E 3e “Measurements with an oscilloscope“

Tasks

1. Characterize the voltages at the different outputs of a generator box regarding the signal form, the frequency (period) and the peak to peak value U_{pp}. Calculate the effective values U_{eff} for the various voltage forms and compare with the reading of a digital multimeter. Check the time base used to determine the frequency of the sinusoidal voltage using the frequency of a function generator which is measured with a digital counter. If necessary determine the corrected value of the time base.

2. Determine the frequency of the sinusoidal voltage from the generator box using Lissajous figures. Measure at least for five different frequency ratios.

3. Using the oscilloscope study the discharging of a capacitor with square voltage pulses applied. Determine the time constant for three different resistances. Compare the experimental results with the calculated values.

4. Measure the impedance of a series circuit consisting of a capacitance C and a resistance R for five different frequencies. Compare the measured and calculated impedance values.

Literature

Physics, P. A. Tipler, 3rd Edition, Vol. 2, Chaps 18-6, 23-2, 24-2, 28-1
Physikalisches Praktikum, Hrsg. W. Schenk, F. Kremer, 13. Auflage, Elektrizitätslehre, 3.0, 3.1

Java-Applet

http://www.phy.ntnu.edu.tw/~hwang/oscilloscope/oscilloscope.html
http://lectureonline.cl.msu.edu/~mmp/kap23/Oscilloscope/app.htm

Instruments and accessories

Oscilloscope, function generator, digital multimeter, generator box, laboratory power supply, digital counter, resistors, capacitor

Keywords for preparation

- Operation principle of an electron beam tube, functional units of a scope
- Time deflection, triggering unit
- Motion of charged particles in electric and magnetic fields,
- Measurements of voltage, current and period with an oscilloscope
- Effective values of current and voltage (sine-, square- and triangular voltage forms)
- Frequency measurements using Lissajous-figures
- RC circuit, charging and discharging of a capacitor (differential equations, time constant)
- Impedance and phase shift of a RC circuit

Remarks

Oscilloscope

At the beginning of the experiment familiarize yourself with the basic principles of a dual channel oscilloscope: horizontal and vertical deflection, AC and DC mode, triggering (level, slope +/-), dual and chopper mode for the dual channel mode, XY mode.

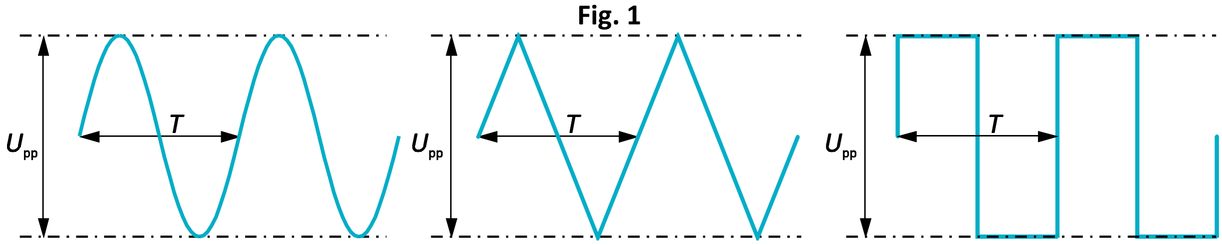
Periodic voltages

Periodic voltages without DC voltage contribution such as sine-, square- or triangular voltages, see Fig. 1, are characterized by the period T, frequency $f = 1/T$, amplitude $\hat{U}$, peak-to-peak value $U_{pp} = 2\hat{U}$ and effective value U_{eff}. The effective value corresponds to the value of a DC voltage which – at a given electrical resistance – leads to the same dissipated power as the AC voltage. The effective voltage can be calculated by averaging the square of the AC voltage U(t):

$$U_{eff} = \sqrt{<U^2>} = \sqrt{\frac{1}{T}\int_0^T U(t)^2 dt}$$

Evaluation of this integral leads to different effective values for the voltage-time characteristics as shown in Fig. 1.

