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Notes on paradigm economy

Gereon Müller

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Abstract It is shown that assuming instances of syncretism to be systematic in the unmarked case may significantly reduce the number of possible inflection classes that can be generated on the basis of a given inventory of markers, without recourse to specific constraints like Carstairs' (*Allomorphy in Inflection*, Croom Helm 1987) Paradigm Economy Principle or Carstairs-McCarthy's (*Language* 70:737–787, 1994) No Blur Principle. If there is always one radically underspecified (i.e., elsewhere) marker per morphological domain, and if there is always one unique marker that is chosen in cases of marker competition, it turns out that there can be at most 2^{n-1} inflection classes for n markers, independently of the number of instantiations of the grammatical category that the markers have to distribute over. The argument relies on the notion of *marker deactivation combinations*.

Keywords Paradigm economy · Syncretism · Inflection classes · Distributed Morphology · Specificity

1 Introduction

In Distributed Morphology (see, e.g., Halle and Marantz (1993, 1994), Noyer (1992), Halle (1997), Harley and Noyer (2003)), paradigms do not exist as genuine objects that, e.g., grammatical constraints can refer to. Rather, paradigms are viewed as epiphenomena—essentially, as empirical generalizations that need to be derived in some way. This is incompatible with a more traditional view according to which paradigms exist as genuine entities in the

G. Müller (✉)
Universität Leipzig, 04107 Leipzig, Germany
e-mail: gereon.mueller@uni-leipzig.de

grammar. A well-known constraint that requires the presence of paradigms as entities of grammatical analysis is the Paradigm Economy Principle proposed in Carstairs (1987). This constraint states that the number of inflection classes for a given inventory of markers in a given grammatical domain is limited by the highest number of allomorphic variation in a paradigm cell. If the notion of paradigm is not available in a theory of inflectional morphology, a constraint like the Paradigm Economy Principle cannot be adopted.[1]

Another constraint that has the effect of restricting the number of possible inflection classes that can be generated on the basis of a given set of inflection markers (for a given grammatical category) is Carstairs-McCarthy's (1994) No Blur Principle. This constraint demands that only one of the allomorphs for a particular cell can fail to unambiguously identify inflection class. As formulated, the constraint again relies on the existence of paradigms. However, Noyer's (2005) Interclass Syncretism Constraint, a constraint that is similar (though not identical) in its effects to the No Blur Principle, is developed within (and ultimately derivable from more basic assumptions of) Distributed Morphology, and thus does without paradigms. The two latter constraints have in common that they are fundamentally incompatible with the idea that natural classes of inflection classes can be referred to by inflection markers, via underspecification with respect to more primitive, decomposed inflection class features, so as to account for instances of syncretism (conceived of broadly as a homophony of markers) that hold across inflection classes ('trans-paradigmatic syncretism'). The reason is that underspecification of inflection markers with respect to inflection class information will automatically give rise to markers that do not

[1] Other constraints that presuppose paradigms include Williams's (1994) Basic Instantiated Paradigm Principle and McCarthy's (2003) constraints in his Optimal Paradigms theory; but see Bobaljik (2002) and Bobaljik (2003), respectively, for arguments against these approaches.

Note incidentally that whereas certain other recent theories of inflectional morphology do envisage a concept of paradigm, it is far from clear whether constraints on paradigms as they have been proposed (including the Paradigm Economy Principle) can be straightforwardly adopted in these approaches. For instance, the concepts of paradigm that are employed in the Minimalist Morphology analyses developed in Wunderlich (1996, 1997), and in Wiese's (1999) stem-and-paradigm approach, are highly abstract ones, involving underspecification, and it is difficult to see how Paradigm Economy could be verified on this basis alone. Similarly, the important building blocks of Stump's (2001) Paradigm Function Morphology are abstract realization rules relying on underspecification of morpho-syntactic features; it is these realizations rules that other constraints (such as the Bidirectional Referral Principle or a Metarule for symmetrical syncretism in Stump (2001, chapt. 4)) may refer to, not the final paradigm resulting from the application of realization (and other) rules.

unambiguously encode inflection class, in violation of No Blur and the Interclass Syncretism Constraint.[2]

Taken together, we can conclude that approaches which do without paradigms and rely on decomposition and underspecification of inflection class features are incompatible with both the Paradigm Economy Principle and the No Blur Principle. Thus, there is a potential danger that such a theory of inflectional morphology is not restrictive enough, in the sense that it fails to systematically narrow down the a priori possible set of inflection classes over a given inventory of markers, to a number that is closer to what can actually be observed in the world's languages.

In view of this state of affairs, two strategies can be pursued. First, one might argue that the question of how inflection classes can be constrained is in fact irrelevant from a synchronic perspective, and assume that the diachronic development of inflection classes, and in particular their status as objects that are derived from some other grammatical elements (e.g., theme vowels, derivational suffixes, etc.) at some point in a language's history, may account for the relatively small number of inflection classes (at least compared to the logical possibilities) in any given domain of any given language. Second, one can try to show that restrictions on the number of possible inflection classes (based on a given marker inventory) follow from independently motivated assumptions, and without invoking specific constraints that explicitly impose restrictions on possible inflection classes (like the Paradigm Economy Principle or the No Blur Principle). I adopt the latter strategy in this paper.

The central assumption that I make use of is that there is a meta-constraint on inflectional systems according to which as many instances of syncretism (in a given grammatical domain) as possible should be assumed to be systematic, and traced back to a single morpho-syntactic specification.[3] This meta-principle can be formulated as in (1) (see Alexiadou and Müller (2007)):

[2] The Interclass Syncretism Constraint is given in (i) (see Noyer (2005, p. 278)).

(i) *Interclass Syncretism Constraint*

Let A and B be distinct inflectional classes, where the morphosyntactic feature [±F] partitions the set of stems in A into two subclasses. If only the [+F] stems in class A share an affix α with the stems in class B, then either: (i) α is a default affix, or (ii) B is the default inflectional class for [+F] stems.

In being incompatible with a decomposition of inflection class features, the Interclass Syncretism Constraint is thus arguably closer to Carstairs-McCarthy's No Blur Principle than to much work in Distributed Morphology where inflection class feature decomposition is assumed; see Halle (1992), Oltra Massuet (1999), and below.

[3] I have nothing to say here about instances of homonymy outside of inflectional morphology.

(1) *Syncretism Principle*
Identity of form implies identity of function (within a certain domain, and unless there is evidence to the contrary).

I assume (1) to be the null hypothesis for both a child acquiring a language, and a linguist investigating it. It is clear that (1) must be confined to smaller domains of grammar (so as to ensure that, e.g., no common source must be sought in English for the use of *s* as a nominal plural marker and as a verbal third person present tense marker); and it is also clear that the possibility of exceptions must not be excluded (there may be historical reasons after all why a domain of inflectional morphology might deviate from what qualifies as optimal design according to (1)). Domain restrictions and exceptions notwithstanding, the Syncretism Principle in (1) brings about a shift of perspective from much recent work in inflectional morphology, in that the burden of proof is not on considering a given instance of syncretism as systematic, but on considering it to be accidental. I will not try to present empirical evidence for (1) in the present paper. However, I would like to contend that analyses that (explicitly or, more often, implicitly) adhere to (1) have successfully been developed in various kinds of theoretical frameworks (among them Distributed Morphology, Minimalist Morphology, and Paradigm Function Morphology and other stem-and-paradigm approaches). It seems fair to conclude, then, that the assumption that something like (1) underlies inflectional systems in natural languages at least qualifies as a legitimate research strategy.[4] Rather, I will provide an argument to the effect that the Syncretism Principle in (1) (in interaction with two simple and widely accepted auxiliary assumptions, concerning presence of an elsewhere marker and uniqueness of marker choice) significantly restricts the number of possible inflection classes all by itself, in a way that is made precise in the formulation of what I call the Inflection Class Economy Theorem in (2):

(2) *Inflection Class Economy Theorem*
Given a set of n inflection markers, there can be at most 2^{n-1} inflection classes, independently of the number of instantiations of grammatical categories that the markers have to distribute over.

Thus, suppose that there is an inventory of three markers ($\{a, b, c\}$) that are to be distributed over four cases (let us call them (*case*) 1, (*case*) 2, (*case*) 3, and (*case*) 4). Then it follows from (2) that these markers can at most be grouped into $2^{3-1} = 4$ inflection classes. This will straightforwardly rule out systems like the one in (3), which has five inflection classes (I–V) based on an inventory of three markers.

[4] See, e.g., Georgi (2006) on the morphological system of argument encoding in Kambera, which looks optimally designed from the of view of the Syncretism Principle. However, also see Carstairs-McCarthy (2004) for an explicit rejection of the idea that optimal design plays a role in morphology.

(3) *An impossible system of inflection classes*

	I	II	III	IV	V
1	a	a	a	a	c
2	b	a	c	c	c
3	b	a	a	a	a
4	b	b	b	c	a

I will proceed as follows. In Sect. 2, I briefly address the Paradigm Economy Principle and the No Blur Principle; I illustrate how these principles restrict the number of inflection classes, and where they may raise problems (in general, or with respect to assumptions made here). In Sect. 3, then, I first introduce the Inflection Class Economy Theorem and its underlying assumptions; after that I illustrate (on the basis of some abstract scenarios) how the Inflection Class Economy Theorem restricts the number of inflection classes; and only then do I show how the Inflection Class Economy Theorem follows from the set of assumptions introduced before. Finally, I go through a couple of abstract examples and show how inflection classes may or may not be generated on the basis of different initial inventories of markers (in the course of doing so, I also come back to (3)).

2 Paradigm economy

2.1 The paradigm economy principle

Assuming as given the set of inflection markers that a language has at its disposal in order to express some grammatical category (like, e.g., case), the question arises how these markers can be grouped into inflection classes (or paradigms—I will use these notions interchangeably in this paper). More specifically, Carstairs (1987) brings up the issue of what the largest number of inflection classes can be on the basis of a given set of inflection markers. An answer is provided by the Paradigm Economy Principle, which can be formulated as in (4) (Carstairs (1987, p. 51)).

(4) *The Paradigm Economy Principle*
When in a given language L more than one inflectional realization is available for some bundle or bundles of non-lexically-determined morpho-syntactic properties associated with some part of speech N, the number of macro-paradigms for N is no greater than the number of distinct ''rival'' macroinflections available for that bundle which is most genereously endowed with such rival realizations.

