O 19e  Specific charge of electron

Tasks

1 Measure the relation between the magnetic flux density and the anode voltage keeping the diameter \( d \) of the circular path of the electrons in the fine beam tube constant (\( d=8 \text{ cm} \))! Determine the specific charge of the electron \( e/m \) by graphic analysis!

2 Determine the radius of the partially observed circular path of the electrons in a uniform magnetic field using the electron beam deflection tube together with the fluorescence screen and the pair of Helmholtz coils! Calculate the ratio \( e/m \)!

3 Determine the ratio \( e/m \) from the deflection of the electron beam in an electric field using the same tube as in task 2 at different anode voltages!

Literature

University Physics, H. Benson, 23.4, 29.6, 29.9
http://www.uni-leipzig.de/~prakphys/aprak/PDF/ratiochargetomass.pdf
http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/maspec.html#c3
http://www.nobel.se/physics/laureates/1906/thomson-bio.html

Accessories

digital multimeter, laboratory power supply, high voltage power supply, fine beam tube with Helmholtz coils, experimental setup to measure the deflection of electrons by magnetic and electric field (crossed fields)

Keywords for preparation

- motion of a point charge in an electric and in a magnetic field
- methods to determine \( e/m \), Thomson’s experiment
- magnetic field in a pair of coils, Helmholtz’s condition
- relativistic analysis of mass, energy and velocity of moving electrons
- Mass in special relativity http://en.wikipedia.org/wiki/Mass_in_special_relativity#The_relativistic_mass_concept

Remarks

Task 1: Measure the dependence of the anode voltage \( U_a \) on the coil current keeping the diameter of the circular electron orbit constant. The ratio \( e/m \) can be determined using the graph \( U_a = f(B) \). The dependence of the magnetic flux density \( B \) between the Helmholtz coils (in the center) and the corresponding coil current can be calculated by the equation

\[
H = \frac{n R^2 I}{\left[R^2 + \left(\frac{a}{2}\right)^2\right]^{\frac{3}{2}}},
\]

where \( a \) is the distance between the coils, \( n \) (\( n = 130 \)) is the number of turns and \( R (R = 15.0 \text{ cm}) \) is the coil radius. From equ. (1) follows using the condition \( b = R \) for the Helmholtz coils

\[
B = 0.716 \frac{n I}{R}.
\]

Task 2: After adjustment of an appropriate coil current the coordinates \( x \) and \( y \) of at least 5 points of the
part of the circular orbit observed on the fluorescence screen have to be measured for a given anode voltage \( U_a \cong 2 \text{ kV} \). Afterwards the electron beam has to be deflected by a uniform magnetic field of similar strength but with opposite direction in order to eliminate inhomogeneity. Then the average value is used for the determination of the radius \( r \) using the plot \( y = f(x^2 + y^2) \). Derive the corresponding equation! The magnetic flux density between the Helmholtz coils \((n = 320, R = 6.8 \text{ cm})\) can be calculated using the equation (2).

**Task 3:** The electron beam has to be deflected at a given anode voltage \( U_a \) and an appropriate voltage \( U_p \) between the deflection plates. Measure the coordinates \( x \) and \( y \) of three points on the path and calculate the ratio \( e/m \) for each pair of values. One gets the required electron velocity \( \nu \) using the so-called ‘velocity filter’ or ‘method of crossed fields’ by J. Thomson.

The distance between the deflection plates is given at work station (as an effective value \( d_{\text{eff}} \)). Realize the measurement at three different voltages in the range 1.5 kV and to 4 kV and find the right way for data analysis to get the ratio \( e/m \).

Explain whether relativistic corrections are required for analysis of the measurements or whether the classic calculation can be used! Calculate the classic and the relativistic velocity at an anode voltage 5 kV numerically at home.

**Safety hints to O 19 ( \( e/m \) )**

The fine beam tube and the electron beam deflection tube are evacuated. Implosion danger exists at strong shocks and pushes! The protection screen shall be put in front of the tubes.

At this experiment dangerous voltages are used. The experimental circuits already are built up. Under no circumstances wires and the plugs must be touched.

Before switching on the high voltage power supply the voltage controllers have to be set on the left stop (then the output voltage is zero). Only the given at workstation high voltages shall be used.

The heater voltages of the tubes to produce the electron beam shall not exceed the given voltages.

**Technical and other information**

**Fine beam tube - Helmholtz coils with holder and measuring device**

http://www.uni-leipzig.de/~prakphys/aprak/PDF/LH-finebeamtube.pdf

This apparatus is used for qualitative and quantitative investigations on electron beams in electric and magnetic fields as well as for the determination of the charge-to-mass ratio \( e/m \) of the electron and also on the electron velocity \( \nu \).

**Safety Notes**

- Vacuum tube! Danger of implosion when subjected to shock.
- Use the fine beam tube only in connection with the holder.
- Lethal voltages! It is advisable to use the safety leads which provide optimum safety.

**Fine beam tube**

Gas filling: \( 1.33 \times 10^{-5} \text{ bar} \)

Electrode system: indirectly heated oxide cathode, Wehnelt cylinder, conically shaped anode with

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<th>Filament voltage and current:</th>
<th>6.3 V, 1 A approx.</th>
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| Anode voltage \( U_a \): | 150 V d. c. to 300 V d. c. |
| Wehnelt voltage \( U_w \): | 10 V max. |

| Pair of deflection plates for electrostatic deflection of the beam (immediately placed after the anode). | |
| Deflection voltage \( U_p \): | 50 V d. c. to 100 V d. c. |
Pair of Helmholtz coils
Number of turns: 130 on each coil
Coil radius \( r \): 150 mm
Distance between the coils \( a \): 150 mm
Maximum current \( I_s \) through the coils: 2 A

Measuring device
It consists of two parts, one with a mirror and the other with two riders, for measuring the diameter of the electron beam when it is made to follow a circular path.

