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THREE TYPES OF POLARITY

1. INTRODUCTION

There can be no doubt that the phenomenon of polarity, though usually the subject of syntactic and semantic study, is essentially of a purely lexical nature.[1] This is evident to anyone who is familiar with the distribution of so-called negative polarity items. The fact that expressions such as *hoeven* and *ook maar iets* in Dutch, *brauchen* and *auch nur irgendetwas* in German, or the English cognates *need* and *anything (at all)* require the presence of a negative element somewhere in the sentence, is a property which is intrinsic to the items in question and must therefore be accounted for in the lexicon. If there is any doubt as to the lexical nature of this phenomenon, it is completely eradicated by the distinction between negative polarity items of the weak and those of the strong type. In order to get a clear view of the content of this distinction, one does well to take the following Dutch examples into consideration.

(1) a At most one child will himself need to justify *Hoogstens* verantwoorden.

één kind zal zich hoeven te

'At most one child need justify himself.'

b *Niemand zal zulk een beproeving hoeven te doorstaan.*
No one will such an ordeal need to go through
'No one need go through such an ordeal.'

c *Weinig handelsreizigers blijken hem te kunnen velen.*
Few salesmen appear him to can abide
'It appears that few salesmen can abide him.'

d *Geen van de leerlingen schijnt haar te kunnen velen.*
None of the students seems her to can abide
'It seems that none of the students can abide her.'

(2) a **Hoogstens zes kinderen hebben ook maar ietsflbemerkt.*
At most six children have anything noticed
'At most six children noticed anything.'

b *Niemand heeft van de regenbui ook maar iets bemerkt.*
No one has of the rain anything noticed
'No one noticed anything of the rain.'

F. Hamm and E. Hinrichs, (eds.), Plurality and Quantification, 177–238.

c **Weinig ouders hebben bijster veel brieven ontvangen.*
Few parents have very many letters received
'Few parents received very many letters.'

d *Geen van de kooplieden toonde zich bijster tevreden.*
None of the merchants showed himself very content
'None of the merchants showed himself very content.'

The contrast between (1) and (2) makes it clear that expressions such as *ook maar iets* and *bijster* place stronger restrictions on their environments than the negative polarity items *hoeven* and *kunnen velen.*[2] We must not suppose that this is a peculiar feature of Dutch, for precisely the same pattern can be found in German, as shown by the sentences in (3) and (4).[3]

(3) a At most one woman will herself to justify need *Höchstens eine*
brauchen.
Frau wird sich zu verantworten

'At most one woman need justify herself.'

b No one will such an ordeal to go through need *Keiner wird*
brauchen.
solch eine Prüfung durchzustehen

'No one need go through such an ordeal.'

c *Nur wenige Kaufleute scheinen dich ausstehen zu können.*
Only a few merchants seem you stand to can
'It seems that only a few merchants can stand you.'

d None of the deputies seems her stand to can *Keine der*
können.
Abgeordneten scheint sie ausstehen zu

'It seems that none of the deputies can stand her.'

(4) a noticed**Höchstens zehn Kinder haben auch nurflirgendetwas*
At most ten children have anything
bemerkt.

'At most ten children noticed anything.'

b *Keiner von diesen Leuten hat auch nur irgendetwas bemerkt.*
None of these people has anything noticed
'None of these people noticed anything.'

c Only a few merchants have very content been*Nur wenige
gewesen.

Kaufleute sind sonderlich zufrieden

'Only a few merchants have been very content.'

d *Kein einziger Lehrer ist sonderlich freundlich gewesen.*
Not one teacher has very friendly been
'Not one teacher has been very friendly.'

Other examples illustrating this remarkable division within the class of negative polarity items include idiomatic expressions such as *lift a finger* (G: *einen Finger rühren*, D: *er een vinger naar uitsteken*), *utter a sound* (G: *einen Muckser von sich geben*, D: *een kik geven*), and *bat an eyelash* (G: *mit der Wimper zucken*, D: *een spier vertrekken*). As the sentences in (5) indicate, such phrases are not satisfied with the presence of a negative constituent of the form *at most n N*. Instead, they require a more prominent negation such as *none of the N* or *none of the n N*.

(5) a None of the fifteen students lifted a finger.
b *At most eighteen porters will lift a finger.
c None of the sixteen children uttered a sound.
d *At most five representatives uttered a sound.
e None of the seven children batted an eyelash.
f *At most eighteen children batted an eyelash.

In German and Dutch, similar patterns can be attested. Parallel to the examples in (5), we find the contrasting sentences in (6) and (7).

(6) a *Keiner der Athleten hat einen Finger gerührt.*
None of the athletes has a finger moved
'None of the athletes lifted a finger.'

b At most one woman has a finger moved**Höchstens eine Frau*
gerührt.

hat einen Finger

'At most one woman lifted a finger'.

c *Kein Junge hat einen Muckser von sich gegeben.*
No boy has a sound of himself given
'No boy uttered a sound.'

d Only few may a sound of themselves give**Nur wenige mögen*
geben.
einen Muckser von sich

'Only a few may utter a sound.'

e *Keiner der Männer hat mit der Wimper gezuckt.*
None of the men has with the eyelash drawn
'None of the men batted an eyelash.'

f **Wenige Athleten haben mit der Wimper gezuckt.*
Few athletes have with the eyelash batted
'Few athletes batted an eyelash.'

(7) a *Niemand heeft er een vinger naar uitgestoken.*
No one has it a finger to stretch out
'No one lifted a finger.'

b **Weinigen zullen er een vinger naar uitsteken.*
Few will it a finger to stretch out
'Few people will lift a finger.'

c *Geen van de aanwezigen heeft een kik gegeven.*
None of the present has a sound given
'None of those present uttered a sound.'

d **Hoogstens zes bedienden zullen een kik geven.*
At most six servants will a sound give
'At most six servants will utter a sound.'

e *Geen van de ukken heeft een spier vertrokken.*
None of the toddlers has a muscle moved
'None of the toddlers batted an eyelash.'

f **Slechts zes ukken mogen een spier vertrekken.*
Only six toddlers may a muscle move
'Only six toddlers may bat an eyelash.'

This state of affairs immediately raises the question as to how such differences should be accounted for. In what follows, I will show that we need to make a distinction between subminimal and minimal negation. Although this difference may not at first seem clear, it finds its origins in the indisputable fact that noun phrases of the forms *no one* and *none of the merchants* embody a stronger type of negation than those of the forms *at most six children* and *few parents.* This becomes apparent when we compare the logical behavior of the expressions in question with that of the sentential prefix *it is not the case that.* By way of illustration we consider two examples.

(8) a It is not the case that Jack ate or Jill ran $\leftrightarrow$
It is not the case that Jack ate and it is not the case that Jill ran

b It is not the case that Jack ate and Jill ran $\leftrightarrow$
It is not the case that Jack ate or it is not the case that Jill ran

One sees immediately that the biconditionals in (8) must both be accepted as valid – a state of affairs which admits no other explanation than that the operation in question is governed by the laws of De Morgan.[4] This observation is important because it has frequently been argued that the logical patterns in (8) characterize the use of negation. Although such a conclusion is correct with respect to the sentential prefix *it is not the case that* and the negative adverb *not*[5], it must be regarded as misleading when it comes to other forms of negation. Not only does natural language contain a variety of negative expressions, their logical behavior is also not the same. In order to convince ourselves of this simple fact, we consider the conditionals in (9).

(9) a Few trees will blossom or will die $\rightarrow$
Few trees will blossom and few trees will die

b Few trees will blossom and few trees will die $\not\rightarrow$
Few trees will blossom or will die

c Few trees will blossom and will die $\not\rightarrow$
Few trees will blossom or few trees will die

d Few trees will blossom or few trees will die $\rightarrow$
Few trees will blossom and will die

From these examples it is clear that the phrase *few trees*, considered as a negative expression, differs substantially from the sentential prefix *it is not the case that.* Of the four conditionals presented in (9), only two are valid: the one in (9a) and the one in (9d). In other words, the logical behavior of noun phrases of the form *few N* is governed by one half of the first law of De Morgan and one half of the second law of De Morgan. In this regard, they are by no means alone, for it requires little reflection to realize that noun phrases of the forms *at most n N*, *not all N*, *only a few N* and *no more than n N* behave in much the same way. What this suggests is that the expressions in question embody a weak form of negation. For that reason they will henceforth be referred to as expressions of subminimal negation – a name which is borrowed from that part of the classical propositional calculus known as subminimal logic.[6]

It turns out that there exists, in fact, a whole hierarchy of negative expressions. For not only do we have phrases of the forms *few N* and *at most n N*, but we also find cases such as *no N*, *none of the N* and *no one.* The latter category differs from the former in that it expresses a stronger form of negation. The following conditionals provide an illustration.

(10) a No man escaped or got killed $\rightarrow$
No man escaped and no man got killed

b No man escaped and no man got killed $\rightarrow$
No man escaped or got killed

c No man escaped and got killed $\not\rightarrow$
No man escaped or no man got killed

d No man escaped or no man got killed $\rightarrow$
No man escaped and got killed

From these examples we may conclude that the noun phrase *no man*, regarded as a negative expression, differs considerably from *few trees.* Of the four conditionals presented in (10), no less than three must be accounted valid: the one in (10a), the one in (10b), and the one in (10d). What this means is that the logical behavior of noun phrases of the form *no N* is determined by the first law of De Morgan as a whole and one half of the second law of De Morgan. We must not suppose that this is a mere accident, for it is easy to see that the property in question also holds of noun phrases of the forms *none of the N*, *neither N* and *no one.* The conclusion must therefore be that expressions of this type embody a stronger form of negation than phrases like *few N* and *at most n N.* For that reason they will henceforth be referred to as expressions of minimal negation.[7]

In order to complete the hierarchy of negation, we must return to the biconditionals in (8). The validity of these examples makes it clear that the sentential prefix *it is not the case that* is governed by both the first and the second law of De Morgan. In the same way, the negative adverb *not* can also be shown to obey the two laws of De Morgan. What this means is that such elements express an even stronger form of negation than noun phrases like *no N* and *none of the N*. As a matter of fact, they will henceforth be referred to as expressions of classical negation.

With this apparatus at our command, we can explain the patterns in (1) through (7). For it is easily established that the distinction between weak and strong forms of negative polarity can be reduced to that between subminimal and minimal forms of negation. By way of illustration we give here a formulation of the laws which govern the occurrence of negative polarity items.

(11) **Laws of negative polarity**

a Only sentences in which an expression of subminimal negation occurs, can contain a negative polarity item of the weak type.

b Only sentences in which an expression of minimal negation occurs, can contain a negative polarity item of the strong type.

According to the first law, the presence of a subminimal negation is a necessary condition for the appearance of negative polarity items of the weak type. On the other hand, the second law stipulates that negative polarity items of the strong type require the presence of a minimal negation. To forestall any misunderstanding, we note that every expression of minimal negation is also an expression of subminimal negation.[8] In order to get a clear view of the domain of application of both laws, one does well to take the following examples into consideration.

(12) a *Niemand schijnt ook maar iets te hebben ondernomen.*
No one seems anything to have undertaken
'No one seems to have undertaken anything.'

b At most eight arrows have anything hit**Hoogstens acht*
geraakt.
pijlen hebben ook maar iets

'At most eight arrows hit anything.'

c *Geen van de geleerden toonde zich bijster tevreden.*
None of the scientists showed himself very content

'None of the scientist showed himself very content.'

d **Weinig ouders hebben bijster veel giften ontvangen.*
Few parents have very many gifts received
'Few parents received very many gifts.'

The contrast between (12a) and (12c), on the one hand, and (12b) and (12d), on the other, proves that the presence of a subminimal negation is not sufficient to justify the occurrence of the negative polarity items *ook maar iets* and *bijster.* Apparently, it is only expressions such as *niemand* and *geen van de geleerden* that are capable of licensing the polarity items in question. That this is by no means a coincidence, is shown by the German sentences in (13).

(13) a None of the merchants has anything undertaken
Keiner der Kaufleute hat auch nur irgendetwas
unternommen.
'None of the merchants undertook anything.'

b noticed**Höchstens sechs Eltern haben auch nurflirgendetwas*
At most six parents have anything
bemerkt.

'At most six parents noticed anything.'

Again, we see that the presence of the subminimal negation *höchstens sechs Eltern* is not sufficient to legitimize the occurrence of *auch nur irgendetwas.* In the same way, the examples in (14) and (15) clearly show that the Dutch negative polarity item *noemenswaardig* and its German counterpart *nennenswert* are incompatible with the weak negations *hoogstens zes ouders* ('at most six parents') and *höchstens sieben Athleten* ('at most seven athletes'). Instead they require the presence of a stronger form of negation – *geen van de wezen* ('none of the orphans') in (14a) and *keiner der anderen Athleten* ('none of the other athletes') in (15a).[9]

(14) a *Geen van de wezen heeft noemenswaardige verliezen*
None of the orphans has appreciable losses
geleden.
suffered
'None of the orphans has suffered any appreciable loss.'

b At most six parents obtain an appreciable result*Hoogstens resultaat.

zes ouders behalen een noemenswaardig

'At most six parents will obtain any appreciable result.'