As a consequence, the number of inflection classes (more precisely: macro-inflection classes; see below) does not exceed the greatest number of allomorphs

for any instantiation of a grammatical category. Thus, a system of inflection classes as in the abstract example in (5) is predicted to be impossible by the Paradigm Economy Principle (see Carstairs-McCarthy (1998)). Here, the greatest number of allomorphic variation for any cell is 2 (e.g., *a* and *f* are possible realizations of the instantiation of a given grammatical category in cell *1; b* and *e* are possible realizations of the instantiation of this grammatical category in cell *2*; and so forth). Therefore, the Paradigm Economy Principle demands that there can be at most two inflection classes on the basis of this marker inventory; a system with four inflection classes, as in (5) (I–IV), is excluded.

(5) *A system that is excluded by the Paradigm Economy Principle*

	I	II	III	IV
1	a	a	f	f
2	b	e	e	e
3	c	c	h	h
4	d	d	d	g

To illustrate that the number of observable inflection classes is typically vastly smaller than what would be predicted if markers could freely re-combine, Carstairs (1987) looks at the system of present indefinite verb inflection in Hungarian; see (6).

(6) *Hungarian present indefinite verb inflection*

	Indicative	Subjunctive
Sg 1	ok, ek, ök, om, em, öm	ak, ek am em
2	(a)sz, (e)sz, ol, el, öl	Ø, ál, él
3	Ø, ik	on, en, ön, ék
Pl 1	unk, ünk	unk, ünk
2	(o)tok, (e)tek, (ö)tök	atok, etek
3	(a)nak, (e)nek	anak, enek

He notes that there could in principle be 276,480 inflection classes, assuming complete independence of distribution of the markers over (macro-) inflection classes (the result of multiplying the numbers of exponents in all of the cells). In actual fact, there are much fewer inflection classes. A superficial analysis might take the inflectional patterns in (7) to be representative of the system of inflection classes.

(7) *Some Hungarian verbs*

Indicative						
		olvasni ‘read’	ülni ‘sit’	enni ‘eat’	érteni ‘understand’	írni ‘write’
Sg	1	olvas-ok	ül-ök	esz-em	ért-ek	ír-ok
	2	olvas-ol	ül-sz	esz-el	ért-esz	ír-sz
	3	olvas-Ø	ül-Ø	esz-ik	ért-Ø	ír-Ø
Pl	1	olvas-unk	ül-ünk	esz-unk	ért-ünk	ír-unk
	2	olvas-tok	ül-tök	esz-tek	ért-etek	ír-tok
	3	olvas-nak	ül-nek	esz-nek	ért-enek	ír-nak
Subjunctive						
Sg	1	olvas-ak	ülj-ek	egy-em	értj-ek	irj-ak
	2	olvas-Ø/-ál	ülj-Ø/-él	egy-él	értj-Ø/-él	írj-Ø/-ál
	3	olvas-on	ülj-en	egy-ek	értj-en	írj-on
Pl	1	olvas-unk	ülj-ünk	egy-ünk	értj-ünk	írj-unk
	2	olvas-atok	ülj-etek	egy-etek	értj-etek	írj-atok
	3	olvas-anak	ülj-enek	egy-enek	értj-enek	írj-anak

However, Carstairs (1987) argues that if one is willing to abstract away from differences that are morpho-phonologically or phonologically predictable, there are only two (macro-) inflection classes: the *normal* conjugation and the *ik* conjugation (each with a back-vowel and a front-vowel version); cf. (8) This is of course fully compatible with the requirements imposed by the Paradigm Economy Principle.

(8) *Hungarian present indefinite conjugations: analysis*

		Indicative		Subjunctive	
		normal	ik	normal	ik
Sg	1	ok	om	ak	am
	2	ol (after sibilants) asz (elsewhere)	ol	Ø/ál	Ø/ál
	3	Ø	ik	on	ék
Pl	1	unk	unk	unk	unk
	2	(o)tok	(o)tok	(o)tok	(o)tok
	3	(a)nak	(a)nak	(a)nak	(a)nak

It is worth noting that the Paradigm Economy Principle crucially relies on the concept of macro-paradigm (or macro-inflection class). To see this, consider first the notion of inflection class proper. Following Aronoff (1994, p. 64), we can assume that an inflection class is "a set of lexemes whose members each select the same set of inflectional realizations". Macro-paradigms (or macro-inflection classes), in contrast, are defined by Carstairs (1987) as in (9).

(9) *Macro-Paradigm*
A macro-paradigm consists of:
a. any two or more similar paradigms whose inflectional differences either can be accounted for phonologically, or else correlate consistently with differences in semantic or lexically determined syntactic properties (like gender);
or
b. any paradigm which cannot be thus combined with other paradigm(s).

Thus, differences between inflection classes that are independently predictable do not create different macro-paradigms. To see why this is needed for the proper functioning of the Paradigm Economy Principle, consider the set of inflection classes for noun inflection in German. The classification in (10) assumes eight inflection classes. It is taken from Alexiadou and Müller (2007); however, there is a similar taxonomy of inflection classes in Carstairs (1986, p. 8).[5]

(10) *German noun inflection*

	I: masc, neut *Hund*$_m$ ('dog'), *Schaf*$_n$ ('sheep')	II: masc *Baum*$_m$ ('tree') *Floß*$_n$ ('raft')	III: neut, masc *Buch*$_n$ ('book'), *Mann*$_m$ ('man')	IV: masc, neut *Strahl*$_m$ ('ray') *Auge*$_n$ ('eye')
nom/sg	Ø	Ø	Ø	Ø
acc/sg	Ø	Ø	Ø	Ø
dat/sg	Ø	Ø	Ø	Ø
gen/sg	(e)s	(e)s	(e)s	(e)s
nom/pl	(e)	''(e)	''er	(e)n
acc/pl	(e)	''(e)	''er	(e)n
dat/pl	(e)n	''(e)n	''ern	(e)n
gen/pl	(e)	''(e)	''er	(e)n

[5] Carstairs (1986) actually has even more inflection classes, including ones with *s* as a plural marker. In (10), the letters *m, f*, and *n* stand for *masculine, feminine*, and *neuter*, respectively.

	V: masc ('weak') *Planet*$_m$ ('planet')	VI: fem *Ziege*$_f$ ('goat')	VII: fem *Maus*$_f$ ('mouse')	VIII: fem *Drangsal*$_f$ ('distress')
nom/sg	Ø	Ø	Ø	Ø
acc/sg	(e)n	Ø	Ø	Ø
dat/sg	(e)n	Ø	Ø	Ø
gen/sg	(e)n	Ø	Ø	Ø
nom/pl	(e)n	(e)n	''(e)	(e)
acc/pl	(e)n	(e)n	''(e)	(e)
dat/pl	(e)n	(e)n	''(e)n	(e)n
gen/pl	(e)n	(e)n	''(e)	(e)

The greatest number of allomorphic variation in (10) is 4, in nominative, accusative, and genitive plural contexts: (*e*), (*''e*), *''er*, (*e*)*n* ('' signals umlaut on the stem; (~) signals a regular morpho-phonological alternation between ə and zero exponence). If the (arguably somewhat exceptional) plural marker *s* is also included as a regular exponent, on a par with the other ones, the greatest number of allomorphic variation would be 5 (but there would of course be additional inflection classes to consider).[6] It follows from this observation that there can be at most 4 (5) macro-inflection classes, given the Paradigm Economy Principle. A first approximation is the system in (11), which recognizes five inflection classes (based on (10), i.e., ignoring *s*).

(11) *Macro-inflection classes for German noun declension*
   a. III (*''e*-plural)
   b. V (so-called 'weak masculines')
   c. IV/VI (*en*-plural; gen/sg *s* for masc/neut; gen/sg Ø for fem)
   d. II/VII (*''e*-plural; gen/sg *s* for masc/neut; gen/sg Ø for fem)
   e. I/VIII (*e*-plural; gen/sg *s* for masc/neut; gen/sg Ø for fem)

Where IV and VI differ, the difference is derivable by invoking gender in addition to inflection class; and the same goes for II and VII, and I and VIII. However, this does not yet suffice: The Paradigm Economy Principle permits four macro-inflection classes for German noun inflection, but there are five inflection classes in (11). Hence, it seems that (11-d) and (11-e) must be combined into a single, even larger macroclass, with umlaut accounted for independently ((morpho-) phonologically). Indeed, Carstairs (1987, p. 58) assumes that stem allomorphy (as with umlaut/non-umlaut alternations) does

[6] The status of the German plural marker *s* is highly controversial. It is considered a default marker by some, and an exceptional exponent that lies outside the regular system of noun inflection by others. For relevant discussion, see, e.g., Wiese (1996), Wunderlich (1999), Clahsen (1999), and Wegener (1999). The classification in the main text follows Müller (2002).

not give rise to different macro-inflection classes (there is thus "a distinction between affixal and non-affixal inflection").

As a further example of how the Paradigm Economy Principle works, and how apparent counter-evidence can be handled by invoking the notion of macro-paradigm, consider noun inflection in Russian. As shown in (12), there are four basic inflection classes (class I contains masculine stems; class II contains mainly feminine stems, but also some masculine stems; class III (the 'i-declension') contains feminine stems; and class IV, which is similar to class I, but still sufficiently different from class I to make postulation of a separate inflection class unavoidable, contains only neuter stems). However, all four classes must be split up into subclasses because of an animacy effect: In the singular of class I, and in the plural of all inflection classes, the genitive form is used in accusative contexts with animate stems, and the nominative is used in accusative contexts with inanimate stems.[7]

(12) *Russian noun inflection*

a. Singular

	Ia/Ib$_m$	IIa/IIb$_{f,m}$	IIIa/IIIb$_f$	IVa/IVb$_n$
nom/sg	Ø	a	Ø	o
acc/sg	*Ø*/*a*	u	Ø	o
dat/sg	u	e	i	u
gen/sg	a	i	i	a
inst/sg	om	oj	ju	om
loc/sg	e	e	i	e

b. Plural

	Ia/Ib$_m$	IIa/IIb$_{f,m}$	IIIa/IIIb$_f$	IVa/IVb$_n$
nom/pl	i	i	i	a
acc/pl	*i/ov*	*i*/Ø	*i/ej*	*a*/Ø
dat/pl	am	am	jam	am
gen/pl	ov	Ø	ej	Ø
inst/pl	ami	ami	jami	ami
loc/pl	ax	ax	jax	ax

Thus, it seems that eight inflection classes must be postulated for Russian noun inflection. However, the greatest number of allomorphic variation for a given case is 4 (in the accusative singular). Again, the solution of this apparent problem for the Paradigm Economy Principle is to hold independently estab-

[7] The relevant markers are set in italics. Interestingly enough, this animacy effect also holds for class IV in the plural, where animate neuters are marked by the genitive exponent Ø in accusative contexts; see Corbett and Fraser (1993), Fraser and Corbett (1994), and Krifka (2003).

lished morpho-syntactic features responsible for the differences, and thereby reduce the number of macro-paradigms. Thus, the variation in accusative singular contexts (in class 1) and accusative plural contexts (in all classes) correlates consistently with differences in semantic properties (viz., animacy), and is thus predictable: This reduces the number of macro-paradigms from 8 to 4. Furthermore, the differences between class I and class IV are also predictable on the basis of independently given information, viz., gender: Hence, the number of macro-paradigms is further reduced from 4 to 3. The result is in accordance with the Paradigm Economy Principle.

