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Quantum Mechanics 2 - Problem Set 14

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Abgabe: The problem set will be discussed in the tutorials on **Thursday, 01.02.2018, 11:00** (German) and **Friday, 02.02.2018, 13:30** (English).

42. Landau levels in graphene

4+4+2 Punkte

The low-energy Hamiltonian around the Dirac points in graphene is

$$H = v_F \begin{pmatrix} -\sigma^* \cdot \mathbf{p} & 0 \\ 0 & \sigma \cdot \mathbf{p} \end{pmatrix},$$

where v_F is the Fermi velocity, $\mathbf{p} = (p_x, p_y)$ the momentum and σ the vector of Pauli matrices. The eigenstates can be written as four-dimensional state vectors with contributions from the K and K' points. That is, we write

$$\chi = \begin{pmatrix} \chi_A \\ \chi_B \\ \chi'_A \\ \chi'_B \end{pmatrix}.$$

(a) Show that the eigenvalue equations decouple into

$$E^2 \chi_A = v_F^2 (p_x + ip_y)(p_x - ip_y)\chi_A,$$
$$E^2 \chi_B = v_F^2 (p_x - ip_y)(p_x + ip_y)\chi_B,$$

and similar for the primed parts of the eigenstates.

(b) Suppose now a magnetic field is switched on. Using the Landau gauge $\mathbf{A} = (-By, 0)$, perform the minimal substitution $\mathbf{p} \to \mathbf{p} - e\mathbf{A}$ in the eigenvalue equations in part (a) and deduce the form of the eigenfunctions.

(c) What does the energy spectrum look like?

43. Unit cell in the presence of a magnetic field *2+5+3 Punkte*

Recall that the operator $\hat{T}_{\mathbf{a}} = e^{\frac{i}{\hbar}\mathbf{a}\cdot\hat{\mathbf{p}}}$ is the generator of translations. For a Hamiltonian with lattice translation symmetry, these operators commute with the Hamiltonian. In a magnetic field this is no longer the case since the vector potential is not translationally invariant. In this problem we will consider a two-dimensional electron gas in the presence of a magnetic field in the z-direction $\mathbf{B} = (0, 0, B)$. The Hamiltonian can be written as

$$\hat{H} = \frac{(\hat{p} - eA(\mathbf{r}))^2}{2m} + V(\mathbf{r}),$$

where $V(\mathbf{r})$ is the periodic lattice potential ($V(\mathbf{r}+\mathbf{a}) = V(\mathbf{r})$ for lattice vectors $\mathbf{a}$) and we use symmetric gauge for the vector potential $\mathbf{A}(\mathbf{r}) = \frac{1}{2}(-By, Bx, 0)$.

(a) Show that a new translation operator

$$\hat{\mathcal{T}}_{\mathbf{a}} = \exp\left\{\frac{i}{\hbar}\mathbf{a}\cdot\left[\hat{\mathbf{p}} + e\mathbf{A}(\mathbf{r})\right]\right\}$$

commutes with the Hamiltonian $\hat{H}$. This translation operator is called a magnetic translation operator.

(b) Show that

$$\hat{\mathcal{T}}_{\mathbf{a}}\hat{\mathcal{T}}_{\mathbf{b}} = \exp\left[\frac{i}{l_0^2}(\mathbf{a}\times\mathbf{b})\cdot\hat{e}_z\right]\hat{\mathcal{T}}_{\mathbf{b}}\hat{\mathcal{T}}_{\mathbf{a}}.$$

Here $l_0 = \sqrt{\frac{\hbar}{eB}}$ is the magnetic length and $\hat{e}_z$ is a unit vector perpendicular to the plane.

(c) We now want to determine the enlarged unit cell such that the magnetic translation operators commute with each other. Let therefore $n\mathbf{a}$ and $m\mathbf{b}$ span an enlarged unit cell in the plane. In this case the magnetic translation operators have to commute with each other. Show that this is only possible if the flux $\Phi = \mathbf{B}\cdot(\mathbf{a}\times\mathbf{b})$ satisfies

$$\frac{\Phi}{\Phi_0} = \frac{l}{mn},$$

with l an integer and $\Phi_0 = h/e$.