Quantum Zeno Effect and Parametric Resonance in Mesoscopic Physics

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As a realization of the quantum Zeno effect, we consider electron tunneling between two quantum dots with one of the dots coupled to a quantum point contact detector. The coupling leads to decoherence and to the suppression of tunneling. When the detector is driven with an ac voltage, a parametric resonance occurs which strongly counteracts decoherence. We propose a novel experiment with which it is possible to observe both the quantum Zeno effect and the parametric resonance in electric transport. [S0031-9007(98)08061-2]

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The interaction of a quantum system with a macroscopic measurement device is known [1,2] to generate decoherence. The frequent repetition of a decohering measurement leads to a striking phenomenon known as the quantum Zeno effect [3]: the suppression of transitions between quantum states. The standard example is a two-level system with a tunneling transition between the two levels. For small times, the probability to tunnel out of one of the two levels is \( t^2 \). With a device that projects the system onto that same level, \( N \) repeated measurements yield the reduced probability \( \sim N(t/N)^2 \). The suppression of tunneling in bound systems has its parallel in systems with unstable states. Here, the quantum Zeno effect predicts the suppression of decay and the enhancement of the lifetime of unstable states.

In spite of considerable theoretical work on the quantum Zeno effect there is only little experimental proof for it. An experimental test using an induced hyperfine transition of Be ions [4] has been reported. Experiments on optical transitions [5] or atomic Bragg scattering [6] have been proposed. However, the observation of the quantum Zeno effect in some of these experiments is hampered by other sources of decoherence not related to the measurement. Further experimental evidence for the existence of the quantum Zeno effect is clearly desirable.

With the aim of stimulating experimental work on the quantum Zeno effect, we investigate the effect in this Letter theoretically for semiconductor microstructures. We study the system shown schematically in Fig. 1. This system is within reach of present-day experimental techniques. Its central elements are two quantum dots (QD) connected via a tunneling barrier. In microstructures, such QDs are realized by a system of barriers which provide a potential well for electrons. The depth of the well is controlled by external gates. The coupled QDs may be viewed as the mesoscopic realization of a double-well potential. We study two possible arrangements: (i) The two dots are weakly coupled to each other but otherwise isolated, with a single (excess) electron on the two dots; (ii) both the upper (\( u \)) and the lower (\( l \)) dot are connected to a pair of external leads. Arrangement (ii) is depicted in Fig. 1. In both cases (i) and (ii), the lower (but not the upper) dot interacts with a quantum point contact (QPC). The QPC may be thought of as a barrier in a wire provided by an external gate. The electrostatic field of an electron on the lower dot changes the transmission through the QPC, and thus the conductance and the current through the wire. This change in current “measures” the location of the electron in the two-dot system. The QPC thus acts as a measuring device.

Arrangement (i) was first studied theoretically by Gurvitz [7]. As expected for the quantum Zeno effect, he found that the interaction with the QPC suppresses the charge oscillations between the two dots. Our work goes considerably beyond that of Ref. [7], and we identify several novel aspects of the effect in coupled quantum dots: For arrangement (i), we show that the application of an ac voltage with frequency \( \omega \) across the QPC leads to parametric resonance and to a strong reduction of decoherence. The resonance occurs when \( \omega \) equals twice the frequency \( \omega_0 \) of the internal charge oscillations in the double-dot system. Moreover, we
show that the power spectrum of the QPC displays a peak which is a clear signal for the quantum Zeno effect. With arrangement (ii), we propose a new transport experiment. With current flowing into the lower dot, we calculate the branching ratio of the current transmitted through the upper and the lower dot, respectively. The coupling to the QPC induces a correction to the branching ratio which is proportional to the decoherence rate. Thus, a measurement of the branching ratio provides a direct signature of dephasing, and of the quantum Zeno effect in the two coupled quantum dots. Our work is motivated by the first experimental demonstration of controlled dephasing in a semiconductor device by Buks et al. [8]. The experiment utilized a QD embedded in one arm of an Aharonov-Bohm interferometer. It was shown that electrons can pass coherently through the device, and an Aharonov-Bohm interferometer. It was shown that with $\mu$ the voltage drop across the QPC, the time between two scattering events in the QPC is given by $\Delta t = \hbar / (2 e \mu)$. During this time, the dynamics of the two-dot system is governed by the tunneling Hamiltonian $\Omega_0 \sigma_z/2$. Combining this fact with Eq. (1), we obtain the master equation

$$\frac{d\rho_{\text{dot}}}{dt} = - \frac{(\Delta T)^2}{8(1 - T)} \frac{e\mu}{\pi \hbar} \sum \frac{i\Omega_0}{2\hbar} [\sigma_x, \rho_{\text{dot}}].$$

A master equation for the double-dot system has previously been derived by Gurvitz [7] using a different method. We note that the factor $1 - T$ is missing in Gurvitz' result.

