Semantics

Greg Kobele

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Review

Semantic Interpretation

Expressions denote in boolean domains

t, et, eet, (et)t, ...

Semantic Combination via Function Application

$$\left[\!\!\left[\begin{array}{ccc} & \bullet & \\ / & & \backslash \\ \alpha & & \beta \end{array} \right]\!\!\right] = \begin{cases} [\![\alpha]\!]\,([\![\beta]\!]) & \textit{if } \alpha : \textit{ab and } \beta : \textit{a} \\ [\![\beta]\!]\,([\![\alpha]\!]) & \textit{if } \alpha : \textit{a and } \beta : \textit{ab} \end{cases}$$

I will write $\alpha \otimes \beta$ to mean $\alpha(\beta)$ or $\beta(\alpha)$, which ever is appropriate

The type $(et)t$

an object of type $(et)t$

- looks at a property
- and says **yes** or **no**

This is called a generalized quantifier

John
is true of a property P iff

someone
is true of a property P iff

everyone
is true of a property P iff

no one
is true of a property P iff

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Boolean Homomorphisms

g is a boolean homomorphism if

1. it distributes over

$$\begin{aligned}\textbf{complement} \quad & g(\neg a) = \neg(g(a)) \\ \textbf{meet} \quad & g(a \wedge b) = g(a) \wedge g(b) \\ \textbf{join} \quad & g(a \vee b) = g(a) \vee g(b)\end{aligned}$$

2. it maps extrema to extrema

$$\begin{aligned}\textbf{top} \quad & g(1) = 1 \\ \textbf{bottom} \quad & g(0) = 0\end{aligned}$$

Proper names revisited

We have shown:

1. all individuals are homomorphisms
2. all homomorphisms are individuals

’Entities’
are exactly those GQs which

- distribute over logical operations
- map extrema to extrema

A purely semantic characterization of proper name denotations

Determiners

Some GQs

- *every boy*
- *some girl*
- *no professor*
- *at least 3 students*
- *most doctors*
- *more doctors than lawyers*
- *between 3 and 12 professors*

Lexical GQs

Some *lexical* GQs

- *John, Mary, Susan, Bill*
- *everybody, nobody, somebody*
- ...?

Are there any?

Proper names
but:

- *this John, every Susan*

Quantifiers
everybody vs *every body*

Most GQs are derived

every boy danced
is built up out of (at least) three parts

dance, boy
are properties

every
maps a property *boy* to a GQ

Determiners

(et)(et)t

- function from properties to GQs
- function from two properties to a truth value

Counting

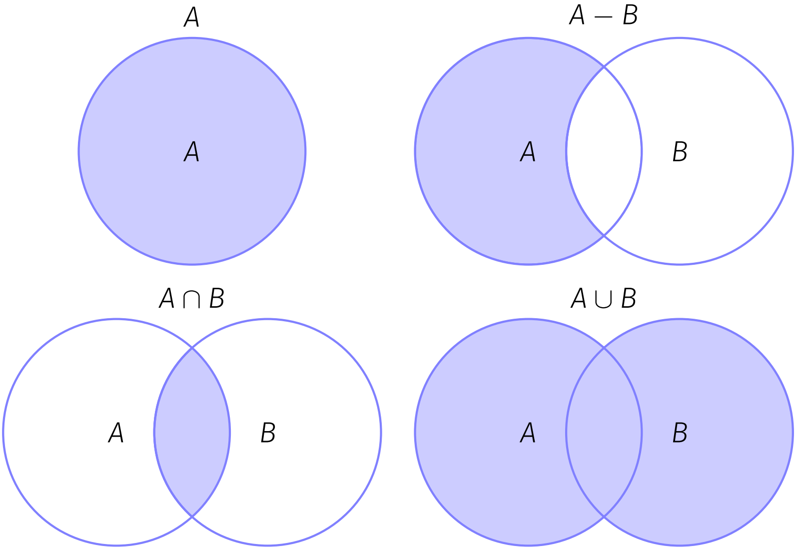
E	et	(et)t	(et)(et)t
n	2^n	$2^{(2^n)}$	$(2^{2^n})^{2^n} = 2^{4^n}$
1	2	4	16
2	4	16	65,536
3	8	256	18,446,744,073,709,551,616

Different Kinds of Determiners

Determiner Denotations

- $some(A)(B) = 1$ iff
  - some element in A which is also in B
  - $A \cap B \neq \emptyset$
- $every(A)(B) = 1$ iff
  - every element in A is also in B
  - $A \subseteq B$
  - $A - B = \emptyset$
  - $A \cap B = A$
- $no(A)(B) = 1$ iff
  - no element in A is also in B
  - $A - B = A$
  - $A \cap B = \emptyset$

Comparing Properties



What do we need to know...

Some students read the book
need to know:

- which students read the book?
- which students did *not* read the book?

Every student read the book
need to know:

- which students read the book?
- which students did *not* read the book?

Most students read the book
need to know:

- which students read the book?
- which students did *not* read the book?

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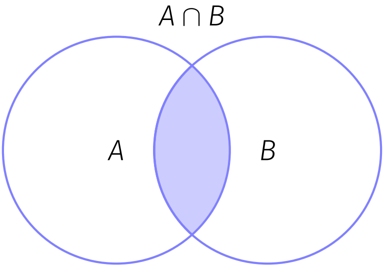
Intersections



Intersectivity

Some DETs only care about $A \cap B$
But then: $D(A)(B) = D(X)(Y)$ whenever $A \cap B = X \cap Y$

call these Intersective

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and so if D is *intersective*
$D(A)(B) = D(A \cap B)(E) = D(E)(A \cap B)$

- *D As* are *Bs*
- *D As* which are *Bs* exist
- *D* things which exist are *As* and *Bs*

Some Intersective Dets

More interesting is:

Theorem
if $D(A)(B) = D(A \cap B)(E)$ then D is intersective

This gives us a test for intersectivity!!!

