

## Today's Lecture (Lecture 7): Ocean surface circulation

### Reference

- ▶ Peixoto and Oort, Sec. 3.1, 3.2, 3.4, 3.5 (discussion of atmosphere is review from last week)
- ▶ Stewart 2008 (linked from web page), Ch. 3–4, 6–12

