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Quantum Mechanics 2 - Problem Set 13

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Abgabe: The problem set will be discussed in the tutorials on **Thursday, 25.01.2018, 11:00** (German) and **Friday, 26.01.2018, 13:30** (English).

39. Zero-energy domain wall state in a lattice model *4+4 Punkte*

Consider the SSH model discussed in the lectures, where the coupling between sites within a unit cell is $-t$ and the coupling between sites in different unit cells is $-s$. The zero-energy domain wall state discussed in the lectures can also be obtained from a lattice model. To illustrate this let's compute the eigenstates and eigenenergies for the two situations illustrated in Fig. 1. In the first case [Fig. 1(a)], we have a finite chain in a trivial phase ($t \neq 0$, $s = 0$) in the absence of domain wall. In the second case [Fig. 1(b)], we have a domain wall between a trivial phase ($t \neq 0$, $s = 0$) and a nontrivial phase ($t = 0$, $s \neq 0$).

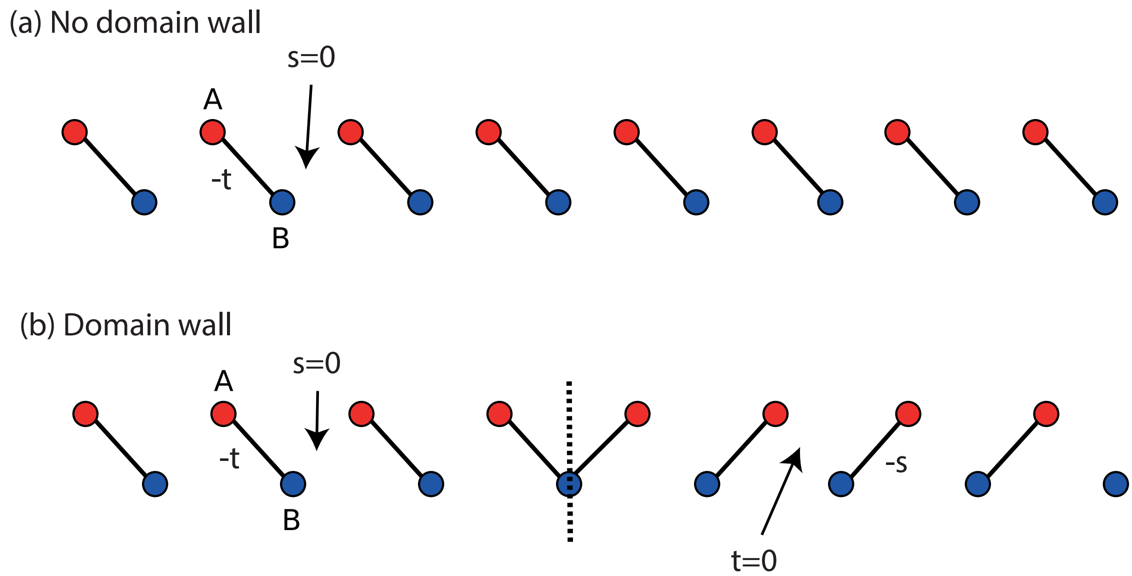

Figure 1: (a) SSH chain in the trivial phase ($t \neq 0$, $s = 0$) in the absence of a domain wall. (b) SSH chain in the presence of a domain wall (vertical dashed line). On the left side of the domain wall the SSH chain is in a trivial phase ($t \neq 0$, $s = 0$) and on the right side of the domain wall the SSH chain is in a nontrivial phase ($t = 0$, $s \neq 0$).

(a) Write down the Hamiltonians in a matrix form (using the states localized at the lattice sites $|j, A\rangle$ and $|j, B\rangle$ as the basis states) for the two situations illustrated in Fig. 1(a) and (b). Notice that in both cases the Hamiltonians are block diagonal. (This means that in these cases there is no need to use Fourier transform to go to the momentum space.)

(b) Find the eigenstates and eigenenergies for the two Hamiltonians.

40. Fractional charge of a domain wall in SSH model *5+2 Punkte*

In the ground state of a fermionic system all states with energy $E < 0$ are occupied and $E > 0$ are unoccupied. The $E = 0$ states may either be occupied or unoccupied.

(a) Use the eigenstates obtained in Problem 39 to calculate the charge localized on each lattice site in the absence and in the presence of the domain wall. In the presence of the domain wall consider the different cases where the domain wall state and the end state are either occupied or unoccupied.