(15) a None of the other athletes was him appreciably better
Keiner der anderen Athleten war ihm nennenswert überlegen.

'None of the other athletes was appreciably better than him.'

b At most seven athletes were him appreciably better
**Höchstens sieben Athleten waren ihm nennenswert* überlegen.

'At most seven athletes were appreciably better than him.'

Similarly, the contrast between the Dutch examples in (16a) and (16b) makes it clear that, of the negative phrases *niet één rechercheur* ('not one detective') and *niet alle kruiers* ('not all porters'), only the first can act as a licencing expression for *ook maar iets.*

(16) a Not one detective will anything accomplish *Niet* bewerkstelligen.

één rechercheur zal ook maar iets

'Not one detective will accomplish anything.'

b Not all porters will anything accomplish**Niet* bewerkstelligen.

alle kruiers zullen ook maar iets

'Not all porters will accomplish anything.'

As a final illustration of the difference between weak and strong negation, we consider the sentences in (17).

(17) a *Niet één leerling toonde zich bijster tevreden.*
Not one student showed himself very content

'Not one student showed himself very content.'

b Only one mother shows herself very content**Slechts één* tevreden.

moeder toont zich bijster

'Only one mother shows herself very content.'

The fact that the occurrence of *bijster* in (17b) leads to an unacceptable result entails that the negative expression *slechts één moeder* does not possess the same properties as *niet één leerling.*

Although these patterns may well seem perplexing at first, in the light of the distinction between subminimal and minimal negation they admit only one explanation: the class of expressions which are capable of licensing the occurrence of the negative polarity items *ook maar iets* and *bijster* is coextensive with the class of minimal negations. This conclusion is corroborated in a surprising manner by the findings of Hoppenbrouwers (1983: 128). From his study of the different uses of negative polarity items in Dutch, it appears that expressions such as *een snars* ('a thing'), *een zier* ('one bit') and *in de verste verte* ('in the slightest') display more or less the same characteristics as *ook maar iets* and *bijster.* As an illustration we consider the following sentences.

(18) a *Geen van de leerlingen heeft er een snars van begrepen.*
None of the students has it a thing of understood
'None of the students understood a thing.'

b At most eight parents have it a thing of understood
**Hoogstens acht ouders hebben er een snars van* begrepen.
'At most eight parents understood a thing.'

c Not one representative will it a bit for feel *Niet één* voelen.
vertegenwoordiger zal er een zier voor
'Not one representative will at all be interested.'

d Only four merchants will it a bit for feel**Slechts vier* voelen.
kooplieden zullen er een zier voor
'Only four merchants will at all be interested.'

e *Niemand heeft de kinderen in de verste verte overtuigd.*
No one has the children in the slightest convinced
'No one has convinced the children in the slightest.'

f Not every speech has him in the slightest convinced
**Niet elk betoog heeft hem in de verste verte* overtuigd.

'Not every speech has convinced him in the slightest.'

Despite the fact that in each of these sentences a negative subject occurs, it is only the minimal negations *geen van de leerlingen*, *niet één vertegenwoordiger* and *niemand* that lead to an acceptable result. In view of the lawlike character of this pattern, it is reasonable to equate the class of licencing expressions for *een snars*, *een zier* and *in de verste verte* with the class of minimal negations.

This state of affairs forces us to make an absolute distinction between negative polarity items of the weak type and those of the strong type. The difference finds its origin in the fact that phrases like *hoeven* and *kunnen velen* place no other restriction on the licencing expression than that it belong to the class of subminimal negations. Negative polarity items which exhibit such behavior will invariably be regarded as weak.

That this class has more members than the expressions *hoeven* and *kunnen velen* alone, is obvious from the work of Hoppenbrouwers (1983). On the basis of his findings, we must conclude that the different uses of phrases as *kunnen schelen* ('care'), *kunnen luchten of zien* ('can stand') and *laten gezeggen* ('be gainsaid') show remarkable similarities with those of *hoeven* and *kunnen velen*. In this regard, the next nine sentences speak for themselves.

(19) a *Het kan geen van de leerlingen iets schelen.*
It can none of the students anything care
'None of the students cares a bit.'

b *Weinig leerlingen kan het echt iets schelen.*
Few students can it really anything care
'Few students really care a bit.'

c **Het kan de meeste onderwijzers iets schelen.*
It can most teachers anything care
'Most teachers care a bit.'

d *Niet één echtgenoot kan hem luchten of zien.*
Not one spouse can him abide
'Not one spouse can abide him.'

e *Slechts één docente kan hem luchten of zien.*
Only one teacher can him abide
'Only one teacher can abide him.'

f **Sommige atleten kunnnen hem luchten of zien.*
Some athletes can him abide
'Some athletes can abide him.'

g *Geen van de vrouwen laat zich iets gezeggen.*
None of the women let herself be gainsaid
'None of the women will be gainsaid.'

h *Niet alle rechters laten zich iets gezeggen.*
Not all judges let themselves be gainsaid
'Not all judges will be gainsaid.'

i **De meeste kinderen laten zich iets gezeggen.*
Most children let themselves be gainsaid
'Most children will be gainsaid.'

From this collection of examples we may deduce that in principle every subminimal negation is capable of acting as a licencing expression for verbs such as *kunnen schelen*, *kunnen luchten of zien* and *laten gezeggen*. Consequently, these phrases must be regarded as negative polarity items of the weak type. In that regard, they clearly differ from expressions like *een snars*, *een zier* and *in de verste verte*, for these can only appear if somewhere in the sentence a minimal negation is present. This means that such constituents place significantly stronger demands on the licencing expression, which is the reason why they will henceforth be referred to as negative polarity items of the strong type.

In order to prevent losing track of these patterns, the available information has been collected in table 1. Although the stock of polarity items surely comprises more than the eighteen expressions mentioned there, it is indisputable that the distinction between weak and strong forms of negative polarity bears a lawlike character. Expressions of the first category are content with a subminimal negation as licencing element, those of the second category require the presence of a minimal negation somewhere in the sentence. It is this opposition that forms the foundation of the laws stated in (11).

Table 1: Eighteen negative polarity items in Dutch, with occasional counterparts in German or English

Weak	**Strong**
hoeven (**G**: brauchen, **E**: need)	ook maar iets (**G**: auch nur irgendetwas, **E**: anything (at all))
kunnen velen (**E**: can abide)	bijster (**G**: sonderlich)
kunnen uitstaan (**E**: can stand, **G**: ausstehen können)	een snars (**E**: a thing)
kunnen schelen (**E**: care)	een zier (**E**: one bit)
kunnen luchten of zien (**G**: ausstehen können, **E**: can stand)	in de verste verte (**G**: im entferntesten, **E**: in the slightest)
laten gezeggen (**G**: sagen lassen, **E**: be gainsaid)	noemenswaardig (**G**: nennenswert)
een oog dichtdoen (**G**: ein Auge zumachen, **E**: sleep a wink)	een spier vertrekken (**G**: mit der Wimper zucken, **E**: bat an eyelash)
een vlieg kwaad doen (**E**: hurt a fly)	een kik geven (**E**: utter a sound, **G**: einen Mucks von sich geben)
er veel mee op hebben (**G**: viel davon halten)	er een vinger naar uitsteken (**G**: einen Finger rühren, **E**: lift a finger)

We must not suppose that our description exhausts the matter. The distinction between weak and strong forms of negative polarity is, as we have seen, a peculiarity which is intrinsic to the expressions in question and must therefore be accounted for in the lexicon. In particular, this entails that phrases such as *kunnen velen* and *kunnen uitstaan* are to be regarded as lexical items with respect to polarity. In that regard, they are by no means alone, for on precisely the same grounds expressions like *kunnen schelen* and *kunnen luchten of zien* must also be considered genuine lexical units. This view of the matter is, indeed, corroborated by the fact that the final verb cluster in Dutch has become petrified in a number of cases. For it is well known that verbs such as *welgevallen* ('befall'), *gezeggen* ('gainsay') and *velen* ('abide') can only occur in the infinitival form, as shown by the examples in (20).

(20) a *U stelt dat sommigen anderen niet kunnen velen.*
You state that some others not can abide
'You state that some cannot abide others.'

b *Zij ontkennen dat hij zich niets laat gezeggen.*
They deny that he himself not let be gainsaid
'They deny that he will not be gainsaid.'

c *U zegt dat Ot zich het vonnis liet welgevallen.*
You say that Ot himself the verdict let befall
'You say that Ot let the verdict befall to him.'

Not only are such sentences incompatible with the notion of V-Raising, originally defended in Evers (1975), but they also show that the behavior of the Dutch verb cluster is in many cases purely lexically determined. It turns out that this is an important observation, among other things, because it suggests a lexicalist solution to the problem of the Dutch verb cluster – a possibility which is also mentioned in Pullum and Gazdar (1982: 501).[10]

Besides the two types of negative polarity discussed so far, there exists a third type which we will refer to as *superstrong* polarity. Elements belonging to this class include the English expression *one bit* and the Dutch adjective *mals* 'tender'. As the ungrammatical examples in (21) and (22) show, such phrases are not content with a subminimal or minimal negation. Instead, they require the presence of the negative adverb *not* (D: *niet*) somewhere in the sentence.[11]

(21) a *Few people were one bit happy about these facts.

b *No linguist was one bit happy about these facts.

c The men weren't one bit happy about these facts.

(22) a **Weinig van zijn oordelen waren mals.*
Few of his opinions were tender
'Few of his opinions were soft.'

b **Niet één van zijn oordelen was mals.*
Not one of his opinions was tender
'Not one of his opinions was soft.'

c *Zijn oordelen waren vaak niet mals.*
His opinions were often not tender
'His opinions often weren't soft.

In terms of the hierarchy of negation expressions, the restrictions on the occurrence of phrases like *one bit* may be described as showing that superstrong polarity items require the presence of an expression of classical negation. Consequently, the laws which govern the occurrence of negative polarity items should be modified as follows.

(23) **Laws of negative polarity**

a Only sentences in which an expression of subminimal negation occurs, can contain a negative polarity item of the weak type.

b Only sentences in which an expression of minimal negation occurs, can contain a negative polarity item of the strong type.

c Only sentences in which an expression of classical negation occurs, can contain a negative polarity item of the superstrong type.

According to the third law, the presence of a classical negation is a necessary condition for the appearance of superstrong polarity items. To forestall any misunderstanding, we note that every expression of classical negation is also an expression of minimal negation.

The remainder of this article can be summarized as follows. In sections 2, 3, 4 and 5, we expound the distinction between subminimal, minimal and classical negation, using the algebraic notions of a monotonic quantifier, a quasi-filter, a quasi-ideal, an ultrafilter and a prime ideal. Section 6 introduces the corresponding functional perspective and discusses in these terms the notion of a monotonic function. Anti-additive and antimorphic functions are discussed in section 7. Finally, in section 8, the laws governing the use of negative polarity items are formulated.

2. TWO TYPES OF MONOTONICITY

In order to get a clear view of the distinction between subminimal and minimal negation, one does well to take the following example into consideration.

(24) At most one villager sang.

At most one villager sang loudly.

Provided that the predicate *sang loudly* applies only to what the predicate *sang* also applies to, the conditional in (24) must be regarded as true. To put it another way, if the state of affairs in the universe is such that the class of individuals who sang loudly is a subset of the class of individuals who sang, then we may legitimately pass from the proposition *At most one villager sang* to *At most one villager sang loudly.* Clearly, this raises the question of how to account for such inferences.

We assume that each verb phrase will receive a subset of some universe U as its semantic value. Such a way of portraying the matter entails that the universe of possible denotations of verb phrases may henceforth be equated with $P(U)$ – that is, the first power set of U. Since the collection $P(U)$ with the usual set-theoretical operations of union, intersection, and complementation can be regarded as a Boolean algebra, we shall from now on speak of the **algebra of verb phrases.**[12] We assume as well that each noun phrase receives a collection of subsets of U as its semantic value. This implies that the universe of possible denotations of noun phrases may be equated with $P(P(U))$ – that is to say, the second power set of U. Since this set also displays the characteristic features of a Boolean algebra, it is said to be the **algebra of noun phrases**.

The algebraic nature of the categories NP and VP enables us to give a precise formulation of a notion which is frequently used in the linguistic literature – to wit, that of a quantifier. Henceforth, what is meant by **a quantifier on a Boolean algebra *B*** is simply a subset of B. Such a definition immediately explains why it is natural to regard noun phrases as quantifiers. For if these expressions receive a collection of subsets of U as their semantic value, they can semantically be equated with a subset of $P(U)$ – that is to say, with a subset of the algebra of verb phrases. However, this entails that noun phrases could just as well be regarded as quantifiers on the algebra of verb phrases.

