# The algebraic structure of morphosyntactic features

Sebastian Bank
sebastian.bank@uni-leipzig.de

Department of Linguistics
University of Leipzig

Features in Phonology, Morphology, Syntax and Semantics
University of Tromsø/CASTL
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## Background: Features in morphological subanalysis

**Present and past tense forms of German *spielen* 'to play'**

| | SG | PL |
|---|---|---|
| 1 | spiel-**e** | spiel-**(e)n** |
| 2 | spiel-**s-t** | spiel-**t** |
| 3 | spiel-**t** | spiel-**(e)n** |

PRESENT

| | SG | PL |
|---|---|---|
| 1 | spiel-**te** | spiel-**te-n** |
| 2 | spiel-**te-s-t** | spiel-**te-t** |
| 3 | spiel-**te** | spiel-**te-n** |

PAST

**Some underspecified marker hypotheses**

/-n/ ↔ [−2 +pl]  /-t/ ↔ [−1]

well-formed feature specification = natural class → **systematic syncretism**

**Some feature decomposition for pronouns**

| | 1 | 12 | 2 | 3 |
|---|---|---|---|---|
| SG | +1 -2 -3 -pl | | -1 +2 -3 -pl | -1 -2 +3 -pl |
| PL | +1 -2 -3 +pl | +1 +2 -3 +pl | -1 +2 -3 +pl | -1 -2 +3 +pl |





## Observation: Two possible kinds of feature algebra

Feature specifications in morphological grammar are...

| nothing but sets of symbols | | representations for sets of things |
|---|---|---|
| [+3] ≠ [−1 −2 +3] ≠ [−1 −2] | | [+3] = [−1 −2 +3] = [−1 −2] |
| [−1] ⋢ [+3] | vs. | [−1] ⊏ [+3] |
| [+1 +pl] ⊓ [+3 +pl] = [+pl] | | [+1 +pl] ⊓ [+3 +pl] = [−2 +pl] |
| [−1] ⊔ [−3] = [−1 −3] | | [−1] ⊔ [−3] = [−1 +2 −3] |
| [+1] ⊔ [−1] ≠ [+2] ⊔ [−2] | | [+1] ⊔ [−1] = ⊥ = [+2] ⊔ [−2] |
| 'autonomy' | | 'extensionalism' |

**Claim of this talk**

Autonomy of feature specification algebra undermines the **restrictiveness** and challenges the **learnability** of morphological grammar.





## Two flavors of feature notations

Given a set of paradigm cells (utterances, contexts)
e.g.
{ 1SG, 1PL, 2SG, 2PL, 3SG.MASC, 3SG.FEM, 3SG.NEUT, 3PL }
or
{ 1SG, 1PL.EXCL, 1PL.INCL 2SG, 2PL, 3SG, 3PL }

**Morphosyntactic feature specifications**

Give formal representation for the meaning of each individual paradigm cell.
Define which sets of paradigm cells correspond to more general meanings.

**Feature-value pairs** (Paradigm Function Morphology, Network Morphology)
{PER:1, NUM:sg}, ...{PER:3, NUM:sg, GEN:neut}, ...{PER:3, NUM:pl}

**Privative/binary features** (Amorphous Morphology, Distributed Morphology)
[+1 −2 −pl], ...[−1 −2 −pl neut], ...[−1 −2 +pl]

# Feature-value pairs

**Features as orthogonal categories of mutually exclusive values**

PER: 1, 2, 3    INCL: yes, no
NUM: sg, pl
GEN: masc, fem, neut

**Cooccurrence restrictions** (as used by Stump 2001)

$\{\text{PER:1}\} \subseteq X \vee \{\text{PER:2}\} \subseteq X \rightarrow \{\text{GEN:}\alpha\} \nsubseteq X$
$\{\text{PER:2}\} \subseteq X \rightarrow \{\text{INCL:yes}\} \subseteq X$
$\{\text{PER:1, INCL:yes}\} \subseteq X \rightarrow \{\text{NUM:pl}\} \subseteq X$
$\{\text{PER:1, NUM:sg}\} \subseteq X \vee \{\text{PER:3}\} \subseteq X \rightarrow \{\text{INCL:no}\} \subseteq X$

**Ordered attribute paths in DATR** (as used by Corbett / Fraser 1993)

TNS ≺ PER ≺ NUM
<past 1 sg>, <present 3>, ...