Concluding so far, given the concept of macro-paradigm (or macro-inflection class), apparent counter-examples to the Paradigm Economy Principle can be explained away. On this view, if a different inflectional pattern can be described by invoking gender features, semantic features (like animacy), phonological features, or if it involves non-affixal inflection, it is irrelevant for paradigm economy: Only those differences count which are absolutely irreducible.

Still, from a more general point of view, there seems to be a potential tension between descriptive adequacy and explanatory adequacy with the Paradigm Economy Principle: Without a concept like that of a macro-paradigm, the Paradigm Economy Principle would be much too restrictive; it would exclude many of the attested inflection patterns in languages with inflection classes. However, if one assumes that the concept of macro-paradigm is relevant for the constraint, the Paradigm Economy Principle's predictive power is reduced.

2.2 No Blur

The No Blur Principle is proposed in Carstairs-McCarthy (1994, p. 742) as a successor to the Paradigm Economy Principle; see (13).

(13) *No Blur Principle*
Within any set of competing inflectional realizations for the same paradigmatic cell, no more than one can fail to identify inflection class unambiguously.

The underlying idea is that there is typically one elsewhere marker that is not specified for inflection class, but no more than that (also see Noyer (2005)). Just like the Paradigm Economy Principle, the No Blur Principle blocks (what looks like) a constant *reuse* of inflectional material in various inflection classes, and thereby constrains the number of possible inflection classes that can be generated on the basis of a given inventory of markers.

The No Blur Principle can be illustrated on the basis of the strong feminine noun declensions in Icelandic (see Carstairs-McCarthy (1994, pp. 740–742)). As shown in (14), there are four strong feminine inflection classes. These are sometimes given names according to the theme vowels (or lack of theme vowels) in Old Norse: Fa (a-stem declension, with a subclass Fa′ that we may ignore in the present context); Fi (i-stem declension); Fc1 (first consonantal declension); and Fc2 (second consonantal declension).

(14) *Strong feminine inflection classes in Icelandic*

	Fa *vél* ('machine')	Fa′ *drottning* ('queen')	Fi *mynd* ('picture')	Fc1 *geit* ('goat')	Fc2 *vík* ('bay')
nom/sg	vél-Ø	drottning-Ø	mynd-Ø	geit-Ø	vík-Ø
acc/sg	vél-Ø	drottning-u	mynd-Ø	geit-Ø	vík-Ø
dat/sg	vél-Ø	drottning-u	mynd-Ø	geit-Ø	vík-Ø
gen/sg	vél-ar	drottning-ar	mynd-ar	geit-ar	vík-ur
nom/pl	vél-ar	drottning-ar	mynd-ir	geit-ur	vík-ur
acc/pl	vél-ar	drottning-ar	mynd-ir	geit-ur	vík-ur
dat/pl	vél-um	drottning-um	mynd-um	geit-um	vík-um
gen/pl	vél-a	drottning-a	mynd-a	geit-a	vík-a

The interesting differences between these inflection classes are all confined to genitive singular and nominative (and accusative) plural contexts; forms with genitive singular markers and forms with nominative plural markers are the "leading forms" ("Kennformen"; see Wurzel (1987)). According to the the No Blur Principle, only one of the allomorphs for a given instantiation of a grammatical category can fail to unambiguously identify an inflection class. The issue is trivial in nominative, accusative, and dative singular contexts (where there is only one default marker *Ø*, abstracting away from *u* in Fa′, which however clearly identifies a (sub)declension); and in dative and genitive plural contexts as well (because, again, there is only one default marker in each case, which is not inflection-class specific: *um, a*). More interestingly, in genitive singular contexts, *ur* is used with inflection class Fc2 (which it unambiguously identifies), and *ar* is the elsewhere marker that is not restricted to any inflection class (and blocked in Fc2 contexts only because there is a more specific marker). Similarly, *ar* is a highly specific nominative (and accusative) plural marker that is confined to inflection class Fa; *ir* is an equally specific nominative (and accusative) plural marker that is confined to inflection class Fi; and *ur* is a more general nominative (and accusative) plural marker that does not bear any inflection class information and thus gets an elsewhere distribution. Since, in both cases, there is only one marker that does not unambiguously identify an inflection class, the No Blur Principle is fully respected by the data in (14).

However, closer scrutiny reveals that the system of Icelandic noun inflection may not be completley unproblematic for the No Blur Principle. This constraint seems to make wrong predictions if the *complete* system of Icelandic noun declension is taken into account (see Kress (1982), Müller (2005)). As shown in (15), there is more than one marker that fails to unambiguously identify inflection class in both genitive singular and nominative plural contexts.[8]

[8] A remark on notation: Ma, Mi, and Mc are the strong masculine declensions; Na is the strong neuter declension; and Mw, Nw, and Fw stand for the three weak declensions.

(15) *The complete system of inflection classes in Icelandic noun inflection*

	I Ma	II Na	III Fa(′)	IV Mi	V Fi	VI Mu	VII Mc	VIII Fc1	IX Fc2	X Mw	XI Nw	XII Fw
nom/sg	ur	Ø	Ø	ur	Ø	ur	ur	Ø	Ø	i	a	a
acc/sg	Ø	Ø	Ø (u)	Ø	Ø	Ø	Ø	Ø	Ø	a	a	u
dat/sg	i	i	Ø (u)	Ø	Ø	i	i	Ø	Ø	a	a	u
gen/sg	s	s	ar	ar	ar	ar	ar	ar	ur	a	a	u
nom/pl	ar	Ø	ar	ir	ir	ir	ur	ur	ur	ar	u	ur
acc/pl	a	Ø	ar	i	ir	i	ur	ur	ur	a	u	ur
dat/pl	um	um	um	um	um	um	um	um	um	um	um	um
gen/pl	a	a	a	a	a	a	a	a	a	a	(n)a	(n)a

As a matter of fact, hardly any of the genitive singular or nominative plural markers in (15) unambiguously identifies and inflection class (the exceptions are *ur* und *u* in the genitive singular, and *Ø* and *u* in the nominative plural).[9] In view of this state of affairs, one might think that the same kind of solution can be suggested for the No Blur Principle as we have seen as a possible reaction to apparent counter-evidence to the Paradigm Economy Principle (and this kind of solution is indeed adopted by Carstairs-McCarthy (1994) for other phenomena): The No Blur Principle only holds for inflection classes of the same gender, not across genders. Still, this does not yet seem to suffice: In masculine nominative plural contexts, neither *ar* nor *ir* unambiguously identifies inflection class: The former marker shows up in Ma and Mw, the latter in Mi and Mu. It is not clear whether this problem can be solved in a way that is not ad hoc.

I take the specific problem just discussed to be indicative of a more general potential problem that is raised by the No Blur Principle (as well as by Noyer's (2005) related Interclass Syncretism Constraint—more precisely, by the assumptions that ultimately derive the latter constraint): Trans-paradigmatic syncretism (i.e., instances of syncretism that affect more than one inflection class) is a recurring pattern of inflectional systems. This pattern has successfully been addressed by standard techniques (going back to Jakobson (1936) and Bierwisch (1967), among others) involving feature decomposition and underspecification (which permits a reference by inflection marker specifications to natural classes of inflection classes). Proposals involving decomposed inflection class features and concomitant underspecification of inflection class information include Halle (1992) (for trans-paradigmatic syncretism in Latvian noun inflection), Oltra Massuet (1999) (for Catalan verb inflection), Wiese (1999) (for German pronominal inflection), Stump (2001) (for Bulgarian verb inflection), Alexiadou and Müller (2007) (for Russian, Greek, and German noun inflection), Müller (2005)

[9] And even the latter is not uncontroversially class-specific if *ur* forms are subanalyzed as a combination of two markers, as suggested in Müller (2005): a marker *u* that bears (underspecified, i.e., non-identifying) inflecion class information, and a general non-obliqueness marker *r*.

(for Icelandic noun inflection), and Trommer (2005) (for Amharic verb inflection); also see the recent proposals in Börjesson (2006), Opitz (2006), and Weisser (2006). In all these approaches, more than one of the inflection markers competing for a given instantiation of a grammatical category fails to unambiguously identify inflection class, in violation of the No Blur Principle.[10]

To sum up, I think that the Paradigm Economy Principle and the No Blur Principle (and the related Interclass Syncretism Constraint) can be viewed as interesting and plausible proposals that reduce the set of logically possible inflection classes (based on a given inventory of markers) to a very small set. However, it seems clear that these constraints constantly face the danger of being *too* restrictive. Severe undergeneration problems can only be avoided by assuming that differences between inflection classes which are

[10] In line with this, I would like to contend that an abstract paradigm like (i), which Carstairs-McCarthy (1991, 2000) presents as a kind of paradigm that should be excluded on general grounds (and which is in fact excluded by the No Blur Principle), may exhibit peculiar properties, but the systematic lack of inflection-class specific exponents is not one of them. Here, I–V are inflection classes, and [1–4] are different instantiations of a grammatical category that is encoded by the markers, e.g., cases.

(i)

	I	II	III	IV	V
[1]	a	b	b	b	b
[2]	c	c	d	d	d
[3]	e	e	e	f	f
[4]	g	g	g	g	h

The system can be characterized as follows: For each case, there are two markers available in the basic inventory. Taking class I as given, class II diverges from class I by changing one cell; class III modifies class II by changing another cell, and so on. It seems that what is strange about (i) is just this exact pattern, and not the richness of trans-paradigmatic syncretism emerging from it. In fact, focussing on just nom/sg, dat/sg, gen/sg, and nom/pl contexts and five select inflection classes in the Icelandic system of noun inflection in (15), we end up with an abstract paradigm that is very similar to the one in (i), except that it lacks the precise "one-cell-at-a-time inflection class divergence pattern" (to use Carstairs-McCarthy's expression) seen in (i). Compare (ii), where a and b stand for the nominative singular markers *Ø* and *ur*, respectively; c and d for the dative singular markers *Ø* and *i*; e and f for the genitive singular markers *ar* and *s*, and g and h for the nominative plural markers *ar* and *ir*. The numbering of the inflection classes is the one in (15). (I abstract away here from genuine case syncretism, which is irrelevant for the argument.)