It is now possible to say how the interpretation of an expression like *at most one villager* differs from that of other noun phrases. For it is easily seen that each set which contains at most one villager, is a member of the quantifier associated with the expression *at most one villager*. This turns out to be important because it entails that the quantifier in question has the property that the conditions $X \in Q$ and $Y \subseteq X \subseteq U$ imply $Y \in Q$. Such quantifiers are commonly referred to as **monotone decreasing quantifiers.**[13] For the sake of clarity we record this in the form of a definition.

(25) **Definition**

Let B be a Boolean algebra. A quantifier Q on B is said to be **monotone decreasing** iff for each two elements X and Y of the algebra B:

if $X \in Q$ and $Y \subseteq X$, then $Y \in Q$.

Noun phrases which invariably receive a monotone decreasing quantifier on the VP-algebra as their semantic value will henceforth be called monotone decreasing noun phrases.

From the fact that the conditional in (24) is valid when the predicate *sang* is true of whatever the predicate *sang loudly* is true of, it follows immediately that noun phrases of the form *at most n N* are monotone decreasing. Similarly, one easily proves that expressions of the forms *not every N*, *no N*, *neither N* and *none of the n N* also have the property of downward monotonicity. For if the predicate *ate fish* only applies to what the predicate *ate* also applies to, then the conditionals in (26) are all valid.

(26)	a	Not every clergymen ate	→	Not every clergymen ate fish
	b	No federal attorney ate	→	No federal attorney ate fish
	c	Neither connoisseur ate	→	Neither connoisseur ate fish
	d	None of the six men ate	→	None of the six men ate fish

Meanwhile, the suspicion arises that noun phrases of the monotone decreasing type have a counterpart. It turns out that this is indeed the case. To pave the way, we begin by considering an example.

(27) At least one villager sang loudly → At least one villager sang

Provided that the predicate *sang* applies to whatever the predicate *sang loudly* applies to, the conditional in (27) must be accounted true. In other words, if the state of affairs in the universe is such that the class of individuals who sang loudly is a subset of the class of individuals who sang, then we may legitimately pass from the proposition *At least one villager sang loudly* to *At least one villager sang.* What this means, is that the quantifier associated with the expression *at least one villager* has the property of being closed under extension: if $X \in Q$ and $X \subseteq Y \subseteq U$, then $Y \in Q$. Such quantifiers are usually called **monotone increasing quantifiers**. For the sake of clarity we give the following definition.

(28) **Definition**

Let B be a Boolean algebra. A quantifier Q on B is said to be **monotone increasing** iff for each two elements X and Y of the algebra B:

if $X \in Q$ and $X \subseteq Y$, then $Y \in Q$.

Noun phrases which invariably receive a monotone increasing quantifier as their semantic value, will accordingly be referred to as monotone increasing noun phrases.

From the fact that the conditional in (27) is valid when the predicate *sang loudly* applies only to what the predicate *sang* also applies to, it follows immediately that noun phrases of the form *at least n N* are monotone increasing. In an analogous manner, one easily proves that expressions of the forms *some N*, *all N*, *the n N* and *both N* are also endowed with the property of upward monotonicity. For if the predicate *ate* is true of whatever the predicate *ate fish* is true of, then the entailments in (29) are all valid.

(29)	a	Some porters ate fish	$\rightarrow$	Some porters ate
	b	All children ate fish	$\rightarrow$	All children ate
	c	The six nuns ate fish	$\rightarrow$	The six nuns ate
	d	Both lawyers ate fish	$\rightarrow$	Both lawyers ate

On the basis of such tests, one can usually arrive at rather trustworthy judgments concerning the presence of monotonicity properties. For the sake of clarity, the outcomes of these tests have been collected in table 2. The forty-four classes of noun phrases mentioned there must all be regarded as being either upward or downward monotonic. Do not suppose that this exhausts the matter, for a more accurate analysis shows that there are several alternative ways to determine whether a given noun phrase possesses the property of downward monotonicity. This turns out to be a consequence of the fact that monotone decreasing quantifiers can be given a number of equivalent characterizations. The next theorem provides the details.

Table 2: Forty-four monotonic noun phrases, with their Dutch counterparts

Monotone increasing	**Monotone decreasing**
every N (**D**: ieder(e) N)	not every N (**D**: niet ieder(e) N)
all N (**D**: alle N)	not all N (**D**: niet alle N)
each N (**D**: elk(e) N)	not each N (**D**: niet elk(e) N)
some N (**D**: sommige N)	no N (**D**: geen N)
sm N (**D**: enkele N)[14]	only a few N (**D**: slechts enkele N)
nearly all N (**D**: vrijwel alle N)	almost no N (**D**: vrijwel geen N)
both N (**D**: beide N)	neither N (**D**: geen van beide N)
at least n N (**D**: minstens n N)	at most n N (**D**: hoogstens n N)
many N (**D**: veel N)	few N (**D**: weinig N)
several N (**D**: verscheidene N)	only n N (**D**: slechts n N)
more than n N (**D**: meer dan n N)	no more than n N (**D**: niet meer dan n N)
the n N (**D**: de n N)	none of the n N (**D**: geen van de n N)
the more than n N (**D**: de meer dan n N)	none of the more than n N (**D**: geen van de meer dan n N)
the N [pl] (**D**: de N [pl])	none of the N (**D**: geen van de N)
the N [sg] (**D**: de N [sg])	not a single N (**D**: geen enkel(e) N)
most N (**D**: de meeste N)	not one N (**D**: niet één N)
everything (**D**: alles)	not everything (**D**: niet alles)
something (**D**: iets)	nothing (**D**: niets)
everyone (**D**: iedereen)	not everyone (**D**: niet iedereen)
someone (**D**: iemand)	no one (**D**: niemand)
not only NP (**D**: niet alleen NP)	only NP (**D**: alleen NP)
proper names	negated proper names

(30) **Theorem**

Let B be a Boolean algebra. The following three statements about a quantifier Q on B are equivalent:

(a) Q is monotone decreasing;

(b) if $X \cup Y \in Q$ then $X \in Q$ and $Y \in Q$;

(c) if $X \in Q$ or $Y \in Q$, then $X \cap Y \in Q$.

With the aid of this result, one can give a precise account of the conditions which have to be fulfilled, if an expression is to be counted as belonging to the class of monotone decreasing noun phrases. The corollary below provides us with the relevant information.

(31) **Corollary**

A noun phrase is monotone decreasing iff the following schemata are logically valid:

(a) NP (VP_1 or VP_2) $\rightarrow$ (NP VP_1 and NP VP_2);

(b) (NP VP_1 or NP VP_2) $\rightarrow$ NP (VP_1 and VP_2).

The significance of these two schemata lies in the fact that each of them gives us both a positive and a negative test for downward monotonicity. Put differently, if one of the two schemata is valid, then the other is valid as well and, therefore, the noun phrase in question must be regarded as being monotone decreasing. If, however, one of the two schemata is invalid, then the other is invalid as well and, consequently, the relevant noun phrase does not possess the property of downward monotonicity. By way of illustration we consider some examples. It is clear, for instance, that the conditionals in (32) and (33) are both valid – a state of affairs which admits no other explanation than that expressions of the forms *few N* and *only NP* belong to the class of monotone decreasing noun phrases.

(32) Few hangmen complained or resisted $\rightarrow$
Few hangmen complained and few hangmen resisted

(33) Only judges resign or only judges get strangled $\rightarrow$
Only judges resign and get strangled

However, if we consider the conditionals in (34) and (35), then it is immediately clear that neither can be accepted as valid.

(34) More than nine nuns prayed or knelt $\not\rightarrow$
More than nine nuns prayed and more than nine nuns knelt

(35) More men than women escaped or died $\not\rightarrow$
More men than women escaped and more men than women died

For even though there may be more than nine nuns who prayed, there need not be any who knelt, in which case the antecedent of the conditional in (34) is true, but its consequent false. In an analogous manner, it is easily shown that the conditional in (35) is invalid. For if two man escaped and one woman died, then the antecedent is true, but the consequent false. We must therefore conclude that noun phrases of the forms *more than n N* and *more N_1 than N_2* are not monotone decreasing.

It should be pointed out in this connection that the class of monotone increasing quantifiers can be characterized in several alternative ways. In order to convince ourselves of this fact, we consider the following theorem.

(36) **Theorem**

Let B be a Boolean algebra. The following three statements about a quantifier Q on B are equivalent:

(a) Q is monotone increasing;
(b) if $X \cap Y \in Q$, then $X \in Q$ and $Y \in Q$;
(c) if $X \in Q$ or $Y \in Q$, then $X \cup Y \in Q$.

In this case, too, we can give a precise specification of the conditions which have to be fulfilled, if an expression is to be regarded as a monotone increasing noun phrase. The next corollary provides the necessary details.

(37) **Corollary**

A noun phrase is monotone increasing iff the following schemata are logically valid:

(a) NP (VP_1 and VP_2) $\rightarrow$ (NP VP_1 and NP VP_2);
(b) (NP VP_1 or NP VP_2) $\rightarrow$ NP (VP_1 or VP_2).

It is easy to see that each of the two schemata gives us both a positive and a negative test for upward monotonicity. As an illustration we note that the conditionals in (38) and (39) are both valid – a circumstance which, in view of the above result, admits of no other explanation than that proper names and definite descriptions are both monotone increasing.

(38) Jonathan called and begged $\rightarrow$
Jonathan called and Jonathan begged

(39) The girl sighed or the girl coughed $\rightarrow$
The girl sighed or coughed

On the other hand, if we take the conditionals in (40) and (41) into consideration, then it is immediately clear that neither can be regarded as valid.

(40) No fireman said good-bye and left $\not\rightarrow$
No fireman said good-bye and no fireman left

(41) Only women got whipped or only women starved $\not\rightarrow$
Only women got whipped or starved

For if there are precisely two firemen, one of whom said good-bye without leaving and one of whom left without saying goodbye, then the antecedent of the conditional in (40) is true, but its consequent false. In an analogous manner, one easily proves that the conditional in (41) is invalid as well. For if those who got whipped can all be regarded as being women, whereas those who starved not only include women, but also men, then the antecedent is true and the consequent false. This state of affairs leads to the conclusion that noun phrases of the forms *no N* and *only NP* are not monotone increasing.

3. QUASI-IDEALS AND QUASI-FILTERS

It would be wrong to suppose that a logical analysis of some depth can confine itself to the distinction between upward and downward monotonic noun phrases. A more accurate inspection shows that the members of each of these two classes exhibit substantial differences. In order to convince ourselves of this, we consider two examples.

(42) No flower will dry up or will fade $\rightarrow$
No flower will dry up and no flower will fade

(43) No flower will dry up and no flower will fade $\rightarrow$
No flower will dry up or will fade

It is clear that the validity of the conditional in (42) is a consequence of the monotone decreasing nature of the expression *no flower*. This explanation does not hold, however, for the reverse proposition in (43). The fact that we have again a valid implication, must instead be attributed to the circumstance that the monotone decreasing quantifier which is associated with *no flower*, is in addition closed under (finite) unions. Such quantifiers will henceforth be called quasi-ideals.[15] More precisely:

(44) **Definition**

Let B be a Boolean algebra. A quantifier Q on B is said to be a **quasi-ideal** iff for each two elements X and Y of the algebra B:

(a) if $X \cup Y \in Q$, then $X \in Q$ and $Y \in Q$;

(b) if $X \in Q$ and $Y \in Q$, then $X \cup Y \in Q$.

Noun phrases which invariably receive a quasi-ideal on the VP-algebra as their semantic value, will accordingly be referred to as quasi-ideals.

From the fact that the conditionals in (42) and (43) are both valid, it follows immediately that expressions of the form *no N* must be regarded as quasi-ideals. This is not to say that the property in question cannot be expressed in a different way. For it is well known that two conditionals, one of which is the reverse of the other, may be replaced equivalently by a biconditional – a state of affairs which entails that, as a proof of *no flower*'s being a quasi-ideal, we may as well point to the validity of the proposition in (45).

(45) No flower will dry up or will fade $\leftrightarrow$

No flower will dry up and no flower will fade

In view of this, it need not surprise us that, as an immediate corollary to the definition in (44), we have the result below.

(46) **Corollary**

A noun phrase is a quasi-ideal iff the following schema is logically valid:

NP (VP_1 or VP_2) $\leftrightarrow$ (NP VP_1 and NP VP_2).

With the aid of this test, it is easily shown that expressions of the forms *neither N* and *none of the N* are also to be regarded as quasi-ideals. To that end, it is sufficient to take the biconditionals in (47) and (48) into consideration.

(47) Neither musician laughs or coughs $\leftrightarrow$
Neither musician laughs and neither musician coughs

(48) None of the boys scoffs or curses $\leftrightarrow$
None of the boys scoffs and none of the boys curses

That both of these sentences have a valid character entails that the monotone decreasing quantifier which is associated with the subject, is in addition closed under (finite) unions. In other words, the two noun phrases in (47) and (48) act semantically as quasi-ideals.

