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Mathematical Methods of Modern Physics - Problem Set 4

Summer Semester 2024

Due: The problem set will be discussed in the seminars on 02.05. and 03.05.

Internet: The problem sets can be downloaded from
https://home.uni-leipzig.de/stp/Mathematical_methods_2_ss24.html

1. Constant functions *1+2+2 Points*

Let Ω be a complex domain and $f : \Omega \to \mathbb{C}$ a holomorphic function. Show that

a) If $f(z)$ is real, then f is constant.
b) If $\arg(f(z)) = \text{const}$, then f is constant.
c) If $|f(z)| = \text{const}$, then f is constant.

2. Holomorphic functions *1+1+1 Points*

Let $u(x, y)$ be the real part of a holomorphic function $f(z) = f(x + iy)$. Determine the function $f(z)$ for

a) $u(x, y) = x^3 - 3xy^2$
b) $u(x, y) = e^x \sin(y)$
c) $u(x, y) = \frac{1}{2}(e^y + e^{-y}) \sin(x)$

3. Derivatives *1+1+1+1 Points*

Use the rules discussed in the lecture to calculate the derivatives of

a) $f(z) = 6z^3 + 8z^2 + iz + 10$ b) $f(z) = \left(z^3 - 3i\right)^{-6}$

b) $f(z) = \frac{z^2-9}{iz^3+2z+\pi}$ d) $f(z) = \frac{(z+2)^2}{(z^2+iz+1)^4}$

4. Extrema of the real and imaginary part *3 Points*

In the lecture, it was shown that the real part $u(x, y)$ and the imaginary part $v(x, y)$ of a holomorphic function satisfy Laplace's equation. Show that neither $u(x, y)$ nor $v(x, y)$ can have a maximum or a minimum in any domain in which f is holomorphic (They can have saddle points only).
A maximum (minimum) in this context is defined as a point that is larger (smaller) than all points in a neighborhood around this point.