(ii)

	III	V	IV	VI	I
[1]	a	a	b	b	b
[2]	c	c	c	d	d
[3]	e	e	e	e	f
[4]	g	h	h	h	g

The availability of (ii) strongly suggests that what is wrong with (i) is not the lack of inflection-class specific markers. Since (ii) is very similar to (i), and since there is no reason to assume that there are meta-grammatical principles that might bring about something like (i) in a non-accidental way, one may speculate that the absence of abstract paradigms of exactly the form in (i) is simply due to the fact that such a pattern is highly unlikely, given a random distribution of markers over cases and inflection classes.

independently derivable (by invoking gender features, phonological features, or semantic features) are somehow irrelevant for the constraints; and this, I believe, may take away something of the constraints' initial predictive power. Furthermore, it has turned out that these constraints are incompatible with the view that paradigms are mere epiphenomena (this holds for the Paradigm Economy Principle, and to some extent also for the No Blur Principle), and with the view that trans-paradigmatic syncretism can be accounted for by invoking class feature decomposition and underspecification. All in all, I would like to conclude that this warrants looking for alternative ways of bringing about paradigm economy. In the next section, I will argue that a version of paradigm economy follows straightforwardly from the Syncretism Principle in (1).

3 Paradigm economy as a theorem

3.1 Claim

In what follows, I will basically presuppose an approach along the lines of Distributed Morphology. However, this is mainly to have a theory in which to frame the discussion. As far as I can see, the issues to be discussed below arise in exactly the same way—and can be addressed in exactly the same way—in alternative morphological theories, such as Minimalist Morphology (Wunderlich (1996, 1997)) or Paradigm Function Morphology (Stump 2001); where appropriate, I will therefore also discuss these latter two approaches. The central claim I would like to advance here is that (16) holds.

(16) *Inflection Class Economy Theorem*
Given a set of n inflection markers, there can be at most 2^{n-1} inflection classes, independently of the number of instantiations of the grammatical category that the markers have to distribute over.

The number of 2^{n-1} inflection classes encodes the powerset of the inventory of markers, minus one radically underspecified marker. I will explain below (in Sect. 3.3) why exactly this number would be relevant. For now, we can conclude that (16) significantly restricts the number of possible inflection classes over a given inventory of inflection markers. For instance, assuming an abstract system with five markers and six instantiations of a grammatical category (e.g., case), the Inflection Class Economy Theorem states that there can at most be sixteen (i.e., $2^{5-1} = 2^4$) inflection classes, out of the 15.625 (i.e., 5^6) that would otherwise be possible.

The Inflection Class Economy Theorem follows under any morphological theory that makes the three assumptions in (17), (18), and (19), which I call 'Syncretism,' 'Elsewhere,' and 'Blocking.' I discuss and try to motivate them in turn, beginning with Syncretism.

(17) *Syncretism*
The Syncretism Principle holds: For each marker, there is a unique specification of morpho-syntactic features.

The Syncretism Principle underlies much recent (and, based on the Jakobsonian tradition, some not so recent) work in inflectional morphology; it provides simple and elegant analyses, and it has been empirically confirmed for a variety of inflectional systems in the world's languages. Assuming Syncretism to be valid is the starting point of the present paper.

The second assumption is that there is always one radically underspecified (elsewhere) inflection marker in any given morphological domain.

(18) *Elsewhere*
There is always one elsewhere marker that is radically underspecified with respect to inflection class (and more generally). Other markers may be underspecified to an arbitrary degree (including not at all).

The concept of underspecification as a means to account for syncretism is employed in most recent theories of inflectional morphology—in Distributed Morphology, Minimalist Morphology, Paradigm Function Morphology, etc.[11]

[11] In Distributed Morphology (see, e.g., Halle and Marantz (1993), Halle (1997)), functional heads in syntax provide fully specified contexts for insertion of vocabulary items; and whereas the former are characterized by fully specified morpho-syntactic features (ignoring impoverishment), the vocabulary items can be (and often are) underspecified with respect to these features; a Subset Principle ensures that a vocabulary item can only be inserted if it does not bear features which contradict those in the functional morpheme in syntax. Similarly, underspecification is considered to be one of the central assumptions of Distributed Morphology (see Wunderlich (1996, 2004)). Even though Minimalist Morphology differs from Distributed Morphology in being an "incremental" approach, where the inflection marker contributes features to the whole word that would otherwise not be present (see Stump (2001) for the terminology), Wunderlich manages to integrate underspecification of inflection markers into the system, and in doing so invokes a Compatibility requirement that has effects which are similar to those of the Subset Principle. Finally, in Paradigm Function Morphology (see Stump (2001)), inflection markers are added to stems by morphological realization rules, which take the abstract form of (i).

(i) $\mathrm{RR}_{n,\tau,C}(<\mathrm{X},\sigma>) = <\mathrm{Y}',\sigma>$

Here, τ is the set of morpho-syntactic features associated with the inflection marker (the inflection marker emerges as the difference between the stem X and the inflected word Y′); τ can be underspecified. In contrast, σ is the set of morpho-syntactic features that the fully inflected word form bears (the analogue to the insertion contexts provided by functional morphemes in Distributed Morphology). Importantly, a constraint on rule/argument coherence ensures that σ is an *extension* of τ; this is comparable to the subset and compatibility requirements of Distributed Morphology and Minimalist Morphology, respectively. Incidentally, I take this convergence of theoretical approaches to be indicative of a more general similarity: Many (though not all) of the differences between approaches like Distributed Morphology, Minimalist Morphology, and Paradigm Function Morphology can ultimately be shown to be differences in notation rather than differences in substance. See Müller (2007b) for discussion.

Similarly, the assumption that there is always one radically underspecified elsewhere marker in inflectional systems is quite common, and well-motivated empirically because it can account for 'discontinuous' occurrences of markers in paradigms (where natural classes captured by non-radical underspecification is unlikely to be involved). Furthermore, and perhaps most importantly, the existence of an elsewhere marker ensures that there are (usually) no paradigmatic gaps in inflectional systems.[12] As soon as underspecification is adopted, and the traditional idea is abandoned that all inflection markers are characterized by fully specified morpho-syntactic specifications, the situation may arise that there is a fully specified context for which there is no marker that fits into it (such that a subset/compatibility/extension relation exists; see footnote 11). Such a situation is avoided on principled grounds if there is always a marker that fits anywhere.[13]

The third and final assumption that must be made to derive the Inflection Class Economy Theorem is Blocking. Every theory of morphology that employs under-specification[14] must somehow ensure that only one marker can be chosen for any given instantiation of a grammatical category (i.e., morpho-syntactic context). The Blocking requirement is often understood in terms of specificity, as in (19).

(19) *Blocking*
Competition of underspecified markers is resolved by choosing the most specific marker: For all (competing) markers α, β, either α is more specific than β, or β is more specific than α.

A Specificity constraint along these lines is adopted in Distributed Morphology (typically as part of the definition of the Subset Principle, see Halle (1997)), in Minimalist Morphology (see Wunderlich (1996, 1997, 2004)), and in Paradigm Function Morphology (Stump (2001)) calls the relevant constraint Panini's Principle). Note that it does not matter how specificity is to be understood exactly—e.g., whether it is determined by simply counting the number of features that characterize an inflection marker (so that one marker can be more

[12] The cases of paradigmatic gaps in inflectional paradigms that do exist are the exception rather than the rule (in contrast to what is the case with derivational morphology). Moreover, existing approaches typically locate the source of these gaps outside the morphological component proper (e.g., in other grammatical domains where the use of certain forms may be blocked; see Halle (1973), Fanselow and Féry (2002)), or attempt to reduce them to speaker-based uncertainties concerning lexical items and whether to apply morphophonological rules to them (see Albright (2003)).

[13] The assumption that there is always one radically underspecified marker that fits into any context is often not made explicit, and then only emerges as a by-product of the feature specifications for inflection markers that is proposed in an analysis. See, however, Stump (2001), who adopts an Identity Function Default rule to this effect.

[14] Or, in fact, any other means that triggers a competition of markers, like non-faithful feature realization by feature-changing impoverishment in Distributed Morphology (see Noyer (1998)) or by optimal violation of faithfulness constraints in recent versions of Minimalist Morphology (see Wunderlich (2004)); or rules of referral in Paradigm Function Morphology (Stump 2001) and related approaches like Network Morphology (see Corbett and Fraser (1993), Baerman et al. (2005)).

specific than another one even though their specifications do not stand in a a subset/extension relation); or by invoking subset/superset relations (so that one marker cannot be more specific than another marker if the two do not compete; see, e.g., Stump's (2001) notion of a "narrower" rule); or by a hierarchy of features (or feature classes).[15]

Taken together, it follows from the three assumptions needed to derive the Inflection Class Economy Theorem that (i) syncretism is systematic in the sense that only one specification of morpho-syntactic features is associated with any given inflection marker (with the qualifications made in (1)); (ii) for any given fully specified context, there is always one inflection marker that fits; and (iii) for any given fully specified context, there is never more than one inflection marker that fits. Thus, Elsewhere and Blocking emerge as two sides of the same coin.[16]

With the Inflection Class Economy Theorem and the assumptions needed to derive it in place, there are two issues that need to be addressed. First, how does the Inflection Class Economy Theorem constrain inflectional systems (and how does it differ in this respect from the constraints discussed in Sect. 2)? And second, how does the Inflection Class Economy Theorem follow as a theorem from Syncretism, Elsewhere, and Blocking? I address the former question first.

3.2 Illustration

The basic question underlying all work on paradigm economy comes in two versions, viz., (20-a) and (20-b):

(20) a. Given an inventory of markers for a certain domain (e.g., noun inflection), how many inflection classes can there be?
b. Given an inventory of markers *with associated features encoding a grammatical category* (*e.g., case*) for a certain domain (e.g., noun inflection), how many inflection classes can there be?