Meanwhile, the suspicion arises that the notion of a quasi-ideal also has a counterpart. This is indeed the case. By way of illustration, we consider the following two conditionals.

(49) All monks rob and kill $\rightarrow$
All monks rob and all monks kill

(50) All monks rob and all monks kill $\rightarrow$
All monks rob and kill

There is no doubt that the validity of the conditional in (49) must be attributed to the monotone increasing nature of the expression *all monks.* This is not the case with the reverse proposition in (50). The fact that we have again a valid entailment, must instead be attributed to the circumstance that the monotone increasing quantifier which acts as the denotation of *all monks*, is also closed under (finite) intersections. Such quantifiers will henceforth be referred to as **quasi-filters** – a term which may not be customary within the theory of Boolean algebras, but which has gained some currency in modal logic.[16] For the sake of clarity we record this in the form of a definition.

(51) **Definition**

Let B be a Boolean algebra. A quantifier Q on B is said to be a **quasi-filter** iff for each two elements X and Y of the algebra B:

(a) if $X \cap Y \in Q$, then $X \in Q$ and $Y \in Q$;
(b) if $X \in Q$ and $Y \in Q$, then $X \cap Y \in Q$.

Noun phrases which are invariably interpreted by means of a quasi-filter on the VP-algebra, will accordingly be called quasi-filters.

The validity of the conditionals in (49) and (50) makes it clear that expressions of the form *all N* are to be regarded as quasi-filters. This is not to say that the property in question cannot manifest itself in a different way. The fact that (50) is the reverse of (49) entails that, as a proof of *all monks*' being a quasi-filter, we might as well have pointed to the validity of the biconditional in (52).

(52) All monks rob and kill $\leftrightarrow$
All monks rob and all monks kill

Once again, then, it need not surprise us that, as an immediate consequence of the definition in (51), we have the corollary below.

(53) **Corollary**

A noun phrase is a quasi-filter iff the following schema is logically valid:

NP (VP_1 and VP_2) $\leftrightarrow$ (NP VP_1 and NP VP_2).

With the aid of this test, one can easily show that proper names and expressions of the form *the n N* also display the characteristic features of quasi-filters. To that end it is sufficient to take the biconditionals in (54) and (55) into consideration.

(54) Themistocles mourns and moans $\leftrightarrow$
Themistocles mourns and Themistocles moans

(55) The nine men grieve and whine $\leftrightarrow$
The nine men grieve and the nine men whine

The fact that each of these biconditionals is valid admits no other explanation than that the monotone increasing quantifier associated with the subject is in addition closed under (finite) intersections. In other words, the two noun phrases in (54) and (55) semantically act as quasi-filters.

We must not suppose that every monotone increasing quantifier is a quasi-filter. Surely not, for a short inspection reveals numerous cases in which the entailment NP (VP_1 and VP_2) $\rightarrow$ (NP VP_1 and NP VP_2) is valid, but its reverse is not. The next two conditionals serve as an illustration.

(56) Some fishermen waved and called $\rightarrow$
Some fishermen waved and some fishermen called

(57) Some fishermen waved and some fishermen called $\not\rightarrow$
Some fishermen waved and called

Evidently, it follows from the monotone increasing nature of the noun phrase *some fishermen* that the conditional in (56) must be accepted as valid. On the other hand, we are by no means justified in passing from the proposition *Some fishermen waved and some fishermen called* to *Some fishermen waved and called.* For if there are two fishermen who waved and two other fishermen who called, then the antecedent of the implication in (57) is true, but its consequent false. This shows that expressions of the form *some N* cannot be regarded as quasi-filters.

By means of a parallel argument it is easily established that not every monotone decreasing quantifier can be classified as a quasi-ideal. Indeed, there are situations in which the conditional NP (VP_1 or VP_2) $\rightarrow$ (NP VP_1 and NP VP_2) is valid, but its reverse is not. The next two sentences are good examples of what we have in mind.

(58) Not all knights rob or kill $\rightarrow$
Not all knights rob and not all knights kill

(59) Not all knights rob and not all knights kill $\rightarrow$
Not all knights rob or kill

It is clear that the conditional in (58) must be accepted as valid in virtue of the downward monotonic nature of the expression *not all knights.* On the other hand, one cannot legitimately pass from the proposition *Not all knights rob and not all knights kill* to *Not all knights rob or kill.* For if the class of knights is such that one half robs and the other half kills, then the antecedent of the conditional in (59) is true, but its consequent false. This suffices to establish that expressions of the form *not all N* are not quasi-ideals.

On the basis of the available tests, we can usually arrive at rather trustworthy judgments when we must decide whether a given monotonic noun phrase is either a quasi-filter or a quasi-ideal. For ease of survey, the outcomes of these tests have been collected in table 3. The twenty-two classes of noun phrases which are mentioned there must all be regarded as being either quasi-filters or quasi-ideals.

Table 3: Twenty-two classes of quasi-filters and quasi-ideals, with their Dutch counterparts

Quasi-filters	**Quasi-ideals**
every N (**D**: ieder(e) N)	no N (**D**: geen N)
all N (**D**: alle N)	neither N (**D**: geen van beide N)
each N (**D**: elk(e) N)	none of the N (**D**: geen van de N)
both N (**D**: beide N)	none of the more than n N (**D**: geen van de meer dan n N)
the n N (**D**: de n N)	none of the N (**D**: geen van de N)
the more than n N (**D**: de meer dan n N)	not a single N (**D**: geen enkel(e) N)
the N [pl] (**D**: de N [pl])	not one N (**D**: niet een N)
the N [sg] (**D**: de N [sg])	nothing (**D**: niets)
everything (**D**: alles)	no one (**D**: niemand)
everyone (**D**: iedereen)	only NP (**D**: alleen NP)
proper names	negated proper names

4. CONSISTENCY AND COMPLETENESS

It is well-known that there are considerable differences between sentential negation, on the one hand, and predicate negation, on the other. Less known is the fact that the logical relationship between both forms of negation depends entirely on the semantical nature of the subject. In order to convince ourselves, we do well to take the following two examples into consideration.

(60) At least two willows do not flower $\rightarrow$
It is not the case that at least two willows flower

(61) Most weeping willows do not flower $\rightarrow$
It is not the case that most weeping willows flower

One sees immediately that the conditional in (60) cannot be regarded as valid. For if the state of affairs is such that of the seven willows only four happen to flower, then the antecedent is true, but the consequent false. On the other hand, it is evident that the conditional in (61) must be accepted as valid. If we are willing to accept the truth of the statement *Most weeping willows do not flower*, then we shall also have to acknowledge that it is not the case that the majority of weeping willows flowers. What this means is that the quantifier associated with the noun phrase *most weeping willows* is invariably **consistent** in nature – consistent in the sense that it cannot contain a given set of individuals as well as the complement of that set. For the sake of clarity we give the following definition.[17]

(62) **Definition**

Let B be a Boolean algebra. A quantifier Q on B is said to be **consistent** iff for each element X of the algebra B:

if $-X \in Q$, then $X \notin Q$.

Noun phrases which invariably receive a consistent quantifier as their semantic value, will accordingly be referred to as consistent noun phrases.

The property of consistency can be formulated in more than one way. Indeed, it is readily established that the law of contraposition allows us to replace the definition in (62) by the alternative characterization in (63).

(63) **Definition**

Let B be a Boolean algebra. A quantifier Q on B is said to be **consistent** iff for each element X of the algebra B:

if $X \in Q$, then $-X \notin Q$.

In spite of this equivalence, we prefer the definition in (62), primarily because it affords us a handy way of establishing the corollary below.

(64) **Corollary**

A noun phrase is consistent iff the following schema is logically valid:

(1) NP (NEG VP) $\rightarrow$ NEG (NP VP)

This simple result is important, because it clearly shows that with a consistent noun phrase as subject the use of predicate negation invariably entails sentence negation.

From the fact that the implication in (61b) is valid, it follows immediately that noun phrases of the form *most N* are consistent. Do not suppose that this exhausts the matter, for one easily proves that proper names and expressions of the forms *both N*, *the n N*, *the N [sg]* and *the N [pl]* must also be regarded as belonging to the class of consistent noun phrases. In the following examples, the use of predicate negation invariably entails sentence negation.

(65) a Themistocles does not mourn →
It is not the case that Themistocles mourns

b Both feet are not ulcerated →
It is not the case that both feet are ulcerated

c The seventeen donkeys do not bray →
It is not the case that the seventeen donkeys bray

d The scientists do not drink coffee →
It is not the case that the scientists drink coffee

e The shopkeeper does not waste time →
It is not the case that the shopkeeper wastes time

Similarly, one easily shows that expressions of the forms *neither N*, *none of the n N* and *none of the N* are also consistent in nature. To this end, it is enough to take the conditionals in (66) into consideration.

(66) a None of the six donkeys does not bray →
It is not the case that none of the six donkeys brays

b Neither foot is not ulcerated →
It is not the case that neither foot is ulcerated

c None of the scientists does not waste time →
It is not the case that none of the scientists wastes time

That each of these entailments is valid, follows from the definitions of the quantifiers corresponding to the noun phrases in question. To give an example, if none of the scientists does not waste time, then it follows that they all waste time, which in turn means that it is not the case that none of them wastes time. Consequently, the conditional in (66c) must be accepted as valid.

The foregoing observations by no means imply that every noun phrase is consistent. The invalid implication in (60) clearly shows that expressions of the form *at least n N* do not belong to this class. In an analogous manner, one easily proves that noun phrases of the forms *all N* and *no N* also cannot be regarded as consistent. The next two examples, if interpreted as cases of regular negation, are both invalid.

(67) All students do not complain $\not\rightarrow$
It is not the case that all students complain

(68) No child does not complain $\not\rightarrow$
It is not the case that no child complains

By way of illustration, it should be noted that, when the universe does not contain any child, the statements *No child complains* and *No child does not complain* are both true, which means that the denial *It is not the case that no child complains* must be regarded as false. Similarly, if the universe happens to lack students, the two statements *All students complain* and *All students do not complain* must both be accepted as true, and hence the denial *It is not the case that all students complain* must be rejected as false.

In this connection, the behavior of partitive noun phrases is rather interesting. It requires no lengthy reflection to see that expressions of the form *at least n of the k N* are consistent iff $n > k/2$. As a special case of the general pattern we have the valid conditional in (69).

(69) At least three of the four children do not complain $\rightarrow$
It is not the case that at least three of the four children complain

On the other hand, if we consider expressions of the form *(exactly) n of the k N*, then the property of consistency appears to manifest itself just in case $n \neq k/2$. That is to say:

(70) Six of the eight children do not complain $\rightarrow$
It is not the case that six of the eight children complain

(71) One of the eight children does not complain $\rightarrow$
It is not the case that one of the eight children complains

Finally, it should be easy to see that noun phrases of the form *at most n of the k N* are consistent only if $n < k/2$. In other words:

(72) At most one of the four children does not complain $\rightarrow$
It is not the case that at most one of the four children complains

The logical behavior of partitive expressions thus appears to show some regularities. These find expression in three general laws concerning the phenomenon of consistency.

(73) **Laws of consistency for partitive expressions**

(1) Expressions of the form *at least n of the k N* are consistent iff $n > k/2$.

(2) Expressions of the form *(exactly) n of the k N* are consistent iff $n \neq k/2$.

(3) Expressions of the form *at most n of the k N* are consistent iff $n < k/2$.

We must therefore conclude that, with a substantial number of partitive subject phrases, the use of predicate negation implies sentence negation.

With the aid of the foregoing test we can usually arrive at rather trustworthy judgments when it comes to deciding whether a given noun phrase does or does not enjoy the property of consistency. For the sake of clarity the outcomes of the test have been collected in table 4. The eighteen classes of noun phrases mentioned there must all be regarded as consistent. Such a catalogue, though at first sight merely of encyclopedic value, is important because we shall soon see that it leads to a coherent and complete account of the relationship between sentence negation and predicate negation.

Table 4: Eighteen classes of consistent noun phrases

most N	the N [sg]
the majority of the N	the N [pl]
at least n of the k N ($n > k/2$)	neither N
(exactly) n of the k N ($n \neq k/2$)	none of the n N ($n > 0$)
at most n of the k N ($n < k/2$)	none of the more than n N ($n > 0$)
both N	none of the no more than n N ($n > 0$)
the n N ($n > 0$)	none of the N
the more than n N ($n > 0$)	proper names
the no more than n N ($n > 0$)	negated proper names

It should be pointed out in this connection that the property of consistency shows a striking resemblance to the logical theorem known as **the law of contradiction**. Indeed, the principle in question is meant to exclude the possibility that two contradictory propositions are both accepted as true. For that reason it is usually stated as '$\sim (p \wedge \sim p)$'.

One sees immediately that the property of consistency is similar to the logical theorem in that it excludes that two sets X and $-X$ both belong to the quantifier. It will become apparent that this state of affairs has far-reaching consequences for our views on the different forms of negation.

