions

Timing

- intuitive and oft-occurring
- not related to rewriting strategies
  (we may have been the only ones to have thought this...)
- best seen as moving from trees to (3rd order) terms
Thank you!