Carstairs (1987) only tries to answer (20-b); but I find (20-a) the arguably more interesting question: It does not presuppose that the specification of a marker for a grammatical category (e.g., with respect to case and/or number) is somehow privileged, i.e., more basic than its inflection class features. Thus, from the perspective of a child acquiring an inflectional system, (20-b) would seem to

[15] In the same way, extrinsic ordering (as, e.g., in Bierwisch (1967)) could be adopted to determine the competition winner; or constraint ranking (Wunderlich 2004); or other mechanisms, like the expanded mode property of certain realization rules that Stump (2001) argues for on the basis of, i.a., Georgian prefixal argument encoding morphology.

[16] Assumptions (ii) and (iii) correspond to the principles of 'Completeness' and 'Uniqueness' in Wunderlich (1996, p. 99); also, taken together they are equivalent to the concept of "parsimoniuous coverage" in Carstairs (1988).

presuppose that there is a stage in the acquisition process where the child has learned a set of markers together with their syntactically relevant features (such as case and number), and then needs to decide how all these markers can be assembled into inflection classes.[17] In contrast, (20-a) suggests that the child is confronted with a set of markers and faces the task of assigning these markers (typically underspecified) specifications, including syntactically relevant features *and* inflection class features that are only important for morphology. In the remainder of this paper, I assume that it is indeed question (20-a) that needs to be addressed by a theory of paradigm economy.

With this in mind, let me introduce a few abstract examples to illustrate the workings of the Inflection Class Economy Theorem. In a system without any restrictions on the number and form of inflection classes, we can observe the following generalization: If, in a given domain (e.g., noun inflection), there are n markers for m instantiations of a grammatical category (e.g., case), the markers can be grouped into n^m distinct inflection classes (i.e., the set of m-tuples over an input set with n members). Thus, suppose that we have a system of noun inflection with three markers and four cases. In an unconstrained system, there should then be eighty-one logically possible inflection classes. This is shown in Table 1.[18]

Here, *a, b*, and *c* stand for the three markers; and all four-letter rows (4-tuples separated by either horizontal space or a line break) correspond to individual inflection classes, with the first marker in a row being used for the first instantiation of case (e.g., nominative), the second one for the second instantiation of case (e.g., accusative), the third one for the third instantiation of case (e.g., dative), and the fourth one for the fourth instantiation of case (e.g., genitive). It seems extremely unlikely that a language can be found in which eighty-one inflection classes have been generated on the basis of three markers and four instantiations of a grammatical category. The Paradigm Economy Principle narrows down the set of possible inflection classes from eighty-one to three. Under present assumptions (where paradigm economy considerations do not take into account the markers' grammatical category specification), the worst case scenario for the Paradigm Economy Principle is that all three markers can be allomorphs for a single case specification (e.g., *a, b*, and *c* can all be accusative markers); still, there can then only be three distinct inflection classes. Note that this reasoning is independent of the number of instantiations of the grammatical category that the exponents have to distribute over (e.g., whether there are two or eight cases).

Next, the No Blur Principle permits maximally nine inflection classes out of the eighty-one classes in Table 1. In the worst case scenario, there is one default marker (say, *a*). One class consists only of default markers (*aaaa*), and all the other inflection classes differ from this class by replacing one of the *a*'s with

[17] Note, however, that this conclusion does not hold for the approach taken in Carstairs-McCarthy (1994).

[18] The generation of the abstract lists in this section and in the following section (as well as of many other, often much more complex lists that were used in developing the present analysis) was greatly facilitated by one of Chris Potts' contributions to the comp4ling toolbox, available on the website of the University of Massachusetts linguistics department: http://web.linguist.umass.edu/~comp4ling/.

Table 1 Three markers, four cases: 81 ($=3^4$) possible inflection classes

a a a a	a b c a	b a b a	b c a a	c a c a	c c b a
a a a b	a b c b	b a b b	b c a b	c a c b	c c b b
a a a c	a b c c	b a b c	b c a c	c a c c	c c b c
a a b a	a c a a	b a c a	b c b a	c b a a	c c c a
a a b b	a c a b	b a c b	b c b b	c b a b	c c c b
a a b c	a c a c	b a c c	b c b c	c b a c	c c c c
a a c a	a c b a	b b a a	b c c a	c b b a	
a a c b	a c b b	b b a b	b c c b	c b b b	
a a c c	a c b c	b b a c	b c c c	c b b c	
a b a a	a c c a	b b b a	c a a a	c b c a	
a b a b	a c c b	b b b b	c a a b	c b c b	
a b a c	a c c c	b b b c	c a a c	c b c c	
a b b a	b a a a	b b c a	c a b a	c c a a	
a b b b	b a a b	b b c b	c a b b	c c a b	
a b b c	b a a c	b b c c	c a b c	c c a c	

either *b* or *c* (*baaa, abaa, aaba, aaab, caaa, acaa, aaca, aaac*), so that all classes respect the No Blur Principle.[19] Adding another class with more than one *b*, or more than one *c*, or a—perhaps minimal—combination of *b*'s and *c*'s (cf. *bbaa*, or *aacc*, or *abca*, etc.) will invariably lead to a violation of the No Blur Principle because either *b* or *c* (or both) will cease to be inflection-class specific.

Note that assuming default markers that are specific with respect to instantiations of a grammatical category (such that, e.g., *a* is the default marker for the first instantiation, *b* for the second, *c* for the third, and perhaps again *a* for the fourth) instead of an extremely general default marker *a* does not change things: This would be compatible with No Blur, but it could not increase the number of possible inflection classes. In the case at hand, the maximal set of inflection classes would include *abca*, *bbca, cbca, aaca, acca, abaa, abba, abcb, abcc*.

In general, the No Blur Principle predicts that there can at most be $((n-1) \times m) + 1$ inflection classes, for n markers and m instantiations of a grammatical category: Every marker except for one—the default marker, hence "–1"—can appear for a given instantiation of a grammatical category only in one inflection class; and "+1" captures a class consisting exclusively of default markers.

Turning finally to the Inflection Class Economy Theorem, we expect that at most four (i.e., 2^{3-1}) classes can exist in the example in Table 1, out of the eighty-one classes that are a priori possible. The predictions made by the

[19] To be sure, such a situation would be unexpected if the Syncretism Principle is adopted since it would not at all be clear how a marker like, e.g., *b* here could be given a unique feature specification. However, No Blur's workings as such do not rely on something like the Syncretism Principle. In fact, as noted above, No Blur is incompatible with non-radical underspecification accounts of transparadigmatic syncretism, i.e., accounts where an inflection marker is associated with a morpho-syntactic specification that captures a natural class of inflection classes which is neither minimal (a single inflection class, derivable without underspecification, where the marker is a class-identifier in Carstairs-McCarthy's terms) nor maximal (all inflection classes, derivable by radical underspecification, where the marker is a class-default).

Paradigm Economy Principle, the No Blur Principle, and the Inflection Class Economy Theorem for the system underlying Table 1 are summarized in (21).

(21) a. Paradigm Economy Principle, worst case scenario:
3 inflection classes: the size of the inventory
b. No Blur Principle, worst case scenario:
9 inflection classes: $((3-1)\times 4)+1$
c. Inflection Class Economy Theorem, worst case scenario:
4 inflection classes: 2^{3-1}

Consider now a second abstract example. This time, there are five inflection markers, and three cases (or other instantiations of a grammatical category). As shown in Table 2, without constraints there could be 125 different inflection classes.

In the worst case scenario under the Paradigm Economy Principle, there could be five different inflection classes. Under No Blur, there could in principle be thirteen inflection classes (e.g., assuming *a* as a default marker, *aaa, baa, aba, aab, caa, aca, aac, daa, ada, aad, eaa, aea, aae*). Given the Inflection Class Economy Theorem, there is a maximum of sixteen inflection classes (viz., 2^{5-1}). Again, this is summarized schematically below:

(22) a. Paradigm Economy Principle, worst case scenario:
5 inflection classes: the size of the inventory
b. No Blur Principle, worst case scenario:
13 inflection classes: $((5-1)\times 3)+1$
c. Inflection Class Economy Theorem, worst case scenario:
16 inflection classes: 2^{5-1}

Let me finally bring up a third, slightly more complex example. Suppose that there are five inflection markers that can be distributed over four instantiations

Table 2 Five markers, three cases: 125 ($=5^3$) possible inflection classes

a a a	a d a	b b a	b e a	c c a	d a a	d d a	e b a	e e a
a a b	a d b	b b b	b e b	c c b	d a b	d d b	e b b	e e b
a a c	a d c	b b c	b e c	c c c	d a c	d d c	e b c	e e c
a a d	a d d	b b d	b e d	c c d	d a d	d d d	e b d	e e d
a a e	a d e	b b e	b e e	c c e	d a e	d d e	e b e	e e e
a b a	a e a	b c a	c a a	c d a	d b a	d e a	e c a	
a b b	a e b	b c b	c a b	c d b	d b b	d e b	e c b	
a b c	a e c	b c c	c a c	c d c	d b c	d e c	e c c	
a b d	a e d	b c d	c a d	c d d	d b d	d e d	e c d	
a b e	a e e	b c e	c a e	c d e	d b e	d e e	e c e	
a c a	b a a	b d a	c b a	c e a	d c a	e a a	e d a	
a c b	b a b	b d b	c b b	c e b	d c b	e d b	e d b	
a c c	b a c	b d c	c b c	c e c	d c c	e a c	e d c	
a c d	b a d	b d d	c b d	c e d	d c d	e a d	e d d	
a c e	b a e	b d e	c b e	c e e	d c e	e a e	e d e	

of a grammatical category (again, let us say, four cases). Now there could in principle be 625 distinct inflection classes on the basis of this inventory, clearly a highly unlikely situation. Consider the list in Table 3.

The Paradigm Economy Principle drastically reduces the number of possible inflection classes. Since there are still only five markers, five is the maximal number of allomorphs for a cell, and consequently there can be no more than five inflection classes. Next, the No Blur Principle predicts that a maximum of seventeen inflection classes should be possible (e.g., *aaaa, baaa, abaa, aaba, aaab, caaa, acaa, aaca, aaac, daaa, adaa, aada, aaad, eaaa, aeaa, aaea, aaae*). Finally, the Inflection Class Economy Theorem states that there can still only be at most sixteen inflection classes for this inventory of markers and for this number of instantiations of a grammatical category. Thus, whereas the number of instantiations of a grammatical category does matter for the No Blur Principle, it turns out to be irrelevant for the Inflection Class Economy Theorem (as well as for the Paradigm Economy Principle): If, say, five markers are distributed over eight instantiations of a grammatical category, No Blur predicts a maximum of thirty-three classes under a worst case scenario, whereas the Inflection Class Economy Theorem still stays at sixteen (and the Paradigm Economy Principle at five). The different predictions made for Table 3 are given in (23).