It requires little reflection to realize that the property of consistency has a counterpart. In order to convince ourselves, we take the following two examples into account.

(74) It is not the case that most tulips flower $\not\rightarrow$
Most tulips do not flower

(75) It is not the case that Seneca plays chess $\rightarrow$
Seneca does not play chess

Clearly, the conditional in (74) cannot be regarded as valid. For if the state of affairs in the universe is such that half of all tulips flowers, then the antecedent is true, but the consequent false. On the other hand, the conditional sentence in (75) surely belongs to the class of valid statements. If we accept the truth of the statement *It is not the case that Seneca plays chess*, then we will also have to accept that Seneca does not play chess. This entails that the quantifiers which are associated with proper names invariably are complete in nature – complete in the sense that it cannot be that neither the complement of a given set nor that set itself is a member of the quantifier in question. For the sake of accuracy we record this in the form of a definition.[18]

(76) **Definition**

Let B be a Boolean algebra. A quantifier Q on B is said to be **complete** iff for each element X of the algebra B:

if $X \notin Q$, then $-X \in Q$.

It is evident that the notion of completeness just introduced is the reversal of the notion of consistency mentioned before. This means that there are alternative characterizations of the property in question. Indeed, it is easily established that the law of contraposition allows us to replace the conditional in (76) by the equivalent statement 'if $-X \notin Q$, then $X \in Q$'. Yet, we shall stick to the original definition, primarily because of the following corollary.

(77) **Corollary**

A noun phrase is complete iff the following schema is logically valid:

(1) NEG (NP VP) $\rightarrow$ NP (NEG VP)

This elementary result is important because it expresses in a lucid way that with a complete noun phrase as subject the use of sentence negation invariably implies predicate negation.

From the fact that the implication in (75) is valid, it follows immediately that proper names are complete. In an analogous way, one easily shows that negated proper names are also complete in nature. This does not exhaust the stock, for a short search produces several other cases of completeness. The next two examples serve as an illustration.

(78) a It is not the case that at least half of all tulips flowers $\rightarrow$
At least half of all tulips does not flower

b It is not the case that at most half of all cows has died $\rightarrow$
At most half of all cows has not died

There can be no doubt that both conditionals are valid. Indeed, if we accept the truth of the statement *It is not the case that at least half of all tulips flowers*, then we shall also have to accept that at least half of all tulips does not flower. Similarly, it is easily established that anyone who accepts the statement *It is not the case that at most half of all cows has died* as true will also be committed to the truth of *At most half of all cows has not died.* Consequently, we must conclude that noun phrases of the forms *at least half of all N* and *at most half of all N* both belong to the class of complete expressions.

For the sake of clarity, these results have been collected in table 5.

Table 5: Four classes of complete noun phrases

at least half of all N	at most half of all N
proper names	negated proper names

One sees immediately that two of the four classes of noun phrases mentioned are also consistent, namely proper names and their negations. This is important, because it follows from the relevant definitions that with a consistent and complete noun phrase as subject the use of sentence negation is equivalent to predicate negation. For that reason, the biconditional in (79) must be regarded as valid.

(79) It is not the case that Themistocles mourns $\leftrightarrow$
Themistocles does not mourn

Indeed, it follows from the completeness of the expression *Themistocles* that the use of sentence negation entails predicate negation. Conversely, the consistent nature of the element in question guarantees that the use of predicate negation implies sentence negation. In this way, we can give a semantic explanation of the at first sight rather intricate relationship between both forms of negation.

It should be noted that the property of completeness bears a close relationship to the familiar logical theorem known as **the law of the excluded middle**. This principle is meant to exclude the possibility that two contradictory propositions are both rejected as false. For that reason it is usually stated as '$p \vee \sim p$'. One sees immediately that the property of completeness is similar to the logical theorem in that it excludes that two sets X and $-X$ both do not belong to the quantifier.

In terms of the properties of consistency and completeness, the relationship between sentence negation and predicate negation can be described adequately. To forestall any misunderstandings, we do well to express this in the form of two general laws concerning the use of both forms of negation.

(80) **Laws of negation**

a The use of sentence negation implies predicate negation just in case the subject of the sentence is complete in nature.

b The use of predicate negation implies sentence negation just in case the subject of the sentence is consistent in nature.

It goes without saying that, in the presence of a subject which is complete *and* consistent, the use of sentence negation is equivalent to predicate negation.

5. PRIME IDEALS AND ULTRAFILTERS

Among the noun phrases that are both consistent and complete, there are some which have the structure of a quasi-ideal. Such quantifiers are usually referred to as **prime ideals** or **maximal ideals**.[19] To be more precise:

(81) **Definition**

Let B be a Boolean algebra. A quantifier Q on B is said to be a **prime ideal** (**maximal ideal**) iff for each two elements X and Y of the algebra B:

(a) $X \cup Y \in Q$ iff $X \in Q$ and $Y \in Q$;

(b) $X \in Q$ iff $-X \notin Q$.

It turns out that the only noun phrases which invariably receive a prime ideal as their semantic value are negated proper names. Expressions of this type will accordingly be called prime ideals.

As an immediate corollary to the definition in (81), we have the following theorem.

(82) **Theorem**

Let B be a Boolean algebra and let Q be a quantifier on B. If Q is a prime ideal, then for each two elements X and Y of the algebra B:

$X \cap Y \in Q$ iff $X \in Q$ or $Y \in Q$.

With the aid of this result, one easily shows that the validity of the schema in (83) is necessary in order that a noun phrase be classified as belonging to the class of prime ideals.

(83) **Corollary**

If a noun phrase is a prime ideal, then the following schema is logically valid:

NP (VP_1 and VP_2) $\leftrightarrow$ (NP VP_1 or NP VP_2).

It is clear that negated proper names act in accordance with the above test. To see this it is sufficient to take the following example into consideration.

(84) Not Themistocles mourns and moans $\leftrightarrow$

Not Themistocles mourns or not Themistocles moans

That the biconditional in (84) must be regarded as valid is a consequence of the fact that the quantifier associated with the negated proper name *not Themistocles* has the structure of a prime ideal.

Among the noun phrases that are both consistent and complete, there are also some which have the structure of a quasi-filter. It is customary to refer to such quantifiers as **ultrafilters** or **maximal filters**.[20] That is to say:

(85) **Definition**

Let B be a Boolean algebra. A quantifier Q on B is said to be an **ultrafilter** (**maximal filter**) iff for each two elements X and Y of the algebra B:

(a) $X \cap Y \in Q$ iff $X \in Q$ and $Y \in Q$;

(b) $X \in Q$ iff $-X \notin Q$.

The only noun phrases which are invariably associated with a quantifier that has the structure of an ultrafilter are proper names. Expressions of this type will accordingly be called ultrafilters.

As an immediate corollary to the definition in (85), we have the following theorem.

(86) **Theorem**

Let B be a Boolean algebra and let Q be a quantifier on B. If Q is an ultrafilter, then for each two elements X and Y of the algebra B:

$X \cup Y \in Q$ iff $X \in Q$ or $Y \in Q$.

With the aid of this result, one easily shows that the validity of the schema in (87) is necessary in order that a noun phrase be classified as belonging to the class of ultrafilters.

(87) **Corollary**

If a noun phrase is an ultrafilter, then the following schema is logically valid:

NP (VP_1 or VP_2) $\leftrightarrow$ (NP VP_1 or NP VP_2).

It is clear that proper names act in accordance with the above test. To see this it is enough to take the following example into consideration.

(88) Themistocles mourns or moans $\leftrightarrow$

Themistocles mourns or Themistocles moans

That the biconditional in (88) must be regarded as valid is a consequence of the fact that the quantifier associated with the proper name *Themistocles* has the structure of an ultrafilter.

6. MONOTONIC FUNCTIONS

Thus far we have assumed that noun phrases must be regarded as quantifiers on the algebra of verb phrases. It is conceivable, however, that the semantic value assigned to expressions of this type is not a quantifier, but rather the characteristic function of a quantifier. Such a conceptual change immediately raises the question to what extent the usual notion of a function lends itself to the classifications discussed above. Surprisingly, this matter appears not to be as complex as one might at first expect. Let us begin by considering the special case of upward monotonicity. The fact that quantifiers of this type are closed under extension, entails that the associated characteristic function is such that assignment of the value 1 to an element X is invariably accompanied with assignment of the value 1 to each element Y which contains X. Put differently, if Q is a monotone increasing quantifier on the algebra of verb phrases, then its characteristic function $K_Q : P(U) \rightarrow 2$ has the property that the conditions $X \subseteq Y$ and $K_Q(X) = 1$ imply $K_Q(Y) = 1$. Do not suppose that this exhausts the matter, for in view of the composition of the Boolean algebra 2 it should be obvious that what the property in question amounts to is that whenever X is contained in Y, $K_Q(X)$ is contained in $K_Q(Y)$.[21] This state of affairs leads to the conclusion that, in general, a function f from a Boolean algebra B to a Boolean algebra B^* can be regarded as monotone increasing, just in case it preserves the inclusion relation. That is to say:

(89) **Definition**

Let B and B^* be two Boolean algebras. A function f from B to B^* is said to be **monotone increasing** iff for each two elements X and Y of the algebra B:

if $X \subseteq Y$, then $f(X) \subseteq f(Y)$.[22]

It is evident that such functions can also be given an alternative characterization. As a matter of fact, corresponding to theorem (36), we have the following result.

(90) **Theorem**

Let B and B^* be two Boolean algebras. The following statements about a function f from B to B^* are equivalent:

(a) f is monotone increasing;

(b) $f(X \cap Y) \subseteq f(X) \cap f(Y)$;

(c) $f(X) \cup f(Y) \subseteq f(X \cup Y)$.

We must not suppose that this exhausts the variety of available functions, for besides monotone increasing functions we can obviously also distinguish monotone decreasing functions. In that case it is the properties of monotone decreasing quantifiers that serve as our point of departure. Indeed, the fact that these collections are closed under inclusion entails that the associated characteristic function is such that assignment of the value 1 to an element Y is invariably accompanied with assignment of the value 1 to each element X which is contained in Y. In other words, if Q is a monotone decreasing quantifier on the algebra of verb phrases, then its characteristic function $K_Q : P(U) \to 2$ possesses the property that the conditions $X \subseteq Y$ and $K_Q(Y) = 1$ imply $K_Q(X) = 1$. In view of the nature of the Boolean algebra 2, it should be clear that what the property in question amounts to is that whenever X is contained in Y, $K_Q(Y)$ is contained in $K_Q(X)$. More precisely:

(91) **Definition**

Let B and B^* be two Boolean algebras. A function f from B to B^* is said to be **monotone decreasing** iff for each two elements X and Y of the algebra B:

if $X \subseteq Y$, then $f(Y) \subseteq f(X)$.

Needless to say, such functions can also be characterized in a different way. Indeed, analogous to theorem (30) we have the following:

(92) **Theorem**

Let B and B^* be two Boolean algebras. The following statements about a function f from B to B^* are equivalent:

(a) f is monotone decreasing;

(b) $f(X \cup Y) \subseteq f(X) \cap f(Y)$;

(c) $f(X) \cup f(Y) \subseteq f(X \cap Y)$.

At this point, the reader may protest that the distinction between monotone increasing and monotone decreasing functions must be regarded as an unnecessary complication. One should keep in mind, however, that the concept of a function is much more general than the rather limited notion of a quantifier. This is clear from the fact that natural languages as a rule have a variety of expressions which cannot be associated with a quantifier, but which nevertheless display the characteristic features of monotonicity. The next two conditionals may serve as an illustration.

(93) a No willow will die slowly $\rightarrow$
No blossoming willow will die slowly

b All jonquils will blossom $\rightarrow$
All fertilized jonquils will blossom

Provided that the noun *blossoming willow* applies only to willows and the noun *fertilized jonquils* only to jonquils, the conditionals in (93) must both be accepted as valid. In other words, if the state of affairs in the universe is such that the class of blossoming willows is a subset of the class of willows and the class of unfertilized jonquils a subset of the class of jonquils, then we may legitimately pass from the propositions *No willow will die slowly* and *All jonquils will blossom* to the propositions *No blossoming willow will pass away* and *All unfertilized jonquils will flower*. These findings show that the logical behavior of the determiners *all* and *no* resembles that of monotone decreasing noun phrases, in spite of the fact that the semantic value of both expressions cannot be treated as a quantifier. In order to account for this resemblance, we must invoke the notion of a monotonic function. Indeed, it is not uncommon to find determiners being portrayed as functions which carry nouns into noun phrases. In accordance with this, such expressions are often identified semantically with a function from the algebra of nouns to the algebra of noun phrases – a state of affairs which admits no other interpretation than that a determiner associates with each element of the power set $P(U)$ a uniquely determined element of the power set $P(P(U))$. Once it is recognized that determiners are to be treated as functions, we have an explanation for the similarities between the logical behavior of monotone decreasing noun phrases and that of expressions like *all* and *no*. For if the semantic value of determiners is functional in nature, then the validity of the conditionals in (93) must be attributed to the circumstance that the functions associated with *all* and *no* are downward monotonic. As an illustration, we consider the case of the determiner *all*. It is easy to see that the monotone decreasing character of this element entails that the extension of the noun phrase *all jonquils* is contained in the extension of *all fertilized jonquils*. Stated differently, each set which is a member of the quantifier associated with *all jonquils* is also a member of the quantifier associated with *all fertilized jonquils*. From this it follows immediately that someone who considers the proposition *All jonquils will blossom* to be true must also accept the truth of *All fertilized jonquils will blossom*. For that reason, we may say that the validity of the conditional in (93b) is a consequence of the monotone decreasing nature of the determiner *all*. Needless to say, such reasoning is not limited to this single case, for in a completely analogous fashion the validity of the implication in (93a) can be attributed to the downward monotonic nature of the determiner *no*.