Since $\rho_{\text{dot}} = \rho_{\text{dot}}^\dagger$ and $\text{Tr} \rho = 1$, we can parametrize $\rho_{\text{dot}}$ by the two quantities $a = 1/2 - \rho_{\text{dot},11}$, $b = \rho_{\text{dot},12}$. Substitution into Eq. (2) yields the equations of motion $(da)/(dt) = \omega_0 \text{Im} b$ and

$$\frac{d^2a}{dt^2} + \frac{(\Delta T)^2}{8(1 - T)} \frac{e\mu}{\pi \hbar} \frac{da}{dt} + \omega_0^2 a = 0.$$  

For a time-independent voltage drop $\mu = \mu_0$ both $a$ and $b$ display damped oscillations $\sim \exp[-\kappa t] \cos \sqrt{\omega_0^2 - \kappa^2} t$ with the damping constant $\kappa = (\Delta T)^2 e \mu / (4\pi \hbar)$. We note that finite damping causes a redshift of the frequency $\sqrt{\omega_0^2 - \kappa^2}$ and, hence, slows down the charge oscillations between the two quantum dots. This suppression of transitions due to measuring with the QPC is a clear demonstration of the quantum Zeno effect. The charge oscillations in the double-dot system modulate the current in the QPC according to $\langle I_{\text{QPC}}(t) \rangle = 2(e^2/h)(I + a(t)\Delta T)$. This modifies the power spectrum of the QPC current and causes a peak with width $2\kappa$ centered at the shifted frequency $\omega_0 \sqrt{1 - \kappa^2/\omega_0^2}$. This peak will be present in addition to standard shot noise.

Interesting new physical aspects arise if $\mu$ has an AC component, $\mu(t) = \mu_0 - \mu_1 \sin \omega t$ where $\mu_0, \mu_1 \geq 0$ and $\mu_1 \leq \mu_0$. According to Eq. (3) this corresponds to a harmonic oscillator with an oscillatory damping constant. Using a simple ansatz for $a(t)$ and neglecting terms of order $O(\mu^2)$, one is led to an equation of the Mathieu type which is known [12] to display parametric resonance close to the frequencies $\omega = 2\omega_0/n$ where $n$ is a positive integer. Parametric resonance is most pronounced for $\omega = 2\omega_0$. The damping near the resonance is strongly reduced, $\kappa = (\Delta T)^2 / (4\pi \hbar) (\mu_0 - 12/\mu_1)$. The resulting time evolution of $a$ near resonance is illustrated in Fig. 2 and compared with the case where $\mu$ is time independent. The resonance condition $\omega = 2\omega_0$ is interpreted as follows: The position of the electron is not measured when $\mu$ is close to zero. The electron uses this time to tunnel from one dot to the other.

The experimental investigation of the phenomena discussed so far would amount to a time-resolved study of the quantum-mechanical measurement process and would
be of considerable interest. Such measurements, e.g., a high-frequency measurement of the power spectrum of the current through the QPC, are difficult to perform, however [13]. A setup which avoids such problems and allows for the observation of the quantum Zeno effect in dc transport is the arrangement (ii) shown in Fig. 1. Here, each dot is coupled to two external leads that allow for the transport of current to and out of the dot. This parallel setup allows for a sensitive observation of the quantum Zeno effect in the ratio of two currents as compared to a serial arrangement suggested in Ref. [7]. We note that the leads themselves do not act as a measuring device since the interactions between electrons on the dots and electrons in the leads are negligibly small (due to screening in the leads). In the Coulomb blockade regime we need to consider only a single energy level in each dot. Both levels are degenerate with energy $E_0 - i/2\Gamma$. The width $\Gamma$ is due to the coupling to the leads. As in arrangement (i) studied earlier, a QPC measures the charge in the lower quantum dot. In the present arrangement (ii) this modifies the transmission through the two-dot system. This modification is found by calculating the joint transition amplitude (technically: the two-particle scattering matrix) for electrons passing both through the QPC and through the two-dot system.