# Privative/binary features

**Feature decomposition** (as used by Anderson 1992; Halle / Marantz 1993)

| | | | |
|---|---|---|---|
| 1.EXCL = [+1 −2] | SG = [−pl] | MASC = [masc] | MASC = [+m −f] |
| 1.INCL = [+1 +2] | PL = [+pl] | FEM = [fem] | FEM = [−m +f] |
| 2 = [−1 +2] | | NEUT = [neut] | NEUT = [−m −f] |
| 3 = [−1 −2] | | | |

**Feature combinations**

| | sg | | pl | |
|---|---|---|---|---|
| **1** | 1SG | [+1 −2 −pl] | 1PL.EXCL | [+1 −2 +pl] |
| **12** | | | 1PL.INCL | [+1 +2 +pl] |
| **2** | 2SG | [−1 +2 −pl] | 2PL | [−1 +2 +pl] |
| **3** | 3SG | [−1 −2 −pl] | 3PL | [−1 −2 +pl] |

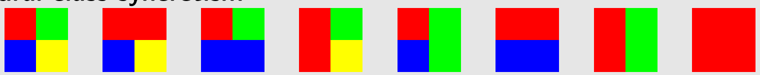




# Natural classes: syncretism vs. accidental homophony

## 15 possible assignments to a 4 cell paradigm (cf. Pertsova 2007)

**Natural class syncretism**

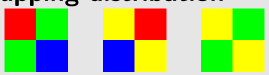


**8**

**Elsewhere syncretism**

**4**

**Overlapping distribution**



**3**

| binary features | cells | possible assignments | natural class syncretism only | percentage |
|---|---|---|---|---|
| 2 | 4 | 15 | 8 | 53.33 |
| 3 | 8 | 4,140 | 146 | 3.53 |
| 4 | 16 | 10,480,142,147 | 61,712 | 0.0006 |

**Features and their possible combinations**

- restrict the sets of paradigm cells that can be part of **systematic syncretism**
- account for the fact that natural class syncretism is **more frequent** than expected if **learners** indistinctively internalized random form-identities





# Formal Concept Analysis

Practical application of **order** and **lattice** theory (Birkhoff 1940) introduced by Wille (1982), elaborated in Gantner & Wille (1999).

Rests upon a Galois connection between two sets: a **set of objects** to describe and a **set of attributes** which each object either has or not (boolean flags).

## Basic elements of Formal Concept Analysis (FCA)

The **formal context** $\langle \mathcal{O}, \mathcal{A}, \mathcal{R} \rangle$
  defines a relation between **objects** and **attributes**.
The **derivation operator** '$'$'
  yields **common attributes** for objects and **common objects** for attributes.
The **concept lattice** $L(\mathcal{O}, \mathcal{A}, \mathcal{R})$
  defines the **relations** and **operations** on objects-attributes pairs.

Provides precise definitions, terminology, and **graphical representations** for the way feature notations are used (mostly implicitly) in linguistics.

Has many more practical applications, algorithms, software tools, etc.,
see `http://www.upriss.org.uk/fca/fca.html`

# Context defines the relation between objects and attributes

Drop feature/value distinction: translate all values into privative features

| | +1 | −1 | +2 | −2 | +3 | −3 | +sg | +pl |
|---|---|---|---|---|---|---|---|---|
| **1s** | × | | | × | | × | × | |
| **1pe** | × | | | × | | × | | × |
| **1pi** | × | | × | | | × | | × |
| **2s** | | × | × | | | × | × | |
| **2p** | | × | × | | | × | | × |
| **3s** | | × | | × | × | | × | |
| **3p** | | × | | × | × | | | × |