Fig. 1



Sine voltage
$U_{eff} = \hat{U}/\sqrt{2} = U_{pp}/\left(2\sqrt{2}\right)$

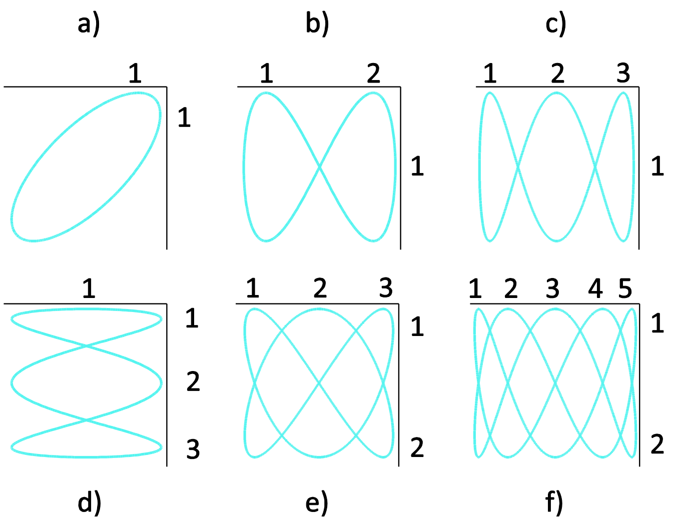
Triangular voltage
$U_{eff} = \hat{U}/\sqrt{3} = U_{pp}/\left(2\sqrt{3}\right)$

Square voltage
$U_{eff} = \hat{U} = U_{pp}/2$

Lissajous figures

Lissajou figures are obtained with an oscilloscope when this is operated in XY mode and when in both channel 1 and 2 voltages are applied. If the ratio of the two frequencies of the voltages is just equal to a rational number, standing figures appear on the oscilloscope (Lissajou figures); if the frequency ratio, however, deviates slightly from a rational number, these figures are moving. In this way it is possible to make small frequency differences visible and measurable. Unknown frequencies can be determined, if these are applied to one channel of the oscilloscope and are superimposed with a voltage of known, but adjustable frequency from a function generator applied to the other channel. When a Lissajou figure is formed, the frequency ratio can be inferred from the form of this figure, see Fig. 2.

Fig. 2



If f_y is the known frequency of the sine wave generator, then the determination of the unknown frequency f_x is possible from Lissajous figures.

a) f_y/f_x = 1:1

b) f_y/f_x = 2:1

c) f_y/f_x = 3:1

d) f_y/f_x = 1:3

e) f_y/f_x = 3:2

f) f_y/f_x = 5:2

Charging and discharging of a capacitor

A series circuit consists of a resistance R and a capacitor C. At $t = 0$ a square voltage is applied to this circuit, such that a current $I(t)$ is flowing that charges the capacitor. The voltage $U_C(t)$ measured across the capacitor is initially zero and increases with time until it approaches the applied voltage U_0. According to Kirchhoff's mesh rule the sum of the voltages across the capacitor ($U_c = Q/C$) and the resistance ($U_R = RI$) is equal to the applied voltage U_0. For the current one has $I = dQ/dt$, and for the charge on the capacitor $Q = CU_C$. This leads to a linear differential equation of first order:

$$\frac{dU_C}{dt} = -\frac{1}{RC}(U_C - U_0)$$

This can be solved by separation of variables leading to a voltage drop across the capacitor of

$$U_C(t) = U_0\left[1 - \exp\left(-\frac{t}{RC}\right)\right]$$

The discharging process is analogous and leads – taking the modified initial contitions into account – to the following equation:

$$U_C(t) = U_0 \exp\left(-\frac{t}{RC}\right)$$

The relaxation time ist defined as the time after which the exponential process has reached the *e*-th fraction (ca. 37%) of the difference between initial and final value. This time characterizes how fast a system recovers. In case of charging/discharging a capacitor the relaxation time is given by

$$\tau_{RC} = RC$$

For practical reasons it is simpler to determine the half-life $\tau_{1/2}$ and to calculate the relaxation time from the half-life using the relation $\tau_{RC} = \tau_{1/2} / \ln(2)$.
In principle the capacitor needs infinitely long to charge or discharge, since the exponential function reaches zero only for infinitely long time spans. In the measurement, however, one obtains a sufficiently good accuracy, when choosing the period of the square voltage such that it is large compared to the relaxation time. If this condition is not fulfilled, the charging starts with a voltage $U_{\text{min}} > 0$ and reaches only a voltage $U_{\text{max}} < U_0$, i.e. the image on the oscilloscope shows only a section of the voltage-time curve $U_C(t)$. This situation is illustrated in Fig. 3.