Hint: If we write the eigenstate $|\Psi(E_n)\rangle$ with energy E_n in the form

$$|\Psi(E_n)\rangle = \sum_j \Big[\psi_{j,A}(E_n)|j,A\rangle + \psi_{j,B}(E_n)|j,B\rangle\Big],$$

the charge localized for example on the site (j, A) is given by

$$q_{jA} = e \sum_{E_n<0} |\psi_{j,A}(E_n)|^2$$

if the zero-energy states $E_n = 0$ are unoccupied. If a zero-energy state is occupied one needs to add the corresponding contribution $e|\psi_{j,A}(E_n = 0)|^2$ to the sum.

(b) By comparing the charge differences in the absence and presence of the domain wall, show that the domain wall carries a charge $\pm e/2$ (localized in the vicinity of the domain wall) depending on whether the zero-energy domain wall state is occupied or unoccupied.

41. Anyons and the Aharonov-Bohm effect *3+2 Punkte*

Consider a two-dimensional electron gas in the presence of a magnetic field. The conductivity tensor is given by

$$\sigma = \begin{pmatrix} 0 & \sigma_{xy} \\ \sigma_{xy} & 0 \end{pmatrix},$$

where $\sigma_{xy} = \nu e^2/h$, with $0 < \nu < 1$, is the Hall conductivity.

(a) Suppose now a flux Φ is turned on adiabatically. Using Faraday's law and that the current density is given by $\mathbf{J} = \sigma\mathbf{E}$, where $\mathbf{E}$ is the induced electric field, show that the charge satisfies $\dot{Q} = \sigma_{xy}\dot{\Phi}$. How does the charge change if the flux changes by $\Phi_0 = h/e$?

(b) Now consider the composite object (quasiparticle) of a flux Φ_0 and charge $q = \nu e$. Determine the mutual exchange statistics of these quasiparticles. When do these composite objects behave as electrons? What do you get for $\nu = 1/3$ and $\nu = 1/5$? These states have been observed in experiments.

Hint: The exchange of the two quasiparticles corresponds to moving one quasiparticle by half a circle and performing a translation. This suggests that the wave function acquires a phase, which is half of the Berry acquired by a charge $q = \nu e$ moving along a path enclosing a magnetic flux Φ_0. Use this reasoning to obtain the exchange statistics of the composite objects for different values of ν. Quasiparticles which acquire a phase different from 0 (bosons) or π (fermions) in the exchange are called anyons.

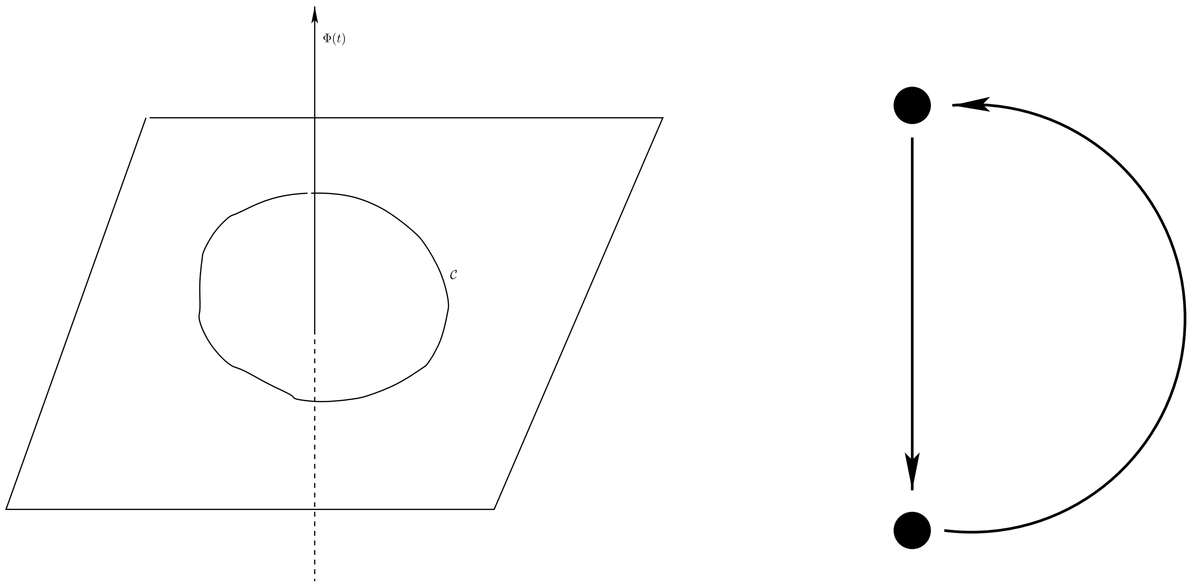

Figure 2: Left: The composite object is made up of a flux enclosed by a path $\mathcal{C}$ and a charge. Right: Illustration of how to exchange two quasiparticles.