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Quantum Mechanics 2 - Problem Set 8

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Abgabe: The problem set will be discussed in the tutorials on **Thursday, 07.12.2017, 11:00** (German) and **Friday, 08.12.2017, 13:30** (English).

25. Time-reversal and rotations

2+3 Punkte

Let $\hat{T}$ denote the time-reversal operator. It is possible to choose the phase convention in the angular momentum states $|j,m\rangle$ so that $\hat{T}|j,m\rangle = i^{2m}|j,-m\rangle$. Consider a rotation operator $\hat{D}(\vec{\varphi}) = e^{-i\vec{\varphi}\cdot\hat{\mathbf{J}}/\hbar}$ with matrix elements $D^{(j)}_{m',m}(\vec{\varphi}) = \langle j,m'|\hat{D}(\vec{\varphi})|j,m\rangle$.

(a) What is the time-reversed state corresponding to $\hat{D}(\vec{\varphi})|j,m\rangle$?

(b) Using the properties of time reversal and rotations, prove that

$$\left(D^{(j)}_{m',m}(\vec{\varphi})\right)^* = (-1)^{m-m'} D^{(j)}_{-m',-m}(\vec{\varphi}).$$

26. Addition of angular momenta

5+3+1 Punkte

Consider two angular momenta $\hat{\mathbf{L}}_1$ and $\hat{\mathbf{L}}_2$ with $l_1 = l_2 = 1$. In this problem we will calculate the eigenvalues and eigenstates of $\hat{\mathbf{L}}^2$. The eigenstates can be written as linear combinations of the 9 basis states

$$|l_1 = 1, l_2 = 1; m_1, m_2\rangle, \qquad \text{with } m_1, m_2 = 1, 0, -1.$$

(a) Construct the 9×9 matrix representation of the operator $\hat{\mathbf{L}}^2$ in this basis.

(b) Calculate the eigenvalues of $\hat{\mathbf{L}}^2$ by diagonalising the matrix.

(c) Calculate the corresponding eigenstates.

Hint: It is possible to make the matrix block-diagonal, as shown in the figure, by making suitable row- and column-operations.

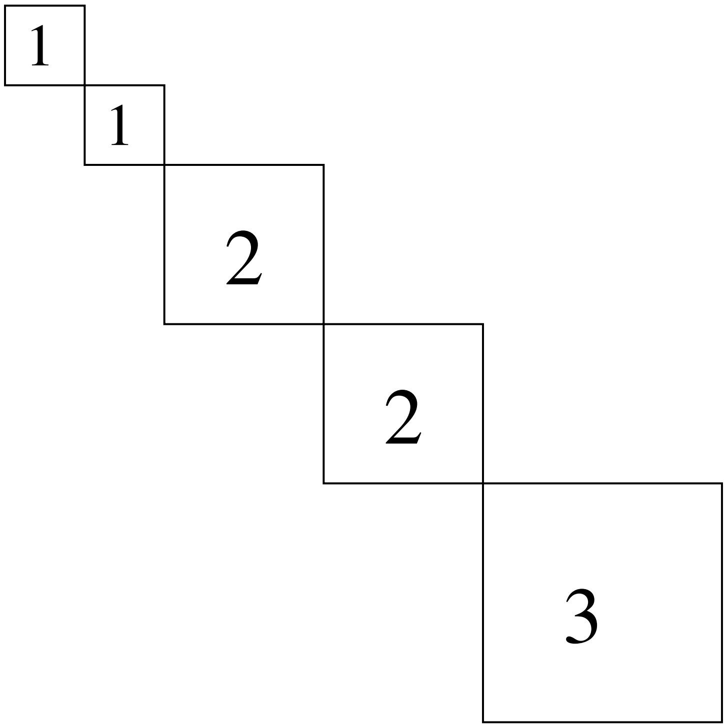

Figure 1: The matrix can be transformed into a block diagonal form.

27. Spin-orbit coupling

2+2+2 Punkte

Consider a particle with orbital angular momentum $\mathbf{L}$ and spin angular momentum $\mathbf{S}$. The total angular momentum is $\mathbf{J} = \mathbf{L} + \mathbf{S}$.

(a) Calculate the expectation value of $\mathbf{L} \cdot \mathbf{S}$ assuming that the particle is in a state $|l, s; j, m\rangle$.

(b) An electron is moving in an electrostatic potential $\phi(r)$. Show that the electric field experienced by the particle is given by

$$\mathbf{E} = -\mathbf{r}\frac{1}{r}\frac{d\phi}{dr}.$$

(c) In the rest frame of the particle, the particle experiences a magnetic field $\mathbf{B} = -\mathbf{v} \times \mathbf{E}/c^2$. Calculate the energy $\frac{e}{m}\mathbf{S} \cdot \mathbf{B}$, where e and m are the electron charge and mass respectively.

Comment: The result found in (c) is off by a factor of two compared to the exact result, which can be obtained using the Dirac equation. The reason is that the simple argument given above assumes a straight-line motion of the particle whereas the potential given above leads to a curved particle trajectory.