$\mathcal{O} = \{\ 1\text{S},\ 1\text{PE},\ 1\text{PI},\ 2\text{S},\ 2\text{P},\ 3\text{S},\ 3\text{P}\ \}$ *objects*

$\mathcal{A} = \{\ \texttt{+1},\ \texttt{-1},\ \texttt{+2},\ \texttt{-2},\ \texttt{+3},\ \texttt{-3},\ \texttt{+sg},\ \texttt{+pl}\ \}$ *attributes*

$\mathcal{R} \subseteq \mathcal{O} \times \mathcal{A} = \{\langle\ 1\text{S},\ \texttt{+1}\ \rangle, \langle\ 1\text{S},\ \texttt{-2}\ \rangle, \ldots \langle\ 3\text{P},\ \texttt{+pl}\ \rangle\}$ *relation*





# Conceptual scaling: contexts for many-valued attributes

**Dichotomic scale**

| | −incl | +incl |
|---|---|---|
| **excl** | × | |
| **incl** | | × |

**Nominal scale**

| | masc | fem | neut |
|---|---|---|---|
| **3.masc** | × | | |
| **3.fem** | | × | |
| **3.neut** | | | × |

**Ordinal scale**

| | × | ×× | ××× |
|---|---|---|---|
| **positive** | × | | |
| **comparative** | × | × | |
| **superlative** | × | × | × |

**Biordinal scale**

| | ×× | × | − | −− |
|---|---|---|---|---|
| **very high** | × | × | | |
| **high** | | × | | |
| **low** | | | × | |
| **very low** | | | × | × |





# From the prime (′) operator to formal concepts

Common attributes $O'$ of $O \subseteq \mathcal{O} := \{a \in \mathcal{A} \mid \forall o \in O : \langle o, a\rangle \in \mathcal{R}\}$

Common objects $A'$ of $A \subseteq \mathcal{A} := \{o \in \mathcal{O} \mid \forall a \in A : \langle o, a\rangle \in \mathcal{R}\}$

Formal concept $\langle O, A\rangle$ with $O' = A$ and $A' = O$ ⟨*extent*, *intent*⟩

| | +1 | −1 | +2 | −2 | +3 | −3 | +sg | +pl |
|---|---|---|---|---|---|---|---|---|
| **1s** | × | | | × | | × | × | |
| **1p** | × | | | × | | × | | × |
| **2s** | | × | × | | | × | × | |
| **2p** | | × | × | | | × | | × |
| **3s** | | × | | × | × | | × | |
| **3p** | | × | | × | × | | | × |

$\langle O'', O'\rangle$ **or** $\langle A', A''\rangle$

$\langle\{\}, \{\ \texttt{+1},\ \texttt{-1},\ \texttt{+2},\ \texttt{-2},\ \texttt{+3},\ \texttt{-3},\ \texttt{+sg},\ \texttt{+pl}\ \}\rangle$ *infimum* ⊥

$\langle\{\ 1\text{S}\ \}, \{\ \texttt{+1},\ \texttt{-2},\ \texttt{-3},\ \texttt{+sg}\ \}\rangle$ *atom*

$\langle\{\ 1\text{S},\ 2\text{S}\ \}, \{\ \texttt{-3},\ \texttt{+sg}\ \}\rangle$

$\langle\{\ 1\text{S},\ 1\text{P},\ 2\text{S},\ 2\text{P}\ \}, \{\ \texttt{-3}\ \}\rangle$ *coatom*