The experiments should be carried out in a darkened room. The deflection plates should be connected to the anode potential in all those experiments which do not require any form of electric beam deflection. Never increase the filament voltage above 6.3 V!

Glow emission of electrons will begin several minutes after switching on. Vary the anode voltage between 150 V and 300 V closely observing the beam until the beam has been focused as best as possible. Focusing can in some cases be further improved by changing the voltage applied to the Wehnelt cylinder.

Adjust rider (b) of the measuring device according to Fig. 2 so that its mirror image will be in line with the electron beam for determination of the diameter when magnetically deflecting the beam.

Fig. 1 Experimental setup (fine beam tube)

![Experimental setup diagram]

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Circuit for the electrode system
- - - - - - - Circuit for deflecting the beam in a homogeneous magnetic field
- - - - - - - Circuit for deflecting the beam in an electric field

Fig. 2 Spacer to measure the diameter \( d \)
Electron beam deflection tube

The electron (high evacuated) beam deflection tube allows investigations on the behavior of cathode rays in both the electric and magnetic fields.
Experiments with electrostatic or magnetic deflection of the electrons yield values corresponding to the order of magnitude of the specific electron charge \( e/m \) and the electron velocity \( v \) respectively.
The electrostatic deflection of the cathode rays is performed by means of two capacitor plates inserted into the tube. The (nearly) homogeneous magnetic field of the pair of Helmholtz coils is used for the magnetic deflection.

Safety Notes
- Plugs (a₂) and (b₁) - cf. Fig. 3 - are to be covered safe to touch by means of two-way plug-adapters.
- The filament voltage must not be increased beyond 6.5 V (danger to damage the heating spiral).
- Do not move the tube when the filament is incandescent.
- Strong mechanical stress exerted on the plastics caps glued over the tube wall, by pressure, pulling or shock should be avoided.
- The plug-pins of the tube should only be connected using one cable as several superposed plugs might cause an inadmissible mechanical stress.

Technical data
Directly heated tungsten cathode
- filament voltage \( U_f \) 6 V d. c. or a. c.
- filament current \( I_f \) approx. 1.35 A
- anode voltage \( U_A \) 1 to 5 kV d. c.
- diameter of the glass bulb approx. 13 cm
- total length of the tube approx. 30 cm

Electrostatic deflection
An electron passing through the (homogeneous) electric field of a plate capacitor (capacitor voltage \( U_p \), distance between the plates \( d \)) with the velocity \( v \) moves on a parabolic trajectory (cf. Fig. 4).

Fig. 3 Electron beam deflection tube mounted on the universal stand

operational parts:
(a) electron gun, consisting of directly heated tungsten incandescent cathode and cylinder-shaped anode
(a₁) pair of sockets connected to the incandescent cathode
(a₂) plug pin connected to the anode
(b) capacitor plates for electrostatic deflection
(b₁) plug pins connected to the capacitor plates
(c) fluorescent screen with graticule divided in centimeters
Fig. 4 Sketch of the deflected electron beam

\[ y = \frac{1}{2} \frac{e}{m} \frac{E}{v^2} x^2 \]  \[ E = \frac{U_p}{d} \]  \[ (3) \]

Because of

\[ v^2 = \frac{2e}{m} U_A \]  \[ (U_A = \text{anode voltage}) \]

it results

\[ y = \frac{x^2}{4d} \frac{U_p}{U_A} \]  \[ (4) \]

By means of an experimental assembly according to Fig. 3 it would be possible in principle

1. to confirm the relation (3) (approximately). For this purpose related couples of values of \( x \) and \( y \) are plotted graphically with \( x^2 \) as abscissa and \( y \) as ordinate and the resulting curve confirms the proportionality \( y \sim x^2 \) from (3).

2. to determine \( \nu \) from (3) when \( x \) and \( y \) are found experimentally and \( e/m \) is assumed as known.

**Electromagnetic deflection - measuring example for determining \( e/m \) and \( \nu \)**

An electron of mass \( m \) and charge \( e \) moving perpendicularly to a magnetic field receives a centripetal force \( \mu_0 H e v \) forcing it into a circular path. The centrifugal force \( \frac{mv^2}{r} \) equilibrates this centripetal force:

\[ \mu_0 H e v = \frac{mv^2}{r} \]  \[ (5) \]

\( v \) = velocity of the electron

\( r \) = radius of curvature

For the velocity \( v \) of the electrons which is determined by the anode voltage \( U_A \) the following equation holds true:

\[ v = \sqrt{\frac{2e}{m} U_A} \]  \[ (6) \]

From (5) and (6) it follows for the specific charge \( e/m \) of the electron:

\[ \frac{e}{m} = \frac{2U_A}{\mu_0^2 H^2 r^2} \]  \[ (7) \]

\( U_A \) can be measured directly (special voltmeter), \( r \) and \( H \) are determined from the experimentally found data and the geometrical dimensions of the tube and the pair of Helmholtz coils in the following way.

**Determination of \( r \)**

The radius of curvature \( r \) of the electron beam visible on the fluorescent screen follows the relation

\[ r^2 = x^2 + (r - y)^2 \]  \[ (6) \]
From equ.(6) follows \( y = \frac{x^2 + y^2}{2r} \) and by graphical analysis \( y \) vs \( (x^2 + y^2) \) the determination of \( r \) is possible.

**Determination of \( H \)**

Data of Helmholtz coils:
- \( n \) = number of turns per coil \( (n = 320) \)
- \( R \) = radius of the coil \( (R = 6.8 \text{ cm}) \)

**Fig. 5** Sketch of the experimental setup using the electron beam deflection tube