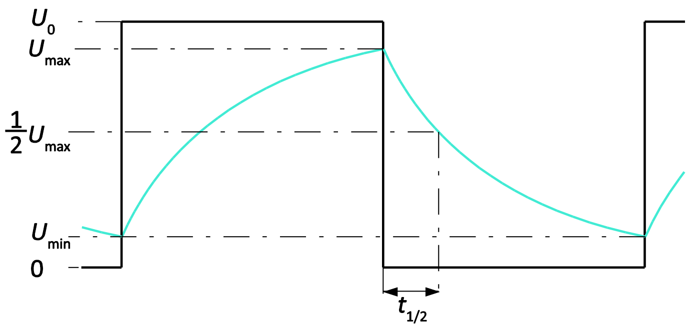


Fig. 3 Voltage-time curve of a capacitor for incomplete charging and discharging.

Measurement of the time constant using the oscilloscope

For the determination of the time constant τ_{RC} with an oscilloscope a RC circuit as shown in Fig. 4 has to be realized. The function generator is set to a square voltage with a frequency of about 100 Hz. At an appropriate setting of the *Y* amplification of the oscilloscope the voltage amplitude has to be chosen in such a way that the square voltage is optimally displayed on the screen.
In order to visualize charging and discharging curves on the oscilloscope screen the dual channel mode is used (with Dual and Add pressed).
From the screen one reads off the half-life $\tau_{1/2}$. For cross-checking calculate the theoretical value τ_{RC}.

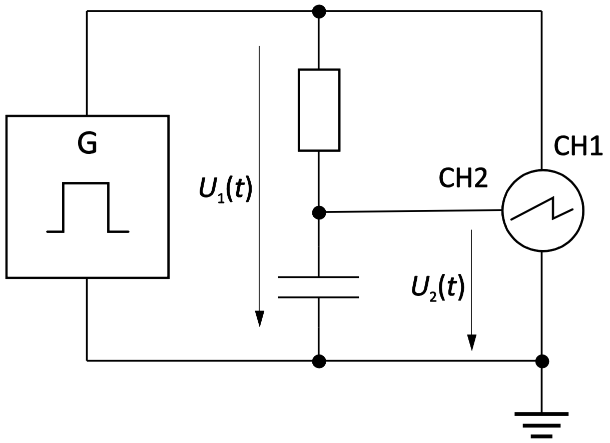


Fig. 4 Circuit diagram: measurement of the time constant of a RC circuit
(RC series circuit; capacitor C = 0.47 µF; decade resistance e.g. R = 3 kΩ)

Since the discharge of the capacitor is only observed to a voltage $U_{min} \neq 0$, one has to carefully adjust the reference lines ($U = U_{max}$ and $U = 0$). To this end the dual channel oscilloscope is used to measure a zero signal without any inputs applied; in this case the zero lines of the two channels must coincide. Further one has to take care that the Y amplification is the same for both channels and that the inputs are connected with dc coupling (DC).

Measurement of impedances

In an AC circuit charging and discharging of a capacitor leads to a reactance

$$Z_C = 1/(2\pi f C) = 1/(\omega C)$$

with the angular frequency $\omega = 2\pi f$. Since for a voltage across a capacitor to develop, first a current has to flow, the voltage maximum of the capacitor voltage appears behind the current maximum, i.e. there will be a phase shift, which in this case is $\varphi = -90\,^{\circ}$.

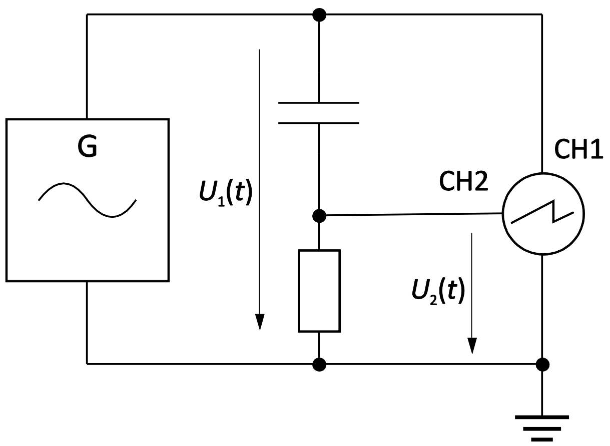


Fig. 5 Circuit diagram: measurement of impedances

Consider a series circuit of a capacitor C and a resistor R (Fig. 5), then the impedance of this circuit is given by

$$Z_{RC} = \sqrt{R^2 + Z_C^2} = \sqrt{R^2 + 1/(\omega C)^2}\ .$$

An AC voltage $U_1(t) = \hat{U}_1 \sin(\omega t)$ applied to this series circuit generates a current $I(t)$ that can be measured by a voltage measurement $U_2(t)$ across the resistor R (Fig. 5), since for a resistor there is no phase shift between voltage and current. Ohm's law yields:

$$I(t) = U_2(t)/R = (\hat{U}_1 / Z_{\mathrm{RC}}) \sin(\omega t + \varphi) = \left[\hat{U}_1 / \sqrt{R^2 + 1/(\omega C)^2} \right] \sin(\omega t + \varphi)$$

with

$$\tan\varphi = -1/(\omega RC) \,.$$

For the RC series circuit a phase shift φ with $0 < -\varphi < 90^{\circ}$ appears between the voltage applied to the circuit and the current.

