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Advanced Quantum Mechanics - Problem Set 2

Winter Term 2023/24

Due Date: Hand in solutions to problems marked with * as a single pdf file using Moodle before the lecture on **Thursday, 26.10.2023, 15:15**. The problem set will be discussed in the tutorials on Monday 30.10.2023 and Wednesday 01.11.2023.

Website: https://home.uni-leipzig.de/stp/Quantum_Mechanics_2_WS2324.html

Moodle: https://moodle2.uni-leipzig.de/course/view.php?id=45746

*1. Simultaneous eigenstates

4+2+2 Points

(a) Let $\hat{A}$ and $\hat{B}$ be two Hermitian operators which commute. Show that there exists a common set of eigenstates of the two operators. Distinguish between the case where the states are non-degenerate and n-fold degenerate.

(b) Assume now that $|\Psi\rangle$ is a simultaneous eigenstate of $\hat{A}$ and $\hat{B}$, and that $\hat{A}$ and $\hat{B}$ anti-commute: $\hat{A}\hat{B} + \hat{B}\hat{A} = 0$. What can you say about the eigenvalues of the two operators?

(c) Give a concrete example of your result from part (b) using the parity and momentum operators.

2. Symmetric double-well potential

3+4 Points

Consider a symmetric rectangular double-well potential

$$V(x) = \begin{cases} \infty, & |x| > a + b, \\ 0, & a < |x| < a + b, \\ V_0, & |x| < a, \end{cases}$$

with $V_0 > 0$.

(a) Determine the general solution of the Schrödinger equation in the different regions, and use suitable boundary conditions to connect these solutions for subregions. Determine the quantization condition for the parameters in your solution.

Hint: Due to the symmetry of the problem you can choose solutions which are also eigenstates of parity.

(b) Assuming very large V_0, calculate the energies of the ground state and the first excited state from the quantization condition derived in a).

3. Translation Operator

1+2 Points

Consider a free particle with Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2m},$$

and define the translation operator $\hat{T}_l$.

(a) Show that $[\hat{H}, \hat{T}_l] = 0$.

(b) Due to the result in (a), the Hamiltonian and translation operator have a common set of eigenstates. For such a state $|k\rangle$, calculate the eigenvalue of $\hat{T}_l$. That is calculate λ_k in the expression $\hat{T}_l|k\rangle = \lambda_k|k\rangle$.