We model the QPC in terms of plane waves with energy $\epsilon_k$, mean density $\rho_F$, and Hamiltonian

$$H_{\text{QPC}} + V = \sum_k \epsilon_k b_k^\dagger b_k + \sum_{k,k'} (U_{kk'} + V_{kk'}d_i^\dagger d_i)b_k^\dagger b_{k'}.$$  \hspace{1cm} (4)

We have included the interaction $V$ with the lower QD. The potential $U_{kk'}$ mimics the external gate defining the QPC while $d_i^\dagger (d_i^\dagger)$ denotes the creation operator for the state on the lower QD (for the QPC plane wave states).

In each of the leads, we consider only one transverse channel labeled by $c = \sigma, \mu$, with $\sigma = l, u$ denoting the lower ($l$) and upper ($u$) lead, and $\mu = +, -, \text{ left and right lead}$. The two-particle scattering matrix is obtained from the Lippmann-Schwinger equation and given by

$$S_{cc',kk'} = \delta_{cc'}\delta_{kk'} - 2\pi i \gamma_c \gamma_{c'} G_{\sigma\sigma',kk'}.$$  \hspace{1cm} (5)

Here, $\gamma_c$ is the matrix element for tunneling from channel $c$ to the adjacent dot. The two-particle Green function $G$ for the joint transition between dots and QPC is given in terms of its inverse

$$(G^{-1})_{\sigma\sigma',kk'} = \left( \frac{G_0^{-1}\delta_{kk'} - U_{kk'}}{\Omega_0/2\delta_{kk'}} \right) \left( \frac{G_0^{-1}\delta_{kk'}U_{kk'} - V_{kk'}}{V_{kk'}} \right).$$ \hspace{1cm} (6)

On the right-hand side, we have explicitly displayed the matrix in $\sigma$ space. Here $G_0^{-1} = E - E_0 - \epsilon_k + i\Gamma/2$ is the inverse propagator of the single Breit-Wigner resonance, and $E$ is the sum energy of the two incoming particles. We have assumed that all $\gamma_c \equiv \gamma$ are identical and used the relation $\Gamma = 4\pi \gamma^*$. For $V = 0$, we can diagonalize the matrix $\epsilon_k \delta_{kk'} + U_{kk'}$ by a unitary transformation in $k$ space, and $G$ reduces to the product of the unit matrix in $k$ space and the two $k$-dependent coupled Breit-Wigner resonances for the double-dot system. Then, all scattering processes are elastic and the branching ratio for the transmission through the upper and lower lead is $T_u^{(0)} / T_l^{(0)} = \Omega_0^2 / \Gamma^2$.

The full two-particle scattering matrix (5) allows for energy exchange between dots and detector: In contrast to the sum energy of the two incoming particles, the energy of electrons in the QPC is not conserved in the scattering process. Such inelastic processes are essential to ensure the unitarity of the $S$ matrix. Physically, the energy exchange allows for a position measurement of the dot electron without violation of the Heisenberg uncertainty relation.

To calculate the transmission and reflection coefficients through the double-dot system, we restrict ourselves to constant scattering potentials $U_{kk'} \equiv U$ and $V_{kk'} \equiv V$. We expand $G$ in Eq. (6) in powers of $V$ and resum the resulting series. We obtain two contributions to $G$. The first is independent of $V$ and describes independent elastic scattering through the QPC and the dots. This term obviously does not contribute to the Zeno effect and is not given here. The second contribution $\tilde{G}$ involves energy exchange $\Omega = \epsilon_k - \epsilon_{k'}$ between dots and QPC. For fixed incident energy $E = E_0 + \epsilon_k$,

$$\tilde{G}_{\sigma\sigma',kk'} = A(\Omega) \left( \frac{\Omega_0^2}{-\Omega_0(2\Omega + i\Gamma)} \right) \left( \frac{i\Omega_0 \Gamma}{i\Gamma(2\Omega + i\Gamma)} \right).$$ \hspace{1cm} (7)

with the amplitude

$$A(\Omega) = \frac{4V}{F_U F_{U+V}(\Gamma + \Omega_0^2)[(2\Omega + i\Gamma)^2 - \Omega_0^2]}.$$ \hspace{1cm} (8)

Here, $F_U = 1 + 2\pi iU \rho_F$. We calculate the transmission and reflection coefficients by adding the two-particle scattering probabilities (not the amplitudes since the paths
tering of observed) and by tracing over the degrees of freedom of the QPC. The resulting expressions are further simplified by using a weak-coupling expansion to second order in $V$. This is the appropriate limit realized experimentally [8]. In this limit the application of a drain source voltage $\mu$ across the QPC is equivalent to the simultaneous scattering of $2e\mu pr$ particles in different longitudinal QPC modes. The total effect of these particles is obtained by multiplying the result for one QPC particle with the number of longitudinal modes.