(23) a. Paradigm Economy Principle, worst case scenario:
5 inflection classes: the size of the inventory
b. No Blur Principle, worst case scenario:
17 inflection classes: $((5-1)\times 4)+1$
c. Inflection Class Economy Theorem, worst case scenario:
16 inflection classes: 2^{5-1}

Concluding so far, the Inflection Class Economy Theorem restricts possible inflection classes in a way that is roughly comparable to the Paradigm Economy and No Blur Principles. It now remains to be shown that it can indeed be derived from the assumptions concerning Syncretism, Elsewhere, and Blocking laid out in the previous section.

3.3 Deriving the inflection class economy theorem

Recall again what the three main assumptions amount to: Syncretism says that (exceptions apart) each marker of the inventory of a given morphological domain is associated with only one morpho-syntactic feature specification; Elsewhere states that there is always one marker that in principle fits into every context of fully specified morpho-syntactic features (which systematically excludes true paradigmatic gaps); and Blocking demands that eventually there is always only one marker that can in fact be used for any fully specified context of morpho-syntactic features (which systematically excludes cases of optionality in exponence).

Table 3 Five markers, four cases: 625 ($= 5^4$) possible inflection classes

a a a a	a c c a	a e e a	b c b a	b e d a	c c a a	c e c a	d b e a	d e b a	e b d a	e e a a
a a a b	a c c b	a e e b	b c b b	b e d b	c c a b	c e c b	d b e b	d e b b	e b d b	e e a b
a a a c	a c c c	a e e c	b c b c	b e d c	c c a c	c e c c	d b e c	d e b c	e b d c	e e a c
a a a d	a c c d	a e e d	b c b d	b e d d	c c a d	c e c d	d b e d	d e b d	e b d d	e e a d
a a a e	a c c e	a e e e	b c b e	b e d e	c c a e	c e c e	d b e e	d e b e	e b d e	e e a e
a a b a	a c d a	b a a a	b c c a	b e e a	c c b a	c e d a	d c a a	d e c a	e b e a	e e b a
a a b b	a c d b	b a a b	b c c b	b e e b	c c b b	c e d b	d c a b	d e c b	e b e b	e e b b
a a b c	a c d c	b a a c	b c c c	b e e c	c c b c	c e d c	d c a c	d e c c	e b e c	e e b c
a a b d	a c d d	b a a d	b c c d	b e e d	c c b d	c e d d	d c a d	d e c d	e b e d	b e b d
a a b e	a c d e	b a a e	b c c e	b e e e	c c b e	c e d e	d c a e	d e c e	e b e e	e e b e
a a c a	a c e a	b a b a	b c d a	c a a a	c c c a	c e e a	d c b a	d e d a	e c a a	e e c a
a a c b	a c e b	b a b b	b c d b	c a a b	c c c b	c e e b	d c b b	d e d b	e c a b	e e c b
a a c c	a c e c	b a b c	b c d c	c a a c	c c c c	c e e c	d c b c	d e d c	e c a c	e e c c
a a c d	a c e d	b a b d	b c d d	c a a d	c c c d	c e e d	d c b d	d e d d	e c a d	c e c d
a a c e	a c e e	b a b e	b c d e	c a a e	c c c e	c e e e	d c b e	d e d e	e c a e	e e c e
a a d a	a d a a	b a c a	b c e a	c a b a	c c d a	d a a a	d c c a	d e e a	e c b a	e e d a
a a d b	a d a b	b a c b	b c e b	c a b b	c c d b	d a a b	d c c b	d e e b	e c b b	e e d b
a a d c	a d a c	b a c c	b c e c	c a b c	c c d c	d a a c	d c c c	d e e c	e c b c	e e d c
a a d d	a d a d	b a c d	b c e d	c a b d	c c d d	d a a d	d c c d	d e e d	e c b d	e e d d
a a d e	a d a e	b a c e	b c e e	c a b e	c c d e	d a a e	d c c e	d e e e	e c b e	e e d e
a a e a	a d b a	b a d a	b d a a	c a c a	c c e a	d a b a	d c d a	e a a a	e c c a	e e e a
a a e b	a d b b	b a d b	b d a b	c a c b	c c e b	d a b b	d c d b	e a a b	e c c b	e e e b
a a e c	a d b c	b a d c	b d a c	c a c c	c c e c	d a b c	d c d c	e a a c	e c c c	e e e c
a a e d	a d b d	b a d d	b d a d	c a c d	c c e d	d a b d	d c d d	e a a d	e c c d	e e e d
a a e e	a d b e	b a d e	b d a e	c a c e	c c e e	d a b e	d c d e	e a a e	e c c e	e e e e
a b a a	a d c a	b a e a	b d b a	c a d a	c d a a	d a c a	d c e a	e a b a	e c d a	
a b a b	a d c b	b a e b	b d b b	c a d b	c d a b	d a c b	d c e b	e a b b	e c d b	
a b a c	a d c c	b a e c	b d b c	c a d c	c d a c	d a c c	d c e c	e a b c	e c d c	
a b a d	a d c d	b a e d	b d b d	c a d d	c d a d	d a c d	d c e d	e a b d	e c d d	
a b a e	a d c e	b a e e	b d b e	c a d e	c d a e	d a c e	d c e e	e a b e	e c d e	
a b b a	a d d a	b b a a	b d c a	c a e a	c d b a	d a d a	d d a a	e a c a	e c e a	
a b b b	a d d b	b b a b	b d c b	c a e b	c d b b	d a d b	d d a b	e a c b	e e e b	

Table 3 Continued

a b b c	a d d c	b b a c	b d c c	c a e c	c d b c	d a d c	d d a c	e a c c	e c e c
a b b d	a d d d	b b a d	b d c d	c a e d	c d b d	d a d d	d d a d	e a c d	e c e d
a b b e	a d d e	b b a e	b d c e	c a e e	c d b e	d a d e	d d a e	e a c e	e c e e
a b c a	a d e a	b b b a	b d d a	c b a a	c d c a	d a e a	d d b a	e a d a	e d a a
a b c b	a d e b	b b b b	b d d b	c b a b	c d c b	d a e b	d d b b	e a d b	e d a b
a b c c	a d e c	b b b c	b d d c	c b a c	c d c c	d a e c	d d b c	e a d c	e d a c
a b c d	a d e d	b b b d	b d d d	c b a d	c d c d	d a e d	d d b d	e a d d	e d a d
a b c e	a d e e	b b b e	b d d e	c b a e	c d c e	d a e e	d d b e	e a d e	e d a e
a b d a	a e a a	b b c a	b d e a	c b b a	c d d a	d b a a	d d c a	e a e a	e d b a
a b d b	a e a b	b b c b	b d e b	c b b b	c d d b	d b a b	d d c b	e a e b	e d b b
a b d c	a e a c	b b c c	b d e c	c b b c	c d d c	d b a c	d d c c	e a e c	e d b c
a b d d	a e a d	b b c d	b d e d	c b b d	c d d d	d b a d	d d c d	e a e d	e d b d
a b d e	a e a e	b b c e	b d e e	c b b e	c d d e	d b a e	d d c e	e a e e	e d b e
a b e a	a e b a	b b d a	b e a a	c b c a	c d e a	d b b a	d d d a	e b a a	e d c a
a b e b	a e b b	b b d b	b e a b	c b c b	c d e b	d b b b	d d d b	e b a b	e d c b
a b e c	a e b c	b b d c	b e a c	c b c c	c d e c	d b b c	d d d c	e b a c	e d c c
a b e d	a e b d	b b d d	b e a d	c b c d	c d e d	d b b d	d d d d	e b a d	e d c d
a b e e	a e b e	b b d e	b e a e	c b c e	c d e e	d b b e	d d d e	e b a e	e d c e
a c a a	a e c a	b b e a	b e b a	c b d a	c e a a	d b c a	d d e a	e b b a	e d d a
a c a b	a e c b	b b e b	b e b b	c b d b	c e a b	d b c d	d d e b	e b b b	e d d b
a c a c	a e c c	b b e c	b e b c	c b d c	c e a c	d b c c	d d e c	e b b c	e d d c
a c a d	a e c d	b b e d	b e b d	c b d d	c e a a	d b c d	d d e d	e b b d	e d d d
a c a e	a e c e	b b e e	b e b e	c b d e	c e a e	d b c e	d d e e	e b b e	e d d e
a c b a	a e d a	b c a a	b e c a	c b e a	c e b a	d b d a	d e a a	e b c a	e d e a
a c b b	a e d b	b c a b	b e c b	c b e b	c e b d	d b d b	d e a b	e b c b	e d e b
a c b c	a e d c	b c a c	b e c c	c b e c	c e b c	d b d c	d e a c	e b c c	e d e c
a c b d	a e d d	b c a d	b e c d	c b e d	c e b d	d b d d	d e a d	e b c d	e d e d
a c b e	a e d e	b c a e	b e c e	c b e e	c e b e	d b d e	d e a e	e b c e	e d e e

As a first, and crucial, step towards deriving the Inflection Class Economy Theorem from these three assumptions, note that the following holds: Since each inflection marker M can only be associated with one specification of morpho-syntactic features (because of Syncretism), it follows that for each inflection marker M and for each inflection class I, it must be the case that M is either *compatible* with I or *incompatible* with I.[20] A marker is compatible with an inflection class I if it bears no inflection class feature, if it bears fully specified inflection class information that completely characterizes I, or (assuming inflection class decomposition and underspecification; see the references above) if it is characterized by a set of underspecified inflection class features that is a subset of (or, in Paradigm Function Morphology, extendable to) the fully specified set of features that characterize the inflection class. We can say that M is *activated* for I if it is compatible with it; and *deactivated* for I if it is incompatible with it.[21]

Next, Blocking ensures that each inflection class can be defined in terms of the markers that are active in it: For all competing markers α and β, it is fixed once and for all by the markers' feature specifications (and independently of inflection classes) that either β is more specific than α, or α is more specific than β.[22] Hence, if the same set of markers is activated for two possible inflection classes I_1 and I_2, I_1 must be identical to I_2. Conversely, since every marker is either activated or deactivated for any given inflection class, it also follows that if the same set of markers is *deactivated* for two inflection classes I_1 and I_2, I_1 and I_2 must be the same inflection class (because the same set of markers is then activated for I_1 and I_2, because a marker /x/ can only have one specification [ξ], and because specificity relations among competing markers are fixed).

