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Advanced Quantum Mechanics - Problem Set 5

Winter Term 2023/24

Due Date: Hand in solutions to problems marked with * as a single pdf file using Moodle before the lecture on **Thursday, 16.11.2023, 15:15**. The problem set will be discussed in the tutorials on Monday 20.11.2023 and Wednesday 22.11.2023.

1. Eigenspinors

4+1 Points

Consider a spin 1/2 system in the presence of an external magnetic field $\boldsymbol{B} = B\hat{\boldsymbol{n}}$, where $\hat{\boldsymbol{n}}$ is a unit vector pointing in an arbitrary direction. The Hamiltonian of this system is given by

$$\hat{H} = -\frac{e}{mc}\hat{\boldsymbol{S}} \cdot \boldsymbol{B},$$

where $e < 0$ is the electron charge, m the electron mass, c the speed of light, and $\hat{\boldsymbol{S}}$ the vector of spin 1/2 operators.

(a) Calculate the eigenvalues and normalized eigenspinors of the Hamiltonian.

(b) Why does the direction of the eigenspinors only depend on $\hat{\boldsymbol{n}}$?

2. Time- and spin-reversal

2+3 Points

(a) Denote the wave function of a spinless particle corresponding to a plane wave in three dimensions by $\psi(\boldsymbol{x}, t)$. Show that $\psi^*(\boldsymbol{x}, -t)$ is the wave function for the plane wave if the momentum direction is reversed.

(b) Let $\chi(\hat{\boldsymbol{n}})$ be the eigenspinor you calculated in the previous problem, with eigenvalue +1. Using the explicit form of $\chi(\hat{\boldsymbol{n}})$ in terms of the polar and azimuthal angles which define $\hat{\boldsymbol{n}}$, verify that the eigenspinor with spin direction reversed is given by $-i\sigma_y\chi^*(\hat{\boldsymbol{n}})$.

*3. Time reversal of a lattice Hamiltonian

2+3+2 Points

In this problem we will consider the effects of time reversal on a lattice Hamiltonian.

(a) First consider the lattice translation operator $\hat{T}_a = e^{-i\hat{p}a}$. How do the eigenvalues of the translation operator transform under time reversal?

(b) Now consider the Hamiltonian

$$H(\boldsymbol{k}) = \sin(k_x)\sigma_x + \sin(k_y)\sigma_y + M\sigma_z,$$

where k_x and k_y are components of the momentum appearing in the eigenvalues of the translation operator and M is a constant. How does this Hamiltonian transform in the case where σ are (i) spin matrices and (ii) some "orbital" matrices (such as in the problem on the SSH model)?

(c) Generalize your result to a Hamiltonian of the form $H(\boldsymbol{k}) = \boldsymbol{d}(\boldsymbol{k}) \cdot \boldsymbol{\sigma}$.