Institut für Theoretische Physik
Universität Leipzig

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Advanced Quantum Mechanics - Problem Set 13

Winter Term 2023/24

Due Date: Hand in solutions to problems marked with * as a single pdf file using Moodle before the lecture on **Thursday, 25.01.2024, 15:15**. The problem set will be discussed in the tutorials on Monday 29.01.2024 and Wednesday 31.01.2024.

Website: https://home.uni-leipzig.de/stp/Quantum_Mechanics_2_WS2324.html

Moodle: https://moodle2.uni-leipzig.de/course/view.php?id=45746

1. Number operator *4 Points*

Consider an operator $\hat{a}$ which satisfies $\{\hat{a}, \hat{a}^\dagger\} = \hat{a}\hat{a}^\dagger + \hat{a}^\dagger a = 1$ and $\{a, a\} = \{a^\dagger, a^\dagger\} = 0$. Show that the operator $\hat{N} = \hat{a}^\dagger \hat{a}$ has eigenvalues 0 and 1. What would you get if the anti-commutator is replaced by a commutator?

*2. Berry phase and the Aharonov-Bohm effect *2+2+1+3 Points*

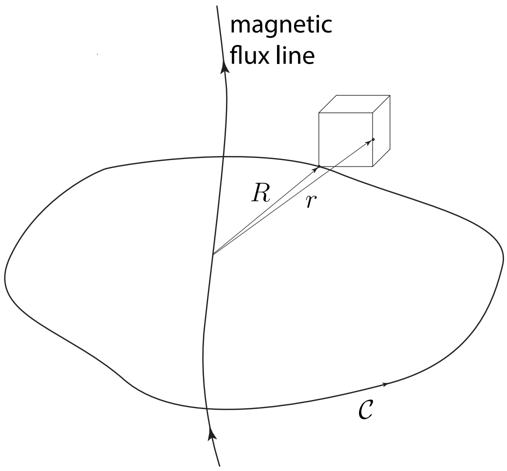

Figure 1: An electron in a box moves around a magnetic flux line. The path of the electron encloses a flux Φ_B.

Consider an electron in a small box moving along a closed loop $\mathcal{C}$, which encloses a magnetic flux Φ_B as shown in Fig. 1. Let $\boldsymbol{R}$ denote the position vector of a point on the box and $\boldsymbol{r}$ the position vector of the electron itself.

(a) Show that if the wave function of the electron in the absence of a magnetic field is $\psi_n(\boldsymbol{r} - \boldsymbol{R})$, then the wave function of the electron in the box at position $\boldsymbol{r}$ is

$$\langle \boldsymbol{r}|n(\boldsymbol{R})\rangle = \exp\left[\frac{ie}{\hbar}\int_{\boldsymbol{R}}^{\boldsymbol{r}} \boldsymbol{A}(\boldsymbol{r'})\cdot d\boldsymbol{r'}\right]\psi_n(\boldsymbol{r}-\boldsymbol{R}).$$

Here $\boldsymbol{A}$ denotes the vector potential. Note that this is only true if the magnetic field inside the box is zero. Why?

(b) Show that

$$\langle n(\boldsymbol{R})|\nabla_{\boldsymbol{R}}|n(\boldsymbol{R})\rangle = -\frac{ie}{\hbar}\boldsymbol{A}(\boldsymbol{R}).$$

(c) Calculate the geometric phase

$$\gamma_n(\mathcal{C}) = i\oint_{\mathcal{C}} \langle n(\boldsymbol{R})|\nabla_{\boldsymbol{R}}|n(\boldsymbol{R})\rangle \cdot d\boldsymbol{R},$$

and comment on your result.

(d) Suppose now an electron moves above or below a very long impenetrable cylinder as shown in the Fig. 2. Inside the cylinder there is a magnetic field parallel to the cylinder axis, taken to be normal to the plane of the figure. Outside the cylinder there is no magnetic field but the particle paths enclose a magnetic flux. Calculate the interference due to the presence of the magnetic flux.

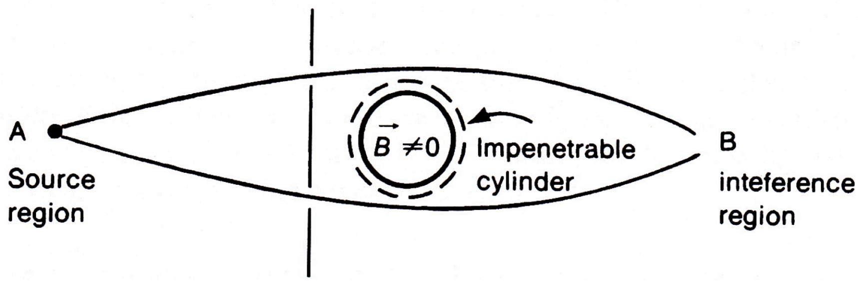

Figure 2: An electron moves either above or below an impenetrable cylinder enclosing a magnetic field parallel to its axis.