In order to determine the maximal number of possible inflection classes on the basis of a given inventory of markers,[23] it now suffices to successively deactivate all possible marker combinations. Starting with the full inventory of markers, we can proceed by successively deactivating all combinations of

[20] This holds independently of whether we assume that the morpho-syntactic features associated with an inflection marker are encoded as a context specification "$\leftrightarrow$ [ξ]", with the marker viewed as a vocabulary item /x/, as in Distributed Morphology; as a lexical entry of an affix, as in Minimalist Morphology; or as the τ-part of a realization rule of exponence in Paradigm Function Morphology (see footnote 11). However, as noted above, for the sake of concreteness I adopt a Distributed Morphology notation here.

[21] However, if a marker is activated for an inflection class I, this does not imply that it will actually be used by I—there may well be more specific markers that block it.

[22] Recall from the discussion of (19) that it does not matter in the present context how exactly specificity is defined, e.g., whether it is defined by the size of the set of morpho-syntactic associated with an exponent, by invoking some notion of feature hierarchy, or by some other means.

[23] The notion of "marker" is to be understood in a somewhat more abstract way that ignores allomorphic variation which is phonologically or morpho-phonologically conditioned (and not morphologically, as with variation determined by inflection class membership). For instance, Halle (1994) argues that the marker realizations *ov* and *ej* for genitive plural in Russian are allomorphs whose choice is morpho-phonologically determined; on this view, there is but a single marker /ov/, accompanied a single underspecified set of morpho-syntactic features (perhaps involving underspecified inflection class features, as suggested in Alexiadou and Müller (2007) in order to account for the fact that this marker exhibits trans-paradigmatic syncretism).

markers, which yields class after class. Thus, the first possibility is that all markers of an inventory can be used (i.e., none is deactivated); this defines a first possible class I_1. The second possibility is that one marker of the inventory, say *a*, is deactivated; this defines a second possible class I_2. Next, another marker, say, *b*, is deactivated; this yields a third possible class I_3. Next, the markers *a* and *b* may be deactivated; this defines a fourth possible class I_4; and so forth, until all subsets of the inventory have been subjected to deactivation, defining a possible inflection class in each case. However, by assumption (Elsewhere), one marker always is the elsewhere (default) marker: It is compatible with all inflection classes because it is radically underspecified; and therefore it cannot be deactivated by definition. Consequently, all possible *marker deactivation combinations* are provided by the *powerset* of the set of all the markers of the inventory minus the elsewhere marker: 2^{n-1}, for *n* markers. Thus, given a set of *n* inflection markers, there can be at most 2^{n-1} marker deactivation combinations. Since marker deactivation combinations fully determine possible inflection classes, it now follows that given a set of *n* inflection markers, there can be at most 2^{n-1} inflection classes.[24]

This reasoning is entirely independent of the number of instantiations of the grammatical category (e.g., the number of cases) that a set of markers needs to distribute over. In contrast to what is the case under the No Blur Principle (but as with the Paradigm Economy Principle), an increase in instantiations of a grammatical category does not induce an increase in possible inflection classes over a given inventory of markers; this information is simply irrelevant for determining the maximal number of inflection classes (as with the Paradigm Economy Principle, and in contrast to what holds under the No Blur Principle). Hence, we end up deriving the statement that given a set of *n* inflection markers, there can be at most 2^{n-1} marker deactivation combinations, *independently of the number of instantiations of the grammatical category that the markers have to distribute over*. This is the Inflection Class Economy Theorem.

It may be useful to illustrate this theorem by a few simple, abstract examples which show select sets of inflection classes that can be generated on the basis of initial marker inventories, and which also show that the restrictions imposed by the Inflection Class Economy Theorem are not just quantitative in nature, but also qualitative.

3.4 Examples

3.4.1 A first example

Consider again Table 1. In order to illustrate the possible marker deactivation patterns, the case categories are now called *1*, *2*, *3*, and *4*. The marker deacti-

[24] Elsewhere thus turns out to be the least important of the three assumptions needed to derive the Inflection Class Economy Theorem: If it did not hold, we could still derive that there cannot be more than 2^n inflection classes if there are *n* markers.

vation combinations are added in (24-c). Given an inventory of three markers, there are $2^{3-1} = 4$ deactivation combinations.

(24) *Example 1 revisited*

a. 3 markers: {a, b, c}

b. 4 cases: 1, 2, 3, 4

c. Deactivation combinations: { {b, c}, {b}, {c}, {} }

As a consequence, of the eighty-one inflection classes that would logically be possible under an unconstrained combination of inflection markers, only four remain, given Syncretism, Elsewhere, and Blocking (i.e., the Inflection Class Economy Theorem). This result holds under any specificity-induced order of the markers, and under any assignment of case features to markers. Let me go through a couple of possible assignments of case features to the three markers of the inventory. Suppose, for instance, that the three markers /a/, /b/, /c/ are characterized by case features as shown in (25-a):[25] /a/ is the elsewhere marker; /b/ is an underspecified marker that fits into contexts with either a case *1* or a case *2* specification; and /c/ is an underspecified marker that fits for all case feature specifications that are not *1*.[26] Given Blocking, the markers must be ordered with respect to specificity. Under simple notions of specificity, /b/ will emerge as more specific than /c/ (but this could just as well be the other way round), and /c/ as more specific than the elsewhere marker /a/; see (25-b). Under these assumptions, the four possible marker deactivation combinations lead to the four inflection classes shown in (25-c). If /b/ and /c/ are deactivated, an inflection class *aaaa* results; if only /b/ is deactivated, *c*'s show up wherever they fit, and where they do not fit, *a* is used: *accc*; if only /c/ is deactivated, we derive

[25] Again, the notation here follows Distributed Morphology; but the same information could just as well be rendered in the notation of Minimalist Morphology, or via realiziation rules in Paradigm Function Morphology.

[26] Assuming decomposition, a specification like [12] can be captured by invoking a decomposed case feature that is shared by cases *1* and *2* (but not by cases *3* and *4*). Things are slightly more involved with specifications like [234]. Incidentally, configurations of this type, where one would not want to assume that a 3-out-of-4 syncretism represents the elsewhere case, seem to occur regularly; compare, e.g., the weak masculine singular declension in German in (10), or the weak feminine and masculine singular declensions in Icelandic in (15) (see Alexiadou and Müller (2007) on German, Müller (2007) on Icelandic). This syncretism can be derived by assuming a third binary case feature, in addition to the two binary case features that are minimally required to derive four cases via cross-classification; alternatively, negation could be employed ($\neg$*1*).

More generally, note that, for present purposes, there is no need whatsoever to impose any restriction on how natural classes can be captured by feature decomposition (see, however, Sect. 3.4 below on disjunction and α notation in marker specifications). That is, even completely unrestricted (and therefore perhaps linguistically implausible) systems of decomposition that introduce an enormous number of abstract, decomposed features in order to classify markers as belonging to a natural class are fully compatible with the present reasoning.

the class *bbaa*; and if no marker is deactivated, *a* does not show up, and the pattern is *bbcc* (given that /b/ is more specific than /c/). This exhausts the possibilities. There is no way that an additional inflection class could arise on the basis of the marker specifications and assumptions about specificity in (25-ab).

(25) *A possible assignment of case specifications to markers*

a. Markers:

(i) /a/ ↔ [ ]
(ii) /b/ ↔ [12]
(iii) /c/ ↔ [234]

b. Specificity:

/b/ > /c/ > /a/

c. Deactivation combinations and inflection classes:

{b, c} → aaaa
{b} → accc
{c} → bbaa
{ } → bbcc

Given these three markers, other case specifications or other specificity relations will produce other possible inflection classes, but not more inflection classes. This is shown in (26), where the three markers have specifications that differ from those in (25). The set of classes in (26-c) is the absolute maximum of classes permitted under Syncretism, Elsewhere, and Blocking (but there can be fewer classes—e.g., if /b/ and /c/ are never simultaneously deactivated, *aaaa* will not be an inflection class in the language).

(26) *Another possible assignment of case specifications to markers*

a. Markers:

(i) /a/ ↔ [ ]
(ii) /b/ ↔ [234]
(iii) /c/ ↔ [4]

b. Specificity:

/c/ > /b/ > /a/

c. Deactivation combinations and inflection classes:

{b, c} → aaaa
{b} → aaac
{c} → abbb
{ } → abbc

Thus, a system like the one in (3), with five inflection classes on the basis of three markers, will never be possible; (3) is repeated here as (27).

(27) *An impossible system of inflection classes*

	I	II	III	IV	V
1	a	a	a	a	c
2	b	a	c	c	c
3	b	a	a	a	a
4	b	b	b	c	a

3.4.2 A second example

As a second illustration, consider again the more complex example in Table 3, with five markers distributed over four cases; cf. (28). There are 625 possible inflection classes in an unconstrained system; but there are only sixteen (2^{5-1}) deactivation combinations. Four possible assignments of markers to cases are listed in (29–32).

(28) *Example 3 revisited*

a. 5 markers: {a, b, c, d, e}
b. 4 cases: 1, 2, 3, 4

(29) *A possible choice*

a. Markers:

(i) /a/ ↔ [ ]
(ii) /b/ ↔ [23]
(iii) /c/ ↔ [14]
(iv) /d/ ↔ [3]
(v) /e/ ↔ [34]

b. Specificity:

/d/ > /e/ > /c/ > /b/ > /a/

c. Deactivation combinations & inflection classes:

{b, c, d, e} → aaaa
{b, c, d} → aaee
{b, c, e} → aada
{b, c} → aade
{b, d, e} → caac
{b, d} → caee
{b, e} → cadc
{b} → cade
{c, d, e} → abba
{c, d} → abee
{c, e} → abda

{c} → abde
{d, e} → cbbc
{d} → cbee
{e} → cbdc
{ } → cbde

In (29), /a/ is again the elsewhere marker, /b/, /c/, and /e/ are underspecified markers that refer to natural classes of cases, and /d/ is a fully specified case marker. There can be at most sixteen inflection classes, based on the sixteen possible marker deactivation combinations.

