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## Mathematical Methods of Modern Physics - Problem Set 3

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*Summer Semester 2024*

**Due:** The problem set will be discussed in the seminars on 25.04. and 26.04.

**Internet:** The problem sets can be downloaded from  
[https://home.uni-leipzig.de/stp/Mathematical\\_methods.2\\_ss24.html](https://home.uni-leipzig.de/stp/Mathematical_methods.2_ss24.html)

### 1. Surjectivity of the exponential function 2 Points

Show that for any  $c \in \mathbb{C}$  there is sequence  $(z_n)$  in  $\mathbb{C}$  with  $\exp(z_n) \xrightarrow{n \rightarrow \infty} c$ , and  $|z_n| \xrightarrow{n \rightarrow \infty} \infty$ .

### 2. Riemann Surfaces 3 Points

Construct the Riemann Surfaces of the function.

$$f(z) = r^{\frac{1}{3}} e^{i\phi/3}, \quad z = r e^{i\phi}$$

### 3. Sequences 1+1+1 Points

Decide whether each of these sequences converges, and if so, find its limit.

$$a) z_n = \frac{i}{n} \quad b) z_n = i(-1)^n \quad c) z_n = \left(\frac{1+i}{4}\right)^n$$

### 4. Continuity and continuous extension 1+1+1+1 Points

Decide whether the following limits are finite and well-defined, i.e. the limit is the same no matter from which direction you approach in the complex plane. If the limit is finite and well-defined, calculate it.

$$a) \lim_{z \rightarrow 2} \frac{z^2 + 3}{iz} \quad b) \lim_{z \rightarrow 3i} \frac{z^2 + 9}{z - 3i} \quad c) \lim_{z \rightarrow i} \frac{z^2 + i}{z^4 - 1}$$

$$d) \lim_{\Delta z \rightarrow 0} \frac{(z_0 - \Delta z)^2 - z_0^2}{\Delta z}$$

### 5. Cauchy Riemann equations 1+1+1 Points

Use the Cauchy-Riemann equations to show that the following functions are nowhere differentiable.

$$a) f(z) = \bar{z} \quad b) g(z) = \operatorname{Re}(z) \quad c) h(z) = 2y - ix, \text{ where } z = x + iy$$