The $V$-dependent corrections to the transmission and reflection coefficients (Table I) arise from both coherent (elastic) and incoherent (inelastic) scattering. Table I shows that measurements with the QPC detector have a twofold effect: (i) They suppress tunneling from the feeding lead into the lower dot, and (ii) they suppress tunneling from the lower into the upper dot. Observation (i) follows from the increase in reflection, and (ii) from the decrease of the branching ratio

$$\frac{T_u}{T_l} = \frac{T_u^{(0)}}{T_l^{(0)}} \left[ 1 - \frac{e\mu}{\pi\Gamma} \frac{(\Delta T)^2}{4\Gamma (1 - T)} \right],$$

where $T_u^{(0)}$, $T_l^{(0)}$, $\Delta T$, and $\Gamma$ were given above. Both effects (i), (ii) have an obvious interpretation as manifestations of the quantum Zeno effect. We note that the second term in the square bracket is up to a factor $\Gamma/4\hbar$ equal to the damping constant found for the isolated double-dot system. The appearance of the damping constant strongly suggests that the parametric resonance discussed above for isolated dots can also be observed in a transport experiment.

We can use our results to see how the interaction with the QPC reduces the transmission through a single dot. To this end, we put $\Omega_0 = 0$ in Eq. (5). Then, the two dots are completely isolated from each other. When a single dot is embedded in an Aharonov-Bohm interferometer, the interaction with the QPC causes a reduction of interference contrast [8]. The modulus of the elastic transmission amplitude is calculated as the square root of the transmission coefficient. At resonance we find

$$|t_{el}| = 1 - \frac{e\mu}{\pi\Gamma} \frac{(\Delta T)^2}{2\Gamma (1 - T)},$$

which is up to a factor of 1/2 in agreement with previous calculations [8,10,11]. The missing factor 1/2 is recovered [14] if one takes account of the Fermi sea in the QPC.

Our results (5),(6) include the effects of the interaction $V$ but not many-body effects due to the presence of other electrons in the leads. To investigate such effects we calculated the two-particle scattering amplitude also using the Lehmann-Symanzik-Zimmerman (LSZ) formalism [15]. The most important modification of Eq. (5) comes from the Pauli principle which reduces the phase space available for scattering and leads to a suppression of dephasing [14]. Equations (5),(7) show that a measurement with the QPC necessarily involves an energy transfer between QPC and dots. Our result (9) applies provided this transfer is not restricted by phase space. Then, we predict dephasing due to the QPC even for zero temperature. This situation is realized if the applied drain source voltages are much larger than the resonance widths, a case realized in the experiment of Ref. [8]. In the opposite regime of small drain source voltages the Fermi surfaces block all inelastic processes. Then, we find [14] that dephasing at $T = 0$ is completely suppressed, in agreement with a recent general theorem [16].

In summary, we have investigated the quantum Zeno effect for a system of two quantum dots coupled to each other and to a QPC, and either isolated from the rest of the world or connected to it by leads. In the first case, charge oscillations from one dot to the other are slowed down and damped. In the second case, the ratio of the transmission coefficients carries similar information as a clear signature of the quantum Zeno effect.

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**TABLE I.** Change of transmission and reflection coefficients of the double-dot system due to coupling with the QPC detector. For each coefficient the first term gives the elastic and the second one the inelastic contribution with $C = \{e\mu (\Omega_0^2/\pi (\Gamma^2 + \Omega_0^2))\} \{4\Gamma T^2/[4\Gamma (1 - T)]\}$.

| $T_u$ | $-4C + C$ |
| $T_l$ | $4C + (1 + 2\Gamma^2/\Omega_0^2)C$ |
| $\Delta T_u$ | $-4C\Gamma^2/\Omega_0^2 + (1 + 2\Gamma^2/\Omega_0^2)C$ |

[16] Y. Imry (to be published).