The characteristics of this RC circuit are measured with the circuit shown in Fig. 5 using a function generator (FG) as voltage source and a dual channel oscilloscope as voltmeter. The oscilloscope is used to measure both the voltage of the function generator as well as the voltage across the resistance R. From the voltage-time display the peak-to-peak values U_{1pp} und U_{2pp} as well as the time delay Δt between the two voltages can be read off. This allows for the calculation of the impedance

$$Z_{\mathrm{RC}} = U_{\mathrm{eff}} / I_{\mathrm{eff}} = R(U_{1pp} / U_{2pp}) \,.$$

Determine the impedance of the RC series circuit (R = 3 kΩ, C = 0.47 μF) for frequencies of 100 Hz and 300 Hz experimentally and compare with the calculated values of the impedances using the known values of resistance, capacitance and frequency.

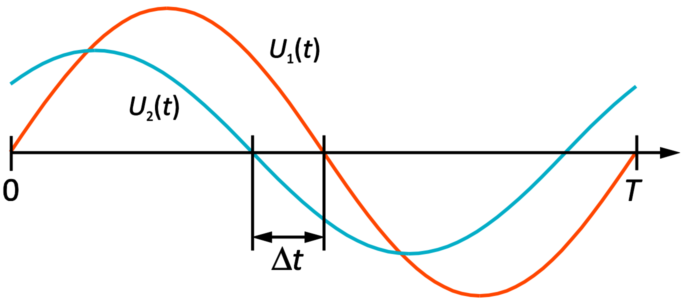


Fig. 6 Time dependence of two sine voltages.

Basic principles of the oscilloscope

Figure 7 shows a block diagram of an oscilloscope. The cathode-ray tube (CRT) is the central element. The electron gun forms a narrow beam of electrons, which passes through two sets of deflection plates on its way to a phosphorescent screen. The screen emits light at the position of the beam impact. The horizontal and vertical amplifiers apply voltages to the deflection plates. The resulting electric fields from these voltages deflect the beam to any position on the screen. Thus, a saw-tooth voltage on the horizontal plates causes the spot to sweep at a constant speed from left to right across the screen making the horizontal position proportional to time. If simultaneously there is a vertical voltage of (e.g.) $U_0 \sin(2\pi t/T)$, a sine wave will be drawn on the screen.
Unless the saw-tooth and the sine voltage are synchronized, successive wave forms will not be superimposed on the screen. In order to overcome this difficulty, the saw-tooth voltage is initiated by a trigger (time base) circuit. When the trigger setting is on “internal”, it starts each saw-tooth sweep at the same voltage level and the corresponding shoulder of the vertical signal, thus the two are synchronized and the pattern on the screen is stationary. (Fig. 8)

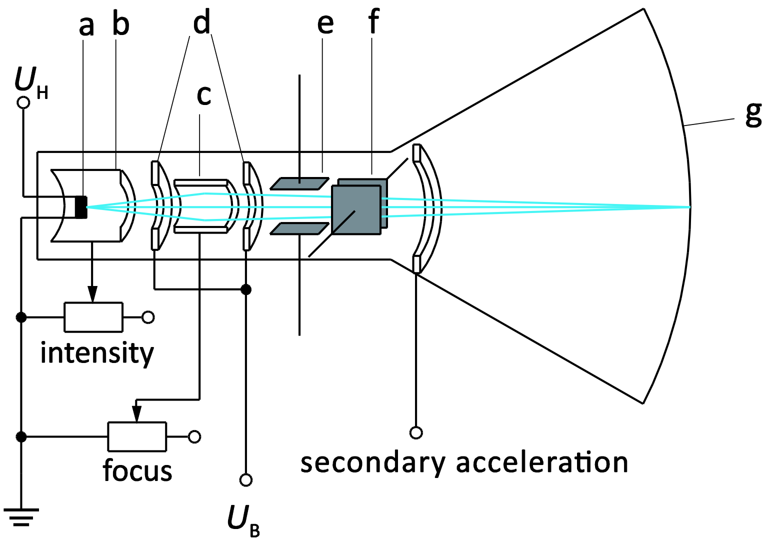


Fig. 7

a	cathode
b	Wehnelt's cylinder electrode
c	electron lens
d	anode
e	vertical deflection plates
f	horizontal deflection plates
g	luminescent screen