Trying it out:

Some student laughed	Some student who laughed exists
Every student laughed	Every student who laughed exists
No student laughed	No student who laughed exists
Most students laughed	Most students who laughed exist

Seat Work

Which of the following are intersective?

- Several
- More than six
- Six out of ten
- Less than six
- At most 60% of the
- Exactly six
- Between one sixth and five sixths of the
- Between six and ten
- Almost 600

Not just intersective, but cardinal

Some DETs only care about the *size of* $A \cap B$
But then: $D(A)(B) = D(X)(Y)$ whenever $|A \cap B| = |X \cap Y|$

call these Cardinal

of course, these are intersective too

Cardinality

Most intersective dets are cardinal

- some, no, several, more than six, between six and ten, ...

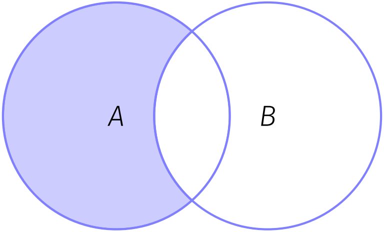
What *isn't* cardinal?!

i.e. what needs to know more about $A \cap B$ than just its size?

- which students read the book
- all students but John read the book
- more male than female students read the book

Co-intersection

$A - B$



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and some only care about $|A - B|$:$\mathbf{B_{block}}$:
these are Co-cardinal

Some Co-intersective Dets

- $\mathit{ALL}(A)(B)$
- $\mathit{ALL\ BUT\ SIX}(A)(B)$
- $\mathit{EVERY} \ldots \mathit{BUT\ JOHN}(A)(B)$

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- $ALL(A)(B)$
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- $EVERY \ldots\ BUT\ JOHN(A)(B)$

which are co-cardinal
and which are not?

Proportions

Some DETs are neither intersective nor co-intersective

- *Most* students smile
- *Nine out of ten* students will pass the exam
- *At most 10% of the* students slept during class
- *Between one third and two thirds of the* students are vegetarians
- *Not one* student *in ten* can answer that question

Proportionality

Some DETs care about comparing $A \cap B$ and A

$D(A)(B) = D(X)(Y)$ whenever $\frac{|A \cap B|}{|A|} = \frac{|X \cap Y|}{|X|}$

call these Proportional

Boolean Naturality

All of these classes of Dets form boolean algebras

- they have a top and a bottom element
- they are closed under the operations
  - i.e. if D, D' are in the same class,
  - then so are $\neg D, D \wedge D', D \vee D'$

Because (co-)intersective Dets depend on one set only

there are the same # of them as functions of type (et)t

E	Det	Int	Co-Int
1	16	4	4
2	65,536	16	16
3	18×10^{18}	256	256

Combining (Co-)Intersective Dets

What happens when we combine different kinds of Dets?

- not necessarily (co-)intersective!

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Conservativity

D is conservative iff

$$D(A)(B) = D(A)(A \cap B)$$

in words:

D A *is* B *iff* D A *is an* A *which is* B

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What happens when we combine different kinds of Dets?

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Theorem

The boolean closure of *intersective* and *co-intersective* Dets are the *conservative* Dets

Conservativity

Claim
All natural language Dets are conservative

A quick check...

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All natural language Dets are conservative

A quick check...

intersective $D(A)(B) = D(X)(Y)$ iff $A \cap B = X \cap Y$

$$\begin{aligned} A \cap B &= (A \cap A) \cap B \\ &= A \cap (A \cap B) \end{aligned}$$

Conservativity

Claim
All natural language Dets are conservative

A quick check...

intersective

co-intersective $D(A)(B) = D(X)(Y)$ iff $A - B = X - Y$

$$\begin{aligned} A - B &= A - ((B - A) \cup (B \cap A)) \\ &= (A - (B - A)) - (B \cap A) \\ &= A - (B \cap A) \\ &= A - (A \cap B) \end{aligned}$$

Claim
All natural language Dets are conservative

A quick check...

intersective

co-intersective

proportional $D(A)(B) = D(X)(Y)$ iff $\frac{|A \cap B|}{|A|} = \frac{|X \cap Y|}{|X|}$

$$\frac{|A \cap B|}{|A|} = \frac{|A \cap (A \cap B)|}{|A|}$$

How big a restriction is conservativity?

E	Det	Int	Co-Int	Cons
1	16	4	4	8
2	65,536	16	16	512
3	18×10^{18}	256	256	13×10^{7}

There are

total $2^{4^{|E|}}$

cons $2^{3^{|E|}}$

Most Dets are not conservative!

Is it even true?

Two possible exceptions:

only *only students drink absinthe*

many$_1$ *many swedes have won nobel prizes* can mean:

1. Of the nobel prize winners, many are swedes
2. many swedes have won nobel prizes

Linguistic Naturality

But what good is it?

A classification is not necessarily useful
some animals have

- white fur on their face, no white fur on face
- green eyes, blue eyes, brown eyes, black eyes
- four legs, two legs, no legs, (three legs)

Our semantic classification
describes allowable inferences

but is there more to it?

Existential There clauses

Good

1. There are many students in my class
2. There are some students in my class
3. There are more than five students in my class
4. There was no student but John in my class

Bad

1. There is every student in my class
2. There are most students in my class
3. There is between one and two thirds of the students in my class

The Existential Question

Which DPs go here:

There are X in my class

Claim
DPs with intersective Dets

Conclusion

- Determiners have type (et)(et)t
- Different kinds:
    - intersective
    - co-intersective
    - proportional
- But always: Conservative