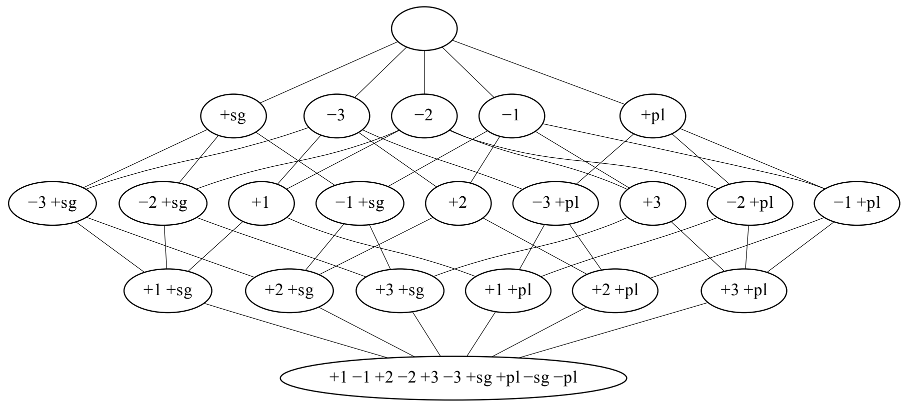
$\langle\{\ 1\text{S},\ 1\text{P},\ 2\text{S},\ 2\text{P},\ 3\text{S},\ 3\text{P}\ \}, \{\}\rangle$ *supremum* ⊤





# Partial order (≤) of super/subconcepts, join (∨), meet (∧)

$\langle O_1, A_1\rangle \leq \langle O_2, A_2\rangle$ when $O_1 \subseteq O_2$ (or equivalently $A_1 \supseteq A_2$)

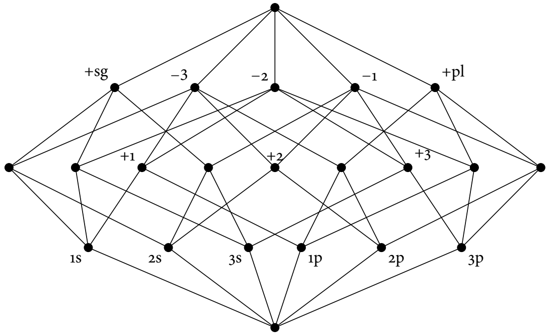



$\bot < \langle\{1\text{S}\}, \{\texttt{+1}, \texttt{-2}, \texttt{-3}, \texttt{+sg}\}\rangle < \langle\{1\text{S}, 2\text{S}\}, \{\texttt{-3}, \texttt{+sg}\}\rangle < \langle\{1\text{S}, 1\text{P}, 2\text{S}, 2\text{P}\}, \{\texttt{-3}\}\rangle < \top$

$\langle\{1\text{P}\}, \{\texttt{+1}, \texttt{-2}, \texttt{-3}, \texttt{+pl}\}\rangle \vee \langle\{3\text{P}\}, \{\texttt{-1}, \texttt{-2}, \texttt{+3}, \texttt{+pl}\}\rangle = \langle\{1\text{P}, 3\text{P}\}, \{\texttt{-2}, \texttt{+pl}\}\rangle$

$\langle\{2\text{S}, 2\text{P}, 3\text{S}, 3\text{P}\}, \{\texttt{-1}\}\rangle \wedge \langle\{1\text{S}, 1\text{P}, 2\text{S}, 2\text{P}\}, \{\texttt{-3}\}\rangle = \langle\{2\text{S}, 2\text{P}\}, \{\texttt{-1}, \texttt{+2}, \texttt{-3}\}\rangle$

# Concept lattice, object concepts, attribute concepts




**Relations and operations**

$[+1] \vee [-1] = \top$ $\quad [+\mathrm{sg}] \vee [+\mathrm{pl}] = \top$ — *tautology*

$[+1] \wedge [-1] = \bot$ $\quad [+1] \wedge [+2] = \bot$ — *contradiction*

$[+1] < [-3] \;\Leftrightarrow\; [+1] \wedge [-3] = [+1] \;\Leftrightarrow\; [+1] \vee [-3] = [-3]$ — *implication*

