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Quantum Field Theory of Many-Particle Systems - Problem Set 4

Summer Semester 2024

Due: The problem set will be discussed in the tutorial on **Friday, 03.05.2024, 13:30**.

Internet: The problem sets can be downloaded from
https://home.uni-leipzig.de/stp/QFT_of_MPS_SS24.html

1. Fermion coherent states

4+3+3+3+3 Punkte

Fermion coherent states and their properties are needed for the derivation of fermionic functional integrals. Consider a fermionic coherent state $|\eta\rangle = e^{-\sum_i \eta_i a_i^\dagger}|0\rangle, \langle\eta| = \langle 0|e^{-\sum_i a_i \bar{\eta}_i}$ and verify the following identities:

(a)

$$\langle\eta|a_i^\dagger = \langle\eta|\bar{\eta}_i,$$

and

$$a_i|\eta\rangle = \eta_i|\eta\rangle,$$

(b)

$$a_i^\dagger|\eta\rangle = -\partial_{\eta_i}|\eta\rangle,$$

and

$$\langle\eta|a_i = \partial_{\bar{\eta}_i}\langle\eta|,$$

(c)

$$\langle\eta|\nu\rangle = e^{\sum_i \bar{\eta}_i \nu_i},$$

(d)

$$\int d(\bar{\eta},\eta)\, e^{-\sum_i \bar{\eta}_i \eta_i}|\eta\rangle\langle\eta| = 1_F.$$

Hint: Proceed in analogy to the proof for bosonic states which was given in lectures, i.e., show that the integral commutes with all operators in Fock space.

(e)

$$\langle n'|\eta\rangle\langle\eta|n\rangle = \langle\zeta\eta|n\rangle\langle n'|\eta\rangle.$$

Here $|n\rangle$ and $|n'\rangle$ are Fock states with the same parity of the number of particles (that is either both have an even or both have an odd number of particles).

2. Green functions in momentum space

Bonus 5+5 Punkte

The time ordered momentum space Green function is defined by the ground state expectation value

$$G(t, \mathbf{k}) = -i \langle \hat{T}_t \hat{a}(t, \mathbf{k}) \hat{a}^\dagger(0, \mathbf{k}) \rangle.$$

Here, we consider non-interacting particles with energy $\epsilon(\mathbf{k})$ and chemical potential μ, and $\hat{a}(t, \mathbf{k})$ and $\hat{a}^\dagger(t, \mathbf{k})$ are annihilation and creation operators in the Heisenberg picture. $\hat{T}_t$ denotes the time ordering operator. Evaluate the Green function for a) non-interacting bosons and b) non-interacting fermions at zero temperature. The ground state for bosons is the vacuum (i.e. $\mu < 0$), and for fermions the Fermi sea (Fermi creation and annihilation operators are defined with respect to the Fermi sea).

Hint: In order to evaluate the time-dependence of the Heisenberg operators it may be useful to recall the Heisenberg equation of motion $-i\hbar\dot{A} = [H, A]$, where A is an operator in the Heisenberg picture (in this case $\hat{a}(t, \mathbf{k})$ or $\hat{a}^\dagger(t, \mathbf{k})$) and H is the Hamiltonian.