(30) *Another possible choice*

a. Markers:

(i) /a/ ↔ [ ]
(ii) /b/ ↔ [ ]
(iii) /c/ ↔ [1]
(iv) /d/ ↔ [2]
(v) /e/ ↔ [34]

b. Specificity:

/c/ > /d/ > /e/ > /b/ > /a/

c. Deactivation combinations & inflection classes:

{b, c, d, e} → aaaa
{b, c, d} → aaee
{b, c, e} → adaa
{b, c} → adee
{b, d, e} → caaa
{b, d} → caee
{b, e} → cdaa
{b} → cdee
{c, d, e} → bbbb
{c, d} → bbee
{c, e} → bdbb
{c} → bdee
{d, e} → cbbb
{d} → cbee
{e} → cdbb
{ } → ~~edee~~

In (30), there are two maximally specific case markers (/c/, /d/), there is one underspecified case marker (/e/), and one marker that is not specified for case at all (/b/; however, given Blocking, this marker must differ from /a/ in some way, e.g., with respect to inflection class information, so that a specificity relation is

established between the two). This time, there are only fifteen instead of sixteen potential inflection classes. The reason is that deactivating /b/ and not deactivating anything produce the same inflection class (to indicate this, one of the two classes is crossed out).

(31) *A third possible choice*

a. Markers:

(i) /a/ ↔ [ ]
(ii) /b/ ↔ [234]
(iii) /c/ ↔ [134]
(iv) /d/ ↔ [123]
(v) /e/ ↔ [123]

b. Specificity:
/d/ > /e/ > /c/ > /b/ > /a/

c. Deactivation combinations & inflection classes:

{b, c, d, e} → aaaa
{b, c, d} → eeea
{b, c, e} → ddda
{b, c} → ~~ddda~~
{b, d, e} → cacc
{b, d} → eeec
{b, e} → dddc
{b} → ~~dddc~~
{c, d, e} → abbb
{c, d} → eeeb
{c, e} → dddb
{c} → ~~dddb~~
{d, e} → cbcc
{d} → ~~eeee~~
{e} → ~~dddc~~
{ } → ~~dddc~~

(31) represents an extreme case: All markers except /a/ are underspecified with respect to case, and their specifications overlap to a significant degree. Now it turns out that the sixteen marker deactivation combinations result in only ten distinct inflection classes; e.g., the inflection class that is generated if /b/ and /c/ are deactivated is identical to the inflection class that is generated if /b/, /c/ and /e/ are deactivated.

(32) *A fourth possible choice*

a. Markers:

(i) /a/ ↔ [ ]
(ii) /b/ ↔ [1]
(iii) /c/ ↔ [2]
(iv) /d/ ↔ [3]
(v) /e/ ↔ [4]

b. Specificity:

/e/ > /d/ > /c/ > /b/ > /a/

c. Deactivation combinations & inflection classes:

{b, c, d, e} → aaaa
{b, c, d} → aaae
{b, c, e} → aada
{b, c} → aade
{b, d, e} → acaa
{b, d} → acae
{b, e} → acda
{b} → acde
{c, d, e} → baaa
{c, d} → baae
{c, e} → bada
{c} → bade
{d, e} → bcaa
{d} → bcae
{e} → bcda
{ } → bcde

Finally, in (32), all markers (except for /a/) bear fully specified case information. Again, there cannot be more than sixteen inflection classes going back to the 16 marker deactivation combinations.[27]

3.4.3 Marker specifications

The Inflection Class Economy Theorem depends on the assumption in (17) (Syncretism), which in turn is based on the notion of a *unique specification of*

[27] The issue of what the decomposed case and inflection class features that encode the deactivation patterns in systems like (29–32) would actually look like is strictly speaking orthogonal to my present concerns. Still, for the case at hand, in the worst case there would have to be four binary inflection class features $[\pm\alpha], [\pm\beta], [\pm\gamma]$ and $[\pm\delta]$ whose cross-classification yields the sixteen inflection classes (with individual markers underspecified as, e.g., $[+\alpha]$); and two abstract grammatical category features (e.g., case features such as [±governed], [±oblique], as in Bierwisch (1967)) would suffice for all systems but (31), where either reference to negated specifications would be necessary, or a third primitive feature would have to be invoked. See footnote 26 and the next subsection.

morpho-syntactic features. As noted above, it is irrelevant for our present purposes exactly what kind of feature system is assumed, and to what extent feature decomposition may apply to a grammatical category (yielding natural classes of instantiations of the grammatical category). However, it is worth pointing out that there are kinds of marker entries that have been proposed in the literature which do not qualify as "unique" in the sense presupposed in (17). An obvious case in point is disjunction or negation in marker specifications (see, e.g., Bierwisch (1967), Wunderlich (1996)) (but only if contradictory feature specifications are involved that cannot be captured by introducing a further abstract feature; see the last footnote). Consider (33), which is modelled on (27) (class V is removed).

(33) *An impossible system of inflection classes requiring disjunction*

	I	II	III	IV
1	a	a	a	a
2	b	a	c	c
3	b	a	a	a
4	b	b	b	c

Here, /a/ is the elsewhere marker, and if disjunctive feature specifications were assumed to be possible, both /b/ and /c/ could be given disjunctive specifications to capture their distribution (/b/ would mark [234] with class I or [4] with classes II and III, and /c/ would mark [2] with classes III and IV or [4] with class IV). However, such an irreducible disjunctive specification is nothing more than a notational device that signals the presence of two different marker specifications in the system. Accordingly, such a situation is blocked by the Syncretism Principle.

The same consequence holds for the use of variables over feature values in marker specifications, i.e., so-called "α notation"; see Chomsky (1965), Chomsky and Halle (1968) for the original concept, and Noyer (1992), Harley (1994), Johnston (1996), Alexiadou and Müller (2007), Börjesson (2006), Georgi (2006), Lahne (2006), and Opitz (2006) on its use in morphology. Consider the following abstract system:

(34) *An impossible system of inflection classes requiring α notation*

	I	II	III	IV
1	a	a	a	b
2	a	a	a	b
3	b	a	a	a
4	b	a	c	c

As before, /a/ is the elsewhere marker; /c/ is a [4] marker for classes III and IV. Assuming a decomposition of inflection class features according to which classes I and IV are characterized by identical features with contrary feature values (e.g., I might be $[+\alpha, -\beta]$, and IV might be $[-\alpha, +\beta]$; see footnote 27),

and assuming a decomposition of cases according to which [12] and [34] each form natural classes (e.g., [12] might be [−obl], [34] might be [+obl]), /b/ could be given a marker specification like [ℵobl, ℵα, −ℵβ], where ℵ is a variable over feature values (+ or −).[28] Evidently, such a notation behaves exactly like a proper disjunction with respect to our present considerations; from the perspective of the Syncretism Principle, it is an elegant abbreviation for the presence of two separate marker specifications (whatever its potential other virtues). Therefore, the system in (34) is incompatible with the Syncretism Principle.

More generally, then, we can conclude that the Inflection Class Economy Theorem not only sets an upper bound for the set of possible inflection classes over a given inventory of inflection markers (for some grammatical domain); it also implies that the inflection classes that it maximally permits are not arbitrary ones—indeed, because of the Syncretism, Elsewhere, and Blocking assumptions, every given inflection class poses tight restrictions on the shape of the remaining ones, and a closely interrelated system arises.

3.4.4 Outlook

To end this paper, let me say something about the scope of the result reported here (viz., that there is a maximum of 2^{n-1} inflection classes for a system comprising n markers). As already noted in the context of introducing the Syncretism Principle (1), it seems unavoidable to conclude that there may be minor imperfections in inflectional systems that can be traced back to historical factors. In particular, these deviations from optimal design show up in the form of isolated markers that cannot be given unique specifications, resulting in cases of non-systematic homophony (which is then sometimes treated by disjunction in the literature, see the preceding subsection). In such a situation, the set of possible inflection classes is mildly increased; it is 2^{n-1+x}, for x additional marker specifications required by unresolved, accidental homophony. Still, given that accidental homophony is the exception rather than the rule, the reduction effect brought about by the Inflection Class Economy Theorem remains considerable.

That said, let me stress again that the 2^{n-1} formula captures worst case scenarios. Overlapping marker specifications reduce the number of possible inflection classes further. Moreover, for an inflectional system to fully exploit the logical possibilities for developing inflection classes as they arise under the Inflection Class Economy Theorem is extremely unlikely—typically, far from all marker deactivation combinations will be employed. Furthermore, it should be kept in mind that the Inflection Class Economy Theorem does not necessarily imply that there may not be additional restrictions that further reduce the number and form of possible inflection classes that can be generated on the basis of a given inventory of markers. Thus, it strikes me as perfectly conceivable that some version of, say, the No Blur Principle could be stipulated on

[28] I use ℵ here as a variable over feature values instead of the standard α because I have already used α as a primitive, decomposed inflection class feature.

top of the present analysis (as long as trans-paradigmatic syncretism derived by underspecification with respect to inflection class remains an option).

Finally, a remark may be due on the effects of certain further operations as they have been introduced in morphological theories. Consider, e.g., the concept of fission that is sometimes assumed in Distributed Morphology (see Halle and Marantz (1993) and Noyer (1992) for two different implementations), or the related concept of rule blocks in stem-and-paradigm accounts (see Anderson (1992), Stump (2001)). Both concepts give rise to instances of subanalysis, in the sense that what may look like a complex marker at first sight turns out to be best analyzed as a sequence of smaller markers, each with its own specifications (see Janda and Joseph (1992) for general remarks, and Bierkandt (2006) for a recent application). Concepts like fission and rule blocks raise many interesting questions;[29] but as long as it is understood that no more than one inflection class can determine a sequence of subanalyzed markers in each case, this is inconsequential for the present approach.[30]

An interesting issue is raised by impoverishment operations in Distributed Morphology, which (in their standard, non-feature changing form) delete morpho-syntactic features after syntax, but before morphological insertion takes place. As shown by Trommer (1999), impoverishment of this type can be reanalyzed as insertion of a highly specific null marker; if vocabulary insertion implies feature discharge, such cases of null marker insertion will remove features before other exponents have a chance to be inserted, and thereby mimic post-syntactic deletion. If so, we may conclude that each impoverishment rule also increases the set of *n*'s (for which the powerset is created) by one.

These qualifications aside, we end up with the conclusion that if languages obey the Syncretism Principle, the number of possible inflection classes that can be generated over a given inventory of markers is drastically reduced.

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[29] They do so, e.g., with respect to the issue of extended exponence in non-inferential theories, such as Distributed Morphology or Minimalist Morphology, which would seem to require invoking contextual features; but see Müller (2007a) for an alternative approach.

[30] This also holds if some markers are highly specific in such a way that they block an otherwise expected marker for another position in the inflectetd word, whereas competing markers do not exert such a blocking effect.

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