That the class of all such expressions has more members than *all* and *no*, is shown by the next two examples.

(94) a At most sixteen beggars have been convicted $\rightarrow$
At most sixteen blind beggars have been convicted

b No more than ten beggars have been tortured $\rightarrow$
No more than ten blind beggars have been tortured

Provided that the noun *blind beggars* applies only to what the noun *beggars* also applies to, the conditionals in (94) must both be accepted as true. Consequently, expressions of the forms *at most n N* and *no more than n N* must also be regarded as belonging to the class of monotone decreasing determiners.

It would surely be wrong to suppose that all determiners are downward monotonic, for it is easy to find expressions which do not have the property in question. The examples which follow speak for themselves.

(95) a Neither pedlar will be prosecuted $\not\rightarrow$
Neither lame pedlar will be prosecuted

b None of the six artists was fined $\not\rightarrow$
None of the six deaf artists was fined

Even if both pedlars are lucky enough not to be prosecuted, we are by no means justified in the conclusion that neither lame pedlar will be prosecuted, for the simple reason that the existence of just two pedlars does not necessarily entail the existence of just two lame pedlars. Similarly, when none of the six artists was fined, we cannot conclude that none of the six deaf artists was fined. This shows that expressions of the forms *neither N* and *none of the n N* do not belong to the class of monotone decreasing determiners.

The foregoing discussion is far from comprehensive, for besides downward monotonic determiners one also finds upward monotonic determiners, judging from the conditionals in (96).

(96) a At least one rich servant cries $\rightarrow$
At least one servant cries

b Not all poor fishermen complain $\rightarrow$
Not all fishermen complain

On the condition that the noun *rich servant* is true only of servants and the noun *poor fishermen* only of fishermen, both conditionals must be considered valid. That is to say, if the universe is such that the class of rich servants is contained in the class of servants and the class of poor fishermen in the class of fishermen, then we may legitimately pass from the propositions *At least one rich servant cries* and *Not all poor fishermen complain* to the propositions *At least one servant cries* and *Not all fishermen complain.* What this shows is that the logical behavior of the determiners *at least one* and *not all* resembles that of monotone increasing noun phrases – something which admits no other explanation than that the functions associated with *at least one* and *not all* are likewise upward monotonic. Indeed, consultation of the relevant definition makes it clear that what this property amounts to is that the extension of the noun phrase *at least one servant* invariably contains the extension of the noun phrase *at least one rich servant.* In other words, each set which is a member of the quantifier associated with *at least one rich servant* must also be regarded as a member of the quantifier associated with *at least one servant.* Needless to say, this implies that someone who considers the proposition *At least one rich servant cries* to be true must also accept the truth of *At least one servant cries.* We may therefore say that the validity of the conditional in (96a) is a consequence of the monotone increasing nature of the determiner *at least one.* A parallel argument shows that the validity of the conditional in (96b) must be attributed to the upward monotonic character of the determiner *not all.*

That the class of all such elements contains more expressions than those of the forms *at least n* and *not all*, is demonstrated by the next two examples.

(97) a Several uninvited guests were treated unfriendly $\rightarrow$
Several guests were treated unfriendly

b More than twelve uninvited guests have collapsed $\rightarrow$
More than twelve guests have collapsed

Provided that the noun *uninvited guests* is true only of what the noun *guests* is also true of, acceptance of the antecedent must in each case lead to acceptance of the consequent. For that reason, expressions of the forms *several* and *more than n* must also be regarded as monotone increasing determiners.

This is of course not to say that all determiners are upward monotonic. Surely not, for natural language has a variety of expressions which cannot be analyzed in such a manner. In order to convince ourselves of this fact, we consider some examples.

(98) Neither lame pedlar will be prosecuted $\not\rightarrow$
Neither pedlar will be prosecuted

(99) None of the six deaf artists was fined $\not\rightarrow$
None of the six artists was fined

Even if both lame pedlars are lucky enough not to be prosecuted, we are by no means justified in the conclusion that neither pedlar will be prosecuted, since the existence of just two lame pedlars does not necessarily entail the existence of just two pedlars. Similarly, when none of the six deaf artists was fined, we cannot conclude that none of the six artists was fined. This shows that expressions of the forms *neither* and *none of the n* cannot be regarded as belonging to the class of monotone increasing determiners.

These and similar findings with regard to the laws which govern the logical behavior of determiners have been collected in table 6. The eighteen classes of expressions which are mentioned there must all be regarded as monotonic. It should be obvious that the logical properties of the determiner may be quite unlike those of the corresponding noun phrase. As a case in point, we consider the expressions *every* and *all*. When the semantical properties of these determiners are compared with those of noun phrases of the forms *every N* and *all N*, it is at once clear that one has downward monotonicity in the first case and upward monotonicity in the second. Things are different, however, when, instead of *every* and *all*, we consider their negations *not every* and *not all*, for now the determiners are monotone increasing and the corresponding noun phrases monotone decreasing.

Table 6: Eighteen classes of monotonic determiners in English, with their Dutch counterparts

Monotone increasing	**Monotone decreasing**
not every (**D**: niet ieder(e))	every (**D**: ieder(e))
not all (**D**: niet alle)	all (**D**: alle)
not each (**D**: niet elk(e))	each (**D**: elk(e))
sm (**D**: enkele)	no (**D**: geen)
at least n (**D**: minstens n)	at most n (**D**: hoogstens n)
some (**D**: sommige)	not a single (**D**: geen enkel(e))
several (**D**: verscheidene)	only n (**D**: slechts n)
more than n (**D**: meer dan n)	no more than n (**D**: niet meer dan n)
some but not all (**D**: sommige maar niet alle)	not one (**D**: niet één)

Even more surprising is the behavior of the composite determiner *some but not all.* By way of illustration, we take the conditional (100) into consideration.

(100) Some but not all contaminated pigs are being destroyed →
Some but not all pigs are being destroyed

There is no doubt that the above implication must be accepted as valid. For if it is true that some but not all contaminated pigs are being destroyed, then it must also be true that some but not all pigs are being destroyed. This entails that the expression in question belongs to the class of monotone increasing determiners. On the other hand, noun phrases of the form *some but not all N* can neither be regarded as upward monotonic, nor as downward monotonic. Consequently, what we are here dealing with is a case in which the determiner possesses the property of upward monotonicity, but the corresponding noun phrase is devoid of any form of monotonicity at all.

We must not suppose that this is an exhaustive treatment of the different possibilities, for it requires little reflection to realize that the reverse situation may also arise. Indeed, natural language has a rather large group of determiners which are evidently not monotonic. Examples include not only expressions of the forms *both*, *neither*, *the n* and *none of the n*, but also elements like *many*, *most* and *(precisely) n*. In order to be convinced of this fact, examine the conditionals in (101) and (102).

(101) Many decayed houses will be pulled down $\not\rightarrow$
Many houses will be pulled down

(102) Many houses will be pulled down $\not\rightarrow$
Many decayed houses will be pulled down

It should be obvious that neither example can be accepted as valid. Indeed, it is easy to see that the intended demolition of many ruinous houses does not preclude the possibility that few houses will be pulled down and, reversely, that the intended demolition of many houses does not rule out the possibility that few decayed houses will be pulled down. For that reason, the expression *many* cannot be regarded as a monotonic determiner, nor can its negative counterpart *few*. In this light, we must also look at examples such as (103) and (104).

(103) Most obsolete engines will be replaced $\not\rightarrow$
Most engines will be replaced

(104) Most engines will be replaced $\not\rightarrow$
Most obsolete engines will be replaced

Using the same reasoning as we employed for the examples in (101) and (102), it is easily proved that the conditionals in (103) and (104) are also invalid. Consequently, the expression *most* cannot be regarded as a monotonic determiner.

Our last example relates to expressions of the form *(precisely) n*. To that end, one does well to take the next two conditionals into consideration.

(105) Precisely three poisoned storks will be killed $\not\rightarrow$
Precisely three storks will be killed

(106) Precisely three storks will be killed $\not\rightarrow$
Precisely three poisoned storks will be killed

It should be obvious that in neither case one can speak of a valid proposition. For if in addition to three poisoned storks two healthy ones will be killed, then the antecedent of (105) is true, but its consequent false. Likewise, the intended death of three storks, among which one poisoned one, is sufficient to reject the conditional in (106) as false. Therefore, no other conclusion can be drawn than that expressions of the form *(precisely) n* also do not belong to the class of monotonic determiners.

With the aid of the preceding test, we can usually arrive at rather trustworthy judgments when we must decide whether a given determiner is monotonic. Some of the outcomes have been collected in table 7. The eighteen classes of determiners mentioned there appear to be devoid of any form of monotonicity. This is not to say that the corresponding noun phrase must also be regarded as being non-monotonic. For it is easily established that determiners of the forms *almost all*, *both*, *many*, *the n*, *the more than n*, *the [sg]*, *the [pl]* and *most* produce a monotone increasing noun phrase. Likewise, one easily proves that determiners of the forms *almost no*, *neither*, *few*, *none of the n*, *none of the more than n* and *none of the* result in a monotone decreasing noun phrase. Only in the case of *(precisely) n*, *all except n*, *an even number* and *an uneven number* is the corresponding noun phrase non-monotonic in nature.

Table 7: Eighteen classes of non-monotonic determiners in English, with their Dutch counterparts

almost all (**D**: vrijwel alle)	neither (**D**: geen van beide)
both (**D**: beide)	few (**D**: weinig)
many (**D**: veel)	none of the n (**D**: geen van de n)
the n (**D**: de n)	none of the more than n (**D**: geen van de meer dan n)
the more than n (**D**: de meer dan n)	none of the (**D**: geen van de)
the [pl] (**D**: de [pl])	(precisely) n (**D**: (precies) n)
the [sg] (**D**: de [sg])	all except n (**D**: op n na alle)
most (**D**: de meeste)	an even number (**D**: een even aantal)
almost no (**D**: vrijwel geen)	an uneven number (**D**: een oneven aantal)

7. ANTI-ADDITIVE, MULTIPLICATIVE AND ANTIMORPHIC FUNCTIONS

So far this short digression on the distinction between monotone increasing and monotone decreasing functions. Now it is necessary to consider the question of how the earlier notion of a quasi-ideal can be given a functional characterization. Surprisingly enough, the matter is not as complex as first expected. This becomes evident when we take the definition of a quasi-ideal into consideration. With the help of the relevant stipulations, one easily establishes that the associated characteristic function is such that the value assigned to the element $X \cup Y$ corresponds to the product of the values which are assigned to X and Y. Put differently, if Q is a quasi-ideal on the algebra of verb phrases, then the characteristic function $K_Q : P(U) \to 2$ has the property that $K_Q(X \cup Y)$ is invariably equal to $K_Q(X) \cap K_Q(Y)$. Such functions are sometimes referred to as anti-additive functions.[23]

(107) **Definition**

Let B and B^* be two Boolean algebras. A function f from B to B^* is said to be **anti-additive** iff for each two elements X and Y of the algebra B:

$f(X \cup Y) = f(X) \cap f(Y)$.

Clearly, noun phrases which are associated with a quasi-ideal must be regarded as anti-additive from a functional point of view. It appears, however, that there are also some determiners which exhibit the behavior of an anti-additive function. By way of illustration, we consider the biconditionals in (108) and (109).

(108) Every goat or donkey will be killed $\leftrightarrow$
Every goat and every donkey will be killed

(109) Not a priest or baker will be fired $\leftrightarrow$
Not a priest and not a baker will be fired

There can be no doubt that both of the preceding propositions are valid. Indeed, anyone who regards one of the members of the biconditionals in (108) and (109) as true, must also accept the truth of the other member – a state of affairs which admits no other interpretation than that the determiners every and not a semantically behave as anti-additive functions. For if one regards the determiner as a functor and the noun as its argument, then the valid schema in (110) results.

(110) (DET (N_1 or N_2)) VP $\leftrightarrow$ (DET (N_1) and DET (N_2)) VP

It is immediately clear that the above scheme can be interpreted in such a way that it satisfies the characteristic requirements of an anti-additive function.