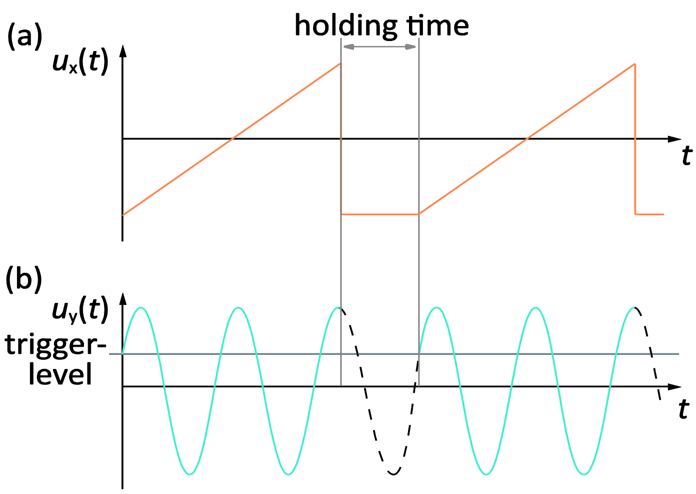


Fig. 8

(a) saw-tooth voltage
(b) sine voltage

Red line is shown on the screen.

The sensitivity of deflection s is defined as the ratio of the deflection b of the electron beam and the applied voltage U_p ; thus

$$s = \frac{b}{U_p} = \frac{L l_p}{2 d U_a} \quad .$$

In the latter equation L is the distance between the center of the deflection plates and the screen, l_P is the length of the deflection plates, d is the space between the parallel deflection plates and U_a is the acceleration voltage (anode voltage). The inverse of s is the deflection coefficient a. It is assumed that $E_p = U_p / d$ for the electric field between the deflection plates (homogeneous field). Then the equation for the sensitivity of deflection might be derived as follows.

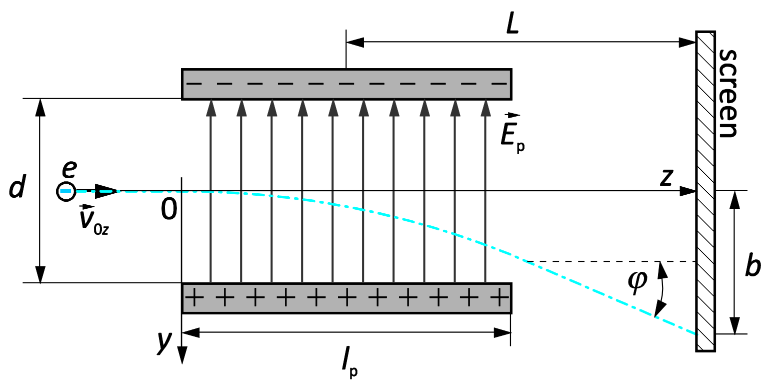


Fig. 9 Deflection of an electron in a homogenous electric field

z-direction: constant velocity, $z = v_{oz}\, t$

$$v_{oz} = \sqrt{\frac{2e}{m_e} U_a}$$

y-direction: constant acceleration, $y = a_y\, t^2/2$

$$a_y = \frac{e}{m_e} E_p$$

path equation:

$$y = \frac{e E_p}{2 m_e v_{oz}^2} z^2 = \frac{U_p}{4 U_a d} z^2$$

angle of deflection φ:

$$\tan\varphi = \frac{v_y}{v_{oz}} = \frac{e E_p}{m_e v_{oz}} t$$

angle of deflection φ after passing the plates ($t = l_p / v_{oz}$):

$$\tan\varphi = \frac{l_p U_p}{2 d U_a} \quad .$$

deflection on the screen : $b = y_A + \left(L - \frac{l_p}{2} \right) \tan\varphi = \frac{e U_p l_p L}{m_e d v_{oz}^2} = \frac{l_p L}{2\ d} \frac{U_p}{U_a}$, where $y_A = y(z = l_p)$.