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Mathematical Methods of Modern Physics - Problem Set 2

Summer Semester 2024

Due: The problem set will be discussed in the seminars on 18.04. and 19.04.

Internet: The problem sets can be downloaded from
https://home.uni-leipzig.de/stp/Mathematical_methods_2_ss24.html

1. Curves in the complex plane

1+2+1+1 Points

Which curves in the complex plane are described by the following equations:

a) $\left|\frac{z-1}{z+1}\right| = 1$

b) $\left|\frac{z-1}{z+1}\right| = 2$

c) $\mathrm{Re}(z^2) = 4$

d) $\bar{z} = z^{-1}$

2. Complex functions

1+1+1+1+1 Points

Write the following functions in the form $w(x+iy) = u(x,y) + iv(x,y)$ and determine their domain of definition

a) $f(z) = 3z^2 + 5z + i + 1$

b) $g(z) = \dfrac{z+i}{z^2+1}$

c) $h(z) = \dfrac{2z^2+3}{|z-1|}$

d) $q(z) = e^z + e^{-\bar{z}}$

e) What is the range of $q(z)$ in d)?

3. Complex sine and cosine

1+2+1+1 Points

The complex sine and cosine can be defined either by their power series or by the complex exponential

$$\sin(z) := \frac{1}{2i}(e^{iz} - e^{-iz})$$
$$\cos(z) := \frac{1}{2}(e^{iz} + e^{-iz})$$

Show that:

a) For $z \in \mathbb{R}$ these definitions give the known real sine and cosine.

b) The equations $\sin(z) = 0$ and $\cos(z) = 0$ have only real solutions.

c) For all $z \in \mathbb{C}$ it is $(\sin(z))^2 + (\cos(z))^2 = 1$.

d) It is $|\sin(z)| \xrightarrow{\mathrm{Im}(z)\to\infty} \infty$ and $|\cos(z)| \xrightarrow{\mathrm{Im}(z)\to\infty} \infty$.