This is not to say that every determiner is anti-additive in nature. Surely not, for natural language has a variety of expressions which do not obey the laws that govern the use of such elements. The next two conditionals serve as an illustration.

(111) Not all oxen or goats have been killed $\not\rightarrow$
Not all oxen and not all goats have been killed

(112) Not all oxen and not all goats have been killed $\rightarrow$
Not all oxen or goats have been killed

It is easy to see that the proposition in (111) is invalid. For if it is the case that all oxen, but not all goats have been killed, then the antecedent of the conditional in (111) is true, but its consequent false. For that reason, the expression *not all* cannot be regarded as belonging to the class of anti-additive determiners. On the other hand, it is a consequence of the monotone increasing nature of the determiner *not all* that the reverse proposition in (112) must be accepted as valid.

The preceding discussion is far from exhaustive, for the general notion of a quasi-filter can also easily be extended to the functional domain. Using the relevant definition, one proves without difficulty that the associated characteristic function is such that the value which is assigned to the element $X \cap Y$ is equal to the product of the values assigned to X and Y separately. In other words, if Q is a quasi-filter on the algebra of verb phrases, then its characteristic function $K_Q : P(U) \rightarrow 2$ has the property that $K_Q(X \cap Y)$ is invariably equal to $K_Q(X) \cap K_Q(Y)$. Functions of this nature are usually called **multiplicative** functions.[24] That is to say:

(113) **Definition**

Let B and B^* be two Boolean algebras. A function f from B to B^* is said to be **multiplicative** iff for each two elements X and Y of the algebra B:

$f(X \cap Y) = f(X) \cap f(Y)$.

It is evident that, from a functional point of view, noun phrases which act as quasi-filters must be regarded as being multiplicative. Curiously enough, it is exceptionally difficult to find a good example of a multiplicative determiner. This may rouse our suspicion as to whether there are any determiners which exhibit the properties of a multiplicative function. Indeed, it seems that natural language excludes such expressions on principle. Given the semantic constraints on the expressive nature of determiners, one can in fact prove that the observed gap is a logical one.[25]

The notion of a prime ideal, introduced in section 5, can likewise be given a functional characterization. With the help of the stipulations in (81), one easily establishes that the associated characteristic function is such that the value assigned to the element $X \cup Y$ corresponds to the product of the values assigned to X and Y, and the value assigned to the element $-X$, to the complement of the value assigned to X. Put differently, if Q is a prime ideal on the algebra of verb phrases, then the characteristic function $K_Q : P(U) \to 2$ has the property that $K_Q(X \cup Y)$ is invariably equal to $K_Q(X) \cap K_Q(Y)$ and that $K_Q(-X)$ is invariably equal to $-K_Q(X)$. Such functions will henceforth be referred to as antimorphic functions.

(114) **Definition**

Let B and B^* be two Boolean algebras. A function f from B to B^* is said to be **antimorphic** iff for each two elements X and Y of the algebra B:

(a) $f(X \cup Y) = f(X) \cap f(Y)$;

(b) $f(-X) = -f(X)$.

We have seen that there are noun phrases whose associated quantifier acts, semantically, as a prime ideal. These expressions must therefore be classified as antimorphic. Within the category of determiners, however, it is difficult to find one whose logical behavior can be characterized as antimorphic. Again, it seems that natural language excludes such expressions on principle.

These findings with respect to the laws which govern the logical behavior of determiners have been collected in table 8. The five determiners mentioned there must all be understood as being anti-additive. It is immediately clear that there can be substantial differences between the behavior of the determiner, on the one hand, and that of the corresponding noun phrase, on the other. Characteristic examples are the expressions *every* and *all*. When we compare the semantical properties of these elements with those of noun phrases of the forms *every* N and

Table 8: Five anti-additive determiners in English, with their Dutch counterparts

every (**D**: ieder(e))	not a (**D**: geen enkel(e))
all (**D**: alle)	not one (**D**: niet één)
no (**D**: geen)	

all N, it turns out that we are dealing with anti-additive expressions in the first case, but multiplicative expressions in the second. Things are different, however, when, in place of *every* and *all*, we consider the universal negations *no*, *not a* and *not one*, for now it is not only the determiner, but also the corresponding noun phrase which is anti-additive in nature. Even more surprising is the behavior of expressions as *both*, *each*, *the n*, *neither* and *none of the n*. In all of these cases, the determiner is clearly non-monotonic. On the other hand, noun phrases of the forms *both N*, *each N* and *the n N* exhibit a multiplicative character, as opposed to those of the forms *neither N* and *none of the n N*, which instead are anti-additive.

This description of the different possibilities is still not complete, for it is also possible that neither the determiner nor the corresponding noun phrase is multiplicative or anti-additive in nature. Expressions which belong to this class include *at least n*, *at most n*, *not all*, *some but not all*, *many*, *few*, *most* and *(precisely) n*. Following our earlier policy, a number of these results have been collected in table 9. On the basis of this survey, it is easy to see that the logical behavior of the determiner is wholly independent of that of the corresponding noun phrase. To forestall any misunderstandings, it should be pointed out that every multiplicative function is also monotone increasing. In an analogous way, one easily establishes that the class of anti-additive functions is a subset of the class of monotone decreasing functions and that the class of antimorphic functions is a subset of the class of anti-additive functions.

In the same way that a determiner can be associated with a function carrying sets of individuals into collections of such sets, a sentential connective can be assigned a function from the algebra of truth values, 2, to the power set algebra $P(2)$. It is not uncommon, for example, to find an expression like *and* being portrayed as a function which maps sentences into so-called adsentences. In accordance with this, the element in question will be associated semantically with a function that assigns the singleton set $\{1\}$ to the truth value 1, and the empty set to the truth

Table 9: Comparison of the logical behavior of determiners with that of the corresponding noun phrases

	Determiner	**Noun phrase**
at least n (**D**: minstens n)	mon. increas.	mon. increas.
some (**D**: sommige)	mon. increas.	mon. increas.
sm (**D**: enkele)	mon. increas.	mon. increas.
no (**D**: geen)	anti-additive	anti-additive
at most n (**D**: hoogstens n)	mon. decreas.	mon. decreas.
not a single (**D**: geen enkel(e))	anti-additive	anti-additive
not every (**D**: niet ieder(e))	mon. increas.	mon. decreas.
not all (**D**: niet alle)	mon. increas.	mon. decreas.
every (**D**: ieder(e))	anti-additive	multiplicative
all (**D**: alle)	anti-additive	multiplicative
some but not all (**D**: sommige maar niet alle)	mon. increas.	non-monotonic
nearly all (**D**: vrijwel alle)	non-monotonic	mon. increas.
both (**D**: beide)	non-monotonic	multiplicative
each (**D**: elk(e))	non-monotonic[26]	multiplicative
many (**D**: veel)	non-monotonic	mon. increas.
the n (**D**: de n)	non-monotonic	mon. increas.
most (**D**: de meeste)	non-monotonic	mon. increas.
almost no (**D**: vrijwel geen)	non-monotonic	mon. decreas.
not each (**D**: niet elk(e))	non-monotonic	mon. decreas.
neither (**D**: geen van beide)	non-monotonic	anti-additive
few (**D**: weinig)	non-monotonic	mon. decreas.
none of the n (**D**: geen van de n)	non-monotonic	anti-additive
(precisely) n (**D**: (precies) n)	non-monotonic	non-monotonic

value 0. What this means is that a sentence of the form *The dog barks and the cat meows* must be regarded as true just in case both of its component sentences are true. It requires little reflection to see that the

connectives *or*, *if* and *without* can be treated in an analogous manner. The following definition provides the necessary details.

(115) **Definition**

$$\begin{array}{llll} And(0) = \emptyset & Or(0) = \{1\} & If(0) = \{0,1\} & Without(0) = \{1\} \\ And(1) = \{1\} & Or(1) = \{0,1\} & If(1) = \{1\} & Without(1) = \emptyset \end{array}$$

From this description it is immediately clear that the connectives *and* and *or* are multiplicative in nature; *if* and *without*, on the other hand, are anti-additive. These and similar findings have been collected in table 10.

Table 10: Comparison of the logical behavior of connectives with that of the corresponding adsentences

	Connective	**Adsentence**
and (**D**: en)	multiplicative	multiplicative
or (**D**: of)	multiplicative	multiplicative
if (**D**: als)	anti-additive	multiplicative
without (**D**: zonder (dat))	anti-additive	multiplicative

On the basis of this survey, it is easy to see that there can be substantial differences between the logical behavior of the connective and that of the corresponding adsentence. Characteristic examples are the expressions *if* and *without*. When we compare the semantical properties of these elements with those of adsentences of the forms *if S* and *without S*, it turns out that we are dealing with anti-additive expressions in the first case, but multiplicative expressions in the second. Things are different, however, when, instead of *if* and *without*, we consider the elements *and* and *or*, for now it is not only the adsentence, but also the corresponding connective which is multiplicative in nature.

8. LAWS OF NEGATIVE POLARITY

Some readers might be inclined to regard the functional perspective just introduced as an unnecessary complication. They should keep in mind that the notion of a function enables us to extend our investigations of polarity phenomena to arbitrary environments. In particular, it becomes clear in this way that the distinction between weak, strong and superstrong forms of negative polarity corresponds with that between monotone decreasing, anti-additive and antimorphic functions. In order to convince ourselves of this fact, we consider first the Dutch examples in (116).

(116) a *Hoogstens een kind zal een opstel hoeven te schrijven.*
At most one child will a paper need to write
'At most one child need write a paper.'

b **Minstens één kind zal een verslag hoeven te schrijven.*
At least one child will a report need to write
'At least one child need write a report.'

The contrast between (116a) and (116b) shows that the presence of the monotone increasing noun phrase *minstens één kind* is not sufficient to justify the occurrence of the weak polarity item *hoeven*. Apparently, it is only monotone decreasing expressions like *hoogstens één kind* that are able to license the element in question. This supposition is confirmed by the pattern which manifests itself in (117).

(117) a *Geen zuigeling zal de proeven hoeven te doorstaan.*
No infant will the tests need to go through
'No infant need go through the tests.'

b **Alle kinderen zullen een test hoeven te ondergaan.*
All children will a test need to undergo
'All children need undergo a test.'

Of the two phrases *geen zuigeling* and *alle kinderen*, it is only the first that can act as a licencing expression for *hoeven* - a state of affairs which must be attributed to the circumstance that *geen zuigeling* is downward monotonic and *alle kinderen* upward monotonic.

However, when we replace *hoeven* by the strong polarity item *ook maar iets*, a clear contrast manifests itself between licencing expressions of the form *hoogstens n N* and those of the form *geen N*. As an illustration we consider the examples in (118).

(118) a *Geen bemiddelaar zal ook maar iets bewerkstelligen.*
No mediator will anything accomplish
'No mediator will accomplish anything.'

b **Hoogstens zes ouders zullen ook maar iets vernemen.*
At most six parents will anything hear
'At most six parents will hear anything.'

The contrast between (118a) and (118b) shows clearly that only the anti-additive expression *geen bemiddelaar* is capable of licencing the occurrence of *ook maar iets*. This is by no means an accident, for as the sentences in (119) show, the strong polarity item *bijster* exhibits exactly the same characteristics as *ook maar iets*.

(119) a *Niet één leerkracht toonde zich bijster verontrust.*
Not one teacher showed himself very disturbed
'Not one teacher showed himself very disturbed.'

b **Slechts één leerling toonde zich bijster ingenomen.*
Only one student showed himself very pleased
'Only one student showed himself very pleased.'

The fact that the occurrence of *bijster* in (119b) produces an unacceptable result proves that the element in question requires the presence of an anti-additive expression elsewhere in the sentence.

In the light of such facts it is absolutely clear that there are certain regularities underlying both forms of negative polarity. These find expression in two general laws concerning the use of negative polarity items.

(120) **Laws of negative polarity**

a Only sentences in which a monotone decreasing expression occurs, can contain a negative polarity item of the weak type.

b Only sentences in which an anti-additive expression occurs, can contain a negative polarity item of the strong type.

According to the first law, the presence of a monotone decreasing expression is a necessary condition for the appearance of negative polarity items of the weak type. On the other hand, the second law stipulates that negative polarity items of the strong type require the presence of an anti-additive expression.

In order to get a clear view of the domain of application of both laws, one does well to take the following examples into consideration.

(121) a *Geen kind dat ook maar iets bevroedt, zal iemand waarschuwen.*
No child who anything suspects will someone warn
'No child who suspects anything will tell someone.'

b *Ieder kind dat ook maar iets vermoedt, zal iemand*
Every child who anything suspects will someone
raadplegen.
consult
'Every child who suspects anything will consult someone.'

The occurrence of the strong polarity item *ook maar iets* is entirely acceptable in both sentences – a state of affairs which must be attributed to the anti-additive nature of the determiners *geen* and *ieder(e)*. However, if the expression in question is part of the main clause, then one of the two sentences becomes ungrammatical, as shown by the contrast in (122).

