Institut für Theoretische Physik
Universität Leipzig

Prof. Dr. B. Rosenow
Dr. C. Drukier
Dr. T. Hyart

Quantum Mechanics 2 - Problem Set 6

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Abgabe: The problem set will be discussed in the tutorials on **Thursday, 23.11.2017, 11:00** (German) and **Friday, 24.11.2017, 13:30** (English). Bachelor students will get bonus points by solving these problems. The exam for the bachelor students will not contain questions about the Dirac equation.

17. Klein tunneling

3+2 Punkte

Consider a scattering of a one-dimensional Dirac fermion with mass m from a potential barrier. The Hamiltonian describing this scattering process is

$$\hat{H} = -\hat{p}\sigma_y + m\sigma_z + V(x)$$

and the potential barrier is given by $V(x) = V_0$ for $0 \leq x \leq a$ and $V(x) = 0$ elsewhere.

(a) Describe how to calculate the transmission probability $T(E)$ assuming that the energy of the incoming particle satisfies $E > m$.

(b) Calculate $T(E)$ for $m = 0$.

Remark: Signatures of Klein tunneling have been experimentally observed in graphene [e.g. N. Stander, B. Huard and D. Goldhaber-Gordon, Physical Review Letters **102**, 026807 (2009) and A. F. Young and P. Kim, Nature Physics **5**, 222 (2009)].

18. Representations of γ matrices

2+1+2 Punkte

The γ matrices can be written as

$$\gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}, \qquad i = 1, 2, 3$$

$$\gamma^0 = \begin{pmatrix} \mathbb{1}_2 & 0 \\ 0 & -\mathbb{1}_2 \end{pmatrix},$$

where σ_i denotes a Pauli matrix and $\mathbb{1}_n$ the $(n \times n)$ unit matrix.

(a) Show that the γ matrices satisfy the Clifford algebra $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}\mathbb{1}_4$, $\mu, \nu = 0, 1, 2, 3$.

(b) A different representation is the Weyl representation where

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{1}_2 \\ \mathbb{1}_2 & 0 \end{pmatrix}.$$

Show that these still satisfy the Clifford algebra.

(c) Using the Clifford algebra show that $\mathrm{Tr}(\gamma^\mu) = 0$, $\mathrm{Tr}(\gamma^\mu\gamma^\nu) = 4\eta^{\mu\nu}$ and $\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho) = 0$.

19. Continuity equation for the Dirac equation *5 Punkte*

Prove that the solutions Ψ of Dirac equation satisfy the continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0,$$

where

$$\mathbf{j} = \Psi^\dagger \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix} \Psi \quad \text{and} \quad \rho = \Psi^\dagger \Psi.$$

20. Free particle solutions of the Dirac equation *5 Punkte*

Calculate the eigenvalues of the free-particle Dirac equation

$$\begin{pmatrix} m & 0 & p & 0 \\ 0 & m & 0 & -p \\ p & 0 & -m & 0 \\ 0 & -p & 0 & -m \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} = E \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix}.$$