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Statistical Mechanics of Deep Learning - Problem set 5

Winter Term 2023/24

Hand in: Friday, 17.11 at 09:00 am, you can upload your solutions to the course webpage on Moodle platform.

10. The Gardner Analysis

3+3+2 Points

(a) Determine the asymptotic behaviour of the quantities describing Gibbs learning for large training set size α. To this end, start from

$$\frac{R}{\sqrt{1-R}} = \frac{\alpha}{\pi} \int \boldsymbol{D}t \, \frac{\exp(-Rt^2/2)}{H(-\sqrt{R}t)}$$

and show that

$$1 - R \sim \left[\frac{\alpha}{\pi} \int \boldsymbol{D}t \, \frac{\exp(-t^2/2)}{H(-t)}\right]^{-2} \sim \frac{1.926}{\alpha^2}$$

Hint : You may use numerical methods to compute the final result.

(b) Use the result you obtain from part (a) to show that

$$\varepsilon \sim \frac{\sqrt{2}}{\int \boldsymbol{D}t \, [\exp(-t^2/2)]/H(t)} \frac{1}{\alpha} \sim \frac{0.625}{\alpha},$$

as follows from

$$\varepsilon = \frac{1}{\pi}\arccos(R)$$

(c) Show also that the quenched entropy for large α is

$$S \sim \frac{1}{2}\ln(1-R) \sim -\ln(\alpha).$$

11. Distribution of a product of Random Numbers

4+2 Points

Generate numerically M random numbers x, each being the product of N independent random numbers equally distributed between 1 and 2 with M between 10^3 and 10^6 and N between 5 and 50.

(a) Approximate the distribution of x by a histogram and compare the evolution with N of the most probable value x_{mp} of x with its average $\langle\langle x \rangle\rangle$ and the typical x value defeind as $x_{typ} := \exp(\langle\langle \ln x \rangle\rangle)$

(b) Can you give an argument, why asymptotically (for $N \to \infty$), the most probable value and x_{typ} should coinside?