(122) a *Geen kind dat iets bevroedt, zal ook maar iemand*
No child who something suspects will anyone
waarschuwen.
warn
'No child who suspects something will tell anyone.'

b **Ieder kind dat iets vermoedt, zal*
Every child who something suspects will
ook maar iemand verwittigen.
anyone notify
'Every child who suspects something will notify anyone.'

This is a consequence of the fact that the anti-additive determiner *ieder(e)* produces multiplicative noun phrases.

Things are different when we take the composite determiner *geen van de* into consideration, for now it is not the main clause, but the relative clause which excludes negative polarity items.

(123) a **Geen van de ukken die ook maar iets zien, zal*
None of the toddlers who anything see will
iemand waarschuwen.
someone warn
'None of the toddlers who see anything will tell someone.'

b *Geen van de ukken die iets zien, zal*
None of the toddlers who see something will
ook maar iemand waarschuwen.
anyone warn
'None of the toddlers who see something will tell anyone.'

It is easy to see that this follows from the non-monotonic nature of the expression *geen van de.* On the other hand, when we replace the occurrence of *geen van de* by *hoogstens zes* in examples such as (124), then the result is in both cases completely unacceptable.

(124) a **Hoogstens zes ukken die ook maar iets zien, zullen*
At most six toddlers who anything see will
iemand roepen.
someone call
'At most six toddlers who see anything will call someone.'

b **Hoogstens zes ukken die iets zien, zullen*
At most six toddlers who something see will
ook maarfliemand roepen.
anyone call
'At most six toddlers who see something will call anyone.'

From this we must conclude that neither the monotone decreasing determiner *hoogstens zes* nor the monotone decreasing noun phrase *hoogstens zes ukken* is capable of triggering negative polarity items of the strong type.

The well-formed occurrence of the strong polarity item *ook maar iets* in (125) can likewise be explained in terms of the anti-additive nature of the connectives *als* en *zonder*.

(125) a *Als het kind ook maar iets bevroedt, zal het iemand*
If the child anything suspects will she someone
waarschuwen.
warn
'If the child suspects anything, she will tell someone.'

b *De man zal iemand waarschuwen zonder*
The man will someone warn without
ook maar iets te bevroeden.
anything to suspect
'The man will inform someone without suspecting anything.'

However, if the polarity item in question is part of the main clause, as in (126), then both sentences become ungrammatical.

(126) a **Als het kind iets bevroedt, zal het*
If the child something suspects will she
ook maar iemand waarschuwen.
anyone warn
'If the child suspects something, she will tell anyone.'

b **De man zal ook maar iemand waarschuwen zonder*
The man will anyone warn without
iets te bevroeden.
something to suspect
'The man will inform anyone without suspecting something.'

This is a consequence of the fact that the anti-additive connectives *als* en *zonder* invariably produce multiplicative adsentences.

We must not suppose that this exhausts the matter, for the super-strong polarity item *one bit* requires the presence of an antimorphic expression, as shown by the examples in (127).

(127) a *Few clergymen liked the performance one bit.
b *No politician liked the performance one bit.
c The men didn't like the performance one bit.

The ungrammaticality of the sentences (127a) and (127b) must be attributed to the fact that neither *few clergymen* nor *no politician* belongs to the class of antimorphic expressions. The negative adverb *not* (*n't*), on the other hand, is antimorphic; hence, the well-formed nature of the sentence in (127c).

As the ungrammatical examples in (128) show, the Dutch adjective *mals* 'tender' is also not content with a monotone decreasing or anti-additive expression. Instead, it requires the presence of the antimorphic adverb *niet* somewhere in the sentence.

(128) a **Weinig van zijn oordelen waren mals.*
Few of his opinions were tender
'Few of his opinions were soft.'

b **Niet één van zijn oordelen was mals.*
Not one of his opinions was tender
'Not one of his opinions was soft.'

c *Zijn oordelen waren vaak niet mals.*
His opinions were often not tender
'His opinions often weren't soft.

In view of the additional constraints on the occurrence of polarity items such as *one bit* and *mals*, we must extend our account as follows.

(129) **Laws of negative polarity**

a Only sentences in which a monotone decreasing expression occurs, can contain a negative polarity item of the weak type.

b Only sentences in which an anti-additive expression occurs, can contain a negative polarity item of the strong type.

c Only sentences in which an antimorphic expression occurs, can contain a negative polarity item of the superstrong type.

We are now in a position to answer the question that has guided us throughout. As the functional characterization in (92) shows, the logical behavior of monotone decreasing expressions is governed by one half of the first law of De Morgan and one half of the second law of De Morgan. This means that the class of monotone decreasing expressions is coextensive with the class of subminimal negations. By reference to (92) and the functional definition in (107) it can likewise be shown that the logical behavior of anti-additive expressions is determined by the first law of De Morgan as a whole and one half of the second law of De Morgan. Accordingly, the class of anti-additive expressions may be equated with the class of minimal negations. Finally, it should be noted that it follows from the set-theoretical theorem in (82) that antimorphic functions have the following property.

(130) **Theorem**

Let B and B^* be two Boolean algebras and let f be a function from B to B^*. If f is antimorphic, then
$f(X \cap Y) = f(X) \cup f(Y)$.

The above result, in combination with the functional characterization of antimorphic functions in (114), is sufficient to prove that the class of antimorphic expressions is identical to the class of classical negations. In other words, the hierarchy of subminimal, minimal, and classical negation, shown in table 11, is a linguistic reflection of the underlying hierarchy of monotone decreasing, anti-additive, and antimorphic functions. This completes our exposition.

Table 11: A hierarchy of negative expressions

Name	**Logical properties**
Expression of subminimal negation	(1) $f(X \cup Y) \subseteq f(X) \cap f(Y)$ (2) $f(X) \cup f(Y) \subseteq f(X \cap Y)$
Expression of minimal negation	(1) $f(X \cup Y) = f(X) \cap f(Y)$ (2) $f(X) \cup f(Y) \subseteq f(X \cap Y)$
Expression of classical negation	(1) $f(X \cup Y) = f(X) \cap f(Y)$ (2) $f(X) \cap f(Y) = f(X \cap Y)$ (3) $f(-X) = -f(X)$

NOTES

[1] The ideas explored in this paper have been presented at several occasions, among them the *Workshop on GPSG and Semantics* at the Technische Universität Berlin February 1989, the *Workshop on Categorial Grammar* at the LSA Summer Institute, University of Arizona, Tucson, the *Dritte Sommerschule der deutschen Gesellschaft für Sprachwissenschaft*, Universität Hamburg, September 1989, the *Jahrestagung der deutschen Gesellschaft für Sprachwissenschaft*, Universität des Saarlandes, Saarbrücken, March 1990, the *Second European Summer School in Language, Logic and Information*, Katholieke Universiteit Leuven, August 1990, and the *Workshop on the Logic of Perceptual Reports*, Università degli studi di Milano Gargnano, September 1990. I am indebted to Jay Atlas, Emmon Bach, Andrea Bonomi, Wojciech Buszkowski, Paolo Casalegno, Elisabet Engdahl, Fritz Hamm Christa Hauenschild, Jack Hoeksema, Hans Kamp, Mark Kas, Ed Keenan, Manfred Krifka, Bill Ladusaw, Dick Oehrle, Víctor Sánchez Valencia, Pieter Seuren, Anna Szabolcsi, Birgit Wesche, Ton van der Wouden, Dietmar Zaefferer, Annie Zaenen and above all, Johan van Benthem for valuable comments on earlier versions of the paper The work reported here is part of a larger project, *Reflections of Logical Patterns in Language Structure and Language Use*, which is supported by the Netherlands Organization for Scientific Research (NWO) within the framework of the PIONIER-program.

[2] The distinction between the sentences in (1) and (2) was first discussed in Zwarts (1981: 39, 123–127), be it from a slightly different point of view. Additional discussion can be found in Hoeksema (1983: 427–432) and Zwarts (1986: 192–195 313–319). It should be clear from the examples in (2) that the English negative polarity expression *anything (at all)* occurs in a larger set of environments than its Dutch counterpart *ook maar iets*. See Ladusaw (1979; 1980; and 1983) for discussion of the English data.

[3] The German examples involving the negative polarity item *sonderlich* (4c and

4d) are due to van Os (1989: 124). Kürschner (1983: 308–327) presents a comprehensive list of German negative polarity items, the only one which has appeared so far. Characteristic examples of the different types of polarity expressions in German can also be found in Krifka (1989). The interested reader may wish to consult Krifka (1990) as well, particularly in connection with the observed relationship between focus and certain classes of polarity items. In fact, this is what leads Krifka to adopt the framework of alternative semantics, developed by Rooth (1985) to account for the behavior of focus-sensitive expressions such as *only*.

[4] The laws in question are, of course, the familiar set-theoretical identities $-(X \cup Y) = -X \cap -Y$ and $-(X \cap Y) = -X \cup -Y$. Quine (1974: 69) speaks in this connection of the first, and the second, law of De Morgan, respectively.

[5] We ignore the fact that, in the intuitionistic propositional calculus, an entirely different interpretation is given to sentential negation. Readers who wish to acquaint themselves with this matter should consult Dummett (1977) and van Dalen (1986), among others.

[6] A description of the calculus of subminimal logic can be found in Hazen (1992) and Dunn (1993).

[7] The calculus of minimal logic was introduced by Johansson (1936). A short description of this system can be found in van Dalen (1986: 297–298), Gamut (1991: 139) and Dunn (1993: 25). Minimal negation is sometimes referred to as regular negation, a term which was introduced in Zwarts (1986: 351). The reader should note that our use of it has nothing to do with the way in which Seuren (1976) uses this notion. As a matter of fact, a significant part of the terminology employed in the present paper finds its origin in the study and classification of so-called *minimal models*, also known as *neighbourhood* or *Scott-Montague models*, in classical modal logic. See Chellas (1980) for a convenient introduction to this matter.

[8] A comparison of the conditionals in (9) and (10) shows that every logical law which governs the behavior of subminimal negation also governs the behavior of minimal negation.

[9] The examples in (15) are due to van Os (1989: 196). Additional discussion of the German polarity item *nennenswert* can be found in Biedermann (1969: 173).

[10] Jack Hoeksema points out that the Dutch adjective *onuitstaanbaar* 'unbearable' acts as a morphological counterpart of the composite verb *kunnen uitstaan*. In particular, the prefix *on-* 'un-' cannot be omitted (**uitstaanbaar*).

[11] The observations on the Dutch expression *mals* are due to Jack Hoeksema and have been discussed in van der Wouden (1994a, 1994b). Bill Ladusaw has supplied the English example *one bit*.

[12] Keenan and Faltz (1985) offer a detailed study of the role of Boolean algebras in the semantical analysis of natural languages.

[13] See Barwise (1979: 62) and Barwise and Cooper (1981: 184–185).

[14] Milsark (1977) uses *sm* to represent unstressed *some*. A very good discussion of the difference between *sommige* and *enkele*, the Dutch counterparts of stressed and unstressed *some*, is found in De Hoop and Kas (1989) and De Hoop (1990).

[15] The notion of an ideal is, of course, familiar from the theory of Boolean algebras. It involves a distinguished type of nonempty subset meeting the conditions in (44). See Sikorski (1969: 11) and Stoll (1974: 207). A detailed study and classification of ideals is provided by Stone (1937). The notion of a quasi-ideal is slightly more general in that it can also involve the empty set.

[16] See Chellas (1980: 215).

[17] See Zwarts (1991: 444).

[18] See Zwarts (1991: 447).

[19] See Stoll (1974: 211).

[20] See Chang and Keisler (1977: 143–144, 166).

[21] The relation of Boolean inclusion in the algebra 2 is defined in such a way that the zero element (0) contains only itself, whereas the unit element (1) contains both itself and the zero element.

[22] Such functions are sometimes said to be isotone. Their monotone decreasing counterparts, defined in (74), are accordingly referred to as antitone functions. See Birkhoff (1967: 3) and Stoll (1974: 55).

[23] See Chang and Keisler (1977: 307).

[24] See Bell and Slomson (1969).

[25] See Zwarts (1993) for a formal proof to this effect, using the assumptions and methods outlined in van Benthem (1986) and Zwarts (1983).

[26] Hoeksema (1986: 36–37) argues that the English determiner *each* differs from its Dutch counterpart *elk(e)* in that it is non-monotonic instead of anti-additive. Unlike *elk(e)*, *each* does not license negative polarity items in relative clauses, as shown by the ungrammaticality of **Each student who ever passed this test was killed by a mysterious disease*. Dutch *elk(e)*, on the other hand, is capable of licencing the strong polarity item *ook maar iets*: *Elk kind dat ook maar iets vermoedt, zal iemand waarschuwen* ('Each child that suspects anything will tell someone'). See Seuren (1985) for additional discussion.

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