$[-1] \wedge [-3] \neq \bot$ **and** $[-1]' \cup [-3]' = \top'$ — *subcontrary*

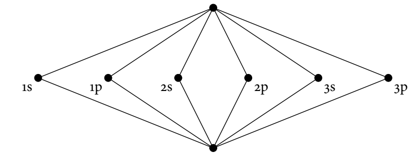
$[+1\,+\mathrm{sg}] \vee [+2\,+\mathrm{pl}] = [-3] \quad \bigvee\{[+1\,+\mathrm{sg}], [+2\,+\mathrm{sg}], [+2\,+\mathrm{pl}]\} = [-3]$ — *intersection*

$[+1] \wedge [+\mathrm{sg}] = [+1\,+\mathrm{sg}] \quad \bigwedge\{[-2], [-3], [+\mathrm{sg}]\} = [+1\,+\mathrm{sg}]$ — *unification*



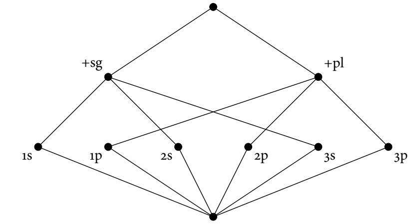


| | I | II | II | IV | V | VI |
|---|---|---|---|---|---|---|
| 1s | × | | | | | |
| 1p | | × | | | | |
| 2s | | | × | | | |
| 2p | | | | × | | |
| 3s | | | | | × | |
| 3p | | | | | | × |

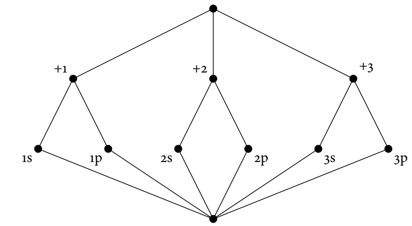



*trivial, nominal scale*

| | I | II | III | IV | V | VI | +sg | +pl |
|---|---|---|---|---|---|---|---|---|
| 1s | × | | | | | | × | |
| 1p | | × | | | | | | × |
| 2s | | | × | | | | × | |
| 2p | | | | × | | | | × |
| 3s | | | | | × | | × | |
| 3p | | | | | | × | | × |



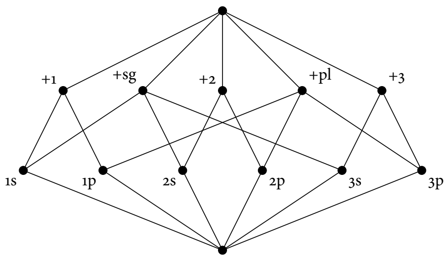

| | I | II | +1 | III | IV | +2 | V | VI | +3 |
|---|---|---|---|---|---|---|---|---|---|
| 1s | × | | × | | | | | | |
| 1p | | × | × | | | | | | |
| 2s | | | | × | | × | | | |
| 2p | | | | | × | × | | | |
| 3s | | | | | | | × | | × |
| 3p | | | | | | | | × | × |



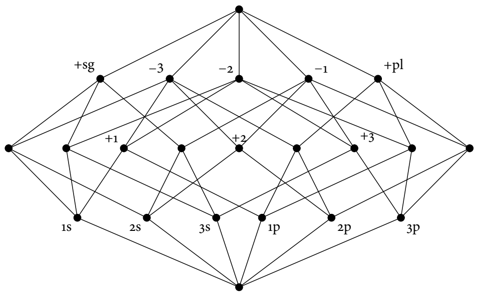





| | +1 | +2 | +3 | +sg | +pl |
|---|---|---|---|---|---|
| 1s | × | | | × | |
| 1p | × | | | | × |
| 2s | | × | | × | |
| 2p | | × | | | × |
| 3s | | | × | × | |
| 3p | | | × | | × |



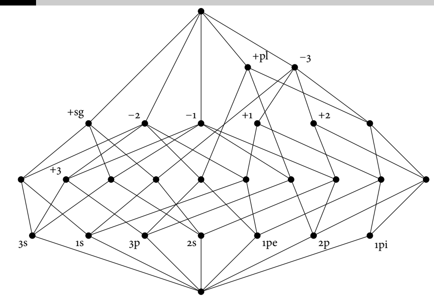

| | +1 | −1 | +2 | −2 | +3 | −3 | +sg | +pl |
|---|---|---|---|---|---|---|---|---|
| 1s | × | | | × | | × | × | |
| 1p | × | | | × | | × | | × |
| 2s | | × | × | | | × | × | |
| 2p | | × | × | | | × | | × |
| 3s | | × | | × | × | | × | |
| 3p | | × | | × | × | | | × |



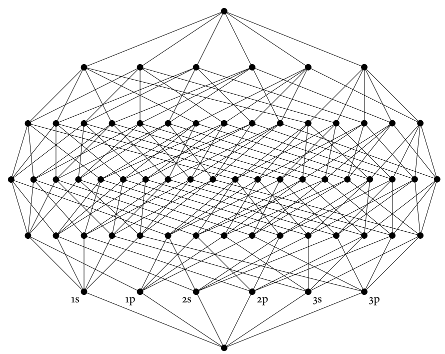





| | +1 | −1 | +2 | −2 | +3 | −3 | +sg | +pl |
|---|---|---|---|---|---|---|---|---|
| 1s | × | | | × | | × | × | |
| 1pe | × | | | × | | × | | × |
| 1pi | × | | × | | | × | | × |
| 2s | | × | × | | | × | × | |
| 2p | | × | × | | | × | | × |
| 3s | | × | | × | × | | × | |
| 3p | | × | | × | × | | | × |




| | $\overline{\mathrm{I}}$ | $\overline{\mathrm{II}}$ | $\overline{\mathrm{III}}$ | $\overline{\mathrm{IV}}$ | $\overline{\mathrm{V}}$ | $\overline{\mathrm{VI}}$ |
|---|---|---|---|---|---|---|
| 1s | | × | × | × | × | × |
| 1p | × | | × | × | × | × |
| 2s | × | × | | × | × | × |
| 2p | × | × | × | | × | × |
| 3s | × | × | × | × | | × |
| 3p | × | × | × | × | × | |




*conjunctive normal form*

*boolean algebra*, $2^6$

## Syncretism, underspecification, and insertion competition

**Present and past tense forms of English 'to be'**

| | SG | PL |
|---|---|---|
| 1 | *am* | *are* |
| 2 | *are* | *are* |
| 3 | *is* | *are* |

PRESENT

| | SG | PL |
|---|---|---|
| 1 | *was* | *were* |
| 2 | *were* | *were* |
| 3 | *was* | *were* |

PAST

**Fully specified**
*am* ↔ [+1 +sg prs]
*is* ↔ [+3 +sg prs]

**Natural class syncretism**
*was* ↔ [−2 −pl pst]

**Elsewhere syncretism**
*were* ↔ [pst]
*are* ↔ [prs]

Insertion with Pāṇinian **blocking** (a.k.a. subset principle, elsewhere principle)

Insert the **most specific** marker(s) whose meaning **subsume** the paradigm cell meaning.

**Insertion of** *was* ↔ [−2 −pl pst]

[−2 −pl pst] ≥ [+1 −pl pst] → ✓, [−2 −pl pst] ≱ [+2 −pl pst] → ✗, [−2 −pl pst] ≥ [+3 −pl pst] → ✓, ...

**Insertion of** *were* ↔ [pst]

*were* ↔ [pst] ≥ *was* ↔ [−2 −pl pst] ≥ [+1 −pl pst] → *was*, *were* ↔ [pst] ≥ [+1 +pl pst] → *were*, ...





## When markers resist blocking: extended exponence

**Agreement affixes of Fox animate intransitive verbs** (Bloomfield 1927)

| | SG | | PL | |
|---|---|---|---|---|
| 1 | *ne-* | | *ne-* | *-pena* |
| 12 | | | *ke-* | *-pena* |
| 2 | *ke-* | | *ke-* | *-pwa* |
| 3 | | *-wa* | | *-wa-gi* |

**Extended exponence**

*-wa* ↔ [+3] ≥ *-gi* ↔ [+3 +pl] ≥ [+3 +pl] ⊁ *-wa-gi*

Markedness of extended exponence hypothesis

The utterance of a subsuming marker does not contribute **information**.
It involves **additional formal machinery** (feature copying, rule blocks, contextual features, marker sensitivity, enrichment) and correspondingly is **harder to learn**.

**Contextual feature solution** (*insertion as feature discharge*, Noyer 1992)

*-gi* ↔ [+pl] / [+3] *discharged features / non-discharged features*





## Masked extended exponence with autonomous features

Present tense verbal agreement affixes of German (Müller 2006)

| | sg | | | pl | |
|---|---|---|---|---|---|
| 1 | [+1 −2 −pl] | *-e* | 1 | [~~±1~~ −2 +pl] | *-n* |
| 2 | [−1 +2 −pl] | *-s-t* | 2 | [−1 +2 +pl] | *-t* |
| 3 | [−1 −2 −pl] | *-t* | 3 | [~~±1~~ −2 +pl] | *-n* |

[±1] → ∅ / [−2 +pl]
/s/ ↔ [+2 −pl]
/n/ ↔ [−2 +pl]
/t/ ↔ [−1]
/e/ ↔ []

Does **not** interpret *t*-insertion in 2SG as extended exponence (but might).

Requires that *t* ↔ [−1] is **not** a superconcept of *s* ↔ [+2 −pl]. *autonomy*

**But** this requires that some paradigm cell is **+2** and **not −1**. *extensionalism*

*Extensionalist analysis*

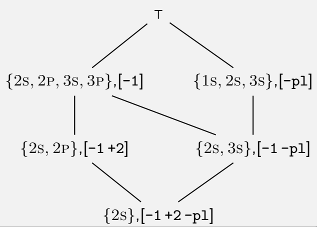
**Extended exponence**

*t* ↔ [−1] ≥ *s* ↔ [+2 −pl] ≥ [+2 −pl] ⊁ *s t*

**Contextual features solution**

/*s*/ ↔ [−pl] / [−1 +2]

predicts functional pressure to change *-s-t* into *-s*








## No masked extended exponence with extensionalism

| | sg | | | pl | |
|---|---|---|---|---|---|
| 1 | [+1 −2 −pl] | *-e* | 1 | [~~±1~~ −2 +pl] | *-n* |
| | | ? | 12 | [+1 +2 +pl] | **-e* |
| 2 | [−1 +2 −pl] | *-s-t* | 2 | [−1 +2 +pl] | *-t* |
| 3 | [−1 −2 −pl] | *-t* | 3 | [~~±1~~ −2 +pl] | *-n* |

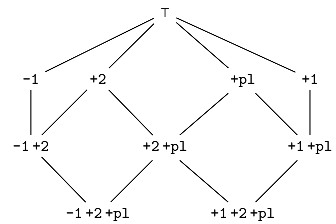
[±1] → ∅ / [−2 +pl]
/s/ ↔ [+2 −pl]
/n/ ↔ [−2 +pl]
/t/ ↔ [−1]
/e/ ↔ []

[+2] ≠ [−1 +2] **only if** there is a [+1 +2] cell

[+2 −pl] ≠ [−1 +2 −pl] **only if** there is a [+1 +2 −pl] cell

However, such an inclusive/augmented reanalysis gives:

(1) a. *Wir spiel-s.
we play-1INCL.MIN
b. *Wir spiel-e.
we play-1INCL.AUG



## Why to avoid autonomous feature algebra?

- cannot replace extended exponence machinery altogether without undermining natural class **restrictivity** by adding features
- introduces superficially equivalent options (analytical **ambiguity**) of exploiting feature autonomy vs. using additional machinery
- results in less specific **predictions** making analyses harder to test
- why prefer a less restrictive theory when a more restrictive version has not yet been falsified?

- if the choice between [+2] and [−1 +2] is only indirectly observable, how can it be learned?
- is there independent evidence for such 'morphomic' features other than the **distributional effects** they have?





## Impoverishment with or without autonomous features

**Autonomy**

A ↔ [+3]  B ↔ [−1 −2 pst]

**Extensionalist**

A ↔ [+3]  B ↔ [pst] / [+3]

| | SG | PL |
|---|---|---|
| 3 | A | A |

PRESENT

| | SG | PL |
|---|---|---|
| 3 | AB | AB |

PAST

pst → ∅ / [+3 +pl pst]

| | SG | PL |
|---|---|---|
| 3 | A | A |

PRESENT

| | SG | PL |
|---|---|---|
| 3 | AB | A |

PAST

−1 −2 +3 → ∅ / [+3 +pl pst]

+3 → ∅ / [+3 +pl pst]

| | SG | PL |
|---|---|---|
| 3 | A | A |

PRESENT

| | SG | PL |
|---|---|---|
| 3 | AB | |

PAST

+3 → ∅ / [+3 +pl pst]

| | SG | PL |
|---|---|---|
| 3 | A | A |

PRESENT

| | SG | PL |
|---|---|---|
| 3 | AB | B |

PAST

*impossible*

only retreat to the general case



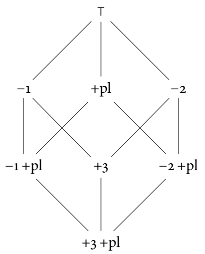


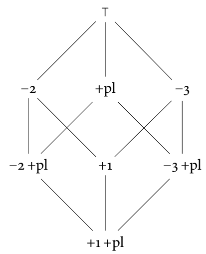
## Feature set subtraction in morphological operations

*feature discharge* (Noyer 1992)

*impoverishment & fission* (Halle / Marantz 1993)

**Impoverishment rule**  [±1] → ∅ / [+pl]  (Frampton 2002)




[+3 +pl] − [+1] = [+3 +pl]

[+3 +pl] − [−1] = [−2 +pl]

{−2 +pl, −2, +pl}

[+1 +pl] − [−1] = [+1 +pl]

[+1 +pl] − [+1] = ? **not −2 or −3**

{−2 +pl, −3 +pl, +pl}

Impoverishment ⇔ feature discharging **Ø-insertion** (Trommer 1999, 2003)





## Subtraction as Ø-insertion without autonomous features

[+3 +pl] − [−1]

| | SG | PL |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |

+3 +pl

| | SG | PL |
|---|---|---|
| 1 | | |
| 2 | Ø | Ø |
| 3 | Ø | Ø |

−1

| | SG | PL |
|---|---|---|
| 1 | | A |
| 2 | Ø | Ø |
| 3 | Ø | ØA |

−2 +pl

| | SG | PL |
|---|---|---|
| 1 | B | B |
| 2 | Ø | Ø |
| 3 | ØB | ØB |

−2

[+1 +pl] − [+1]

| | SG | PL |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |

+1 +pl

| | SG | PL |
|---|---|---|
| 1 | Ø | Ø |
| 2 | | |
| 3 | | |

+1

| | SG | PL |
|---|---|---|
| 1 | Ø | ØC |
| 2 | | |
| 3 | | C |

−2 +pl

| | SG | PL |
|---|---|---|
| 1 | Ø*D | Ø*D |
| 2 | | |
| 3 | *D | *D |

−2

Regarding subtraction as insertion without form-change

- makes various (possibly overly powerful) formalisms more **restricted**
- allows for a consistent **information-based** interpretation

# Conclusion

- if features are more than abbreviations for observable distributional facts, even simple formalisms can acquire considerable **power**
- at least in some cases it is undesirable to use this extra power – not before there is evidence that it is really needed
- Formal Concept Analysis provides the terminology and the tools to spot and disassemble such 'feature tricks'
- learnability might raise fundamental objections against them
- for the most part feature autonomy can be avoided by always using the **most specific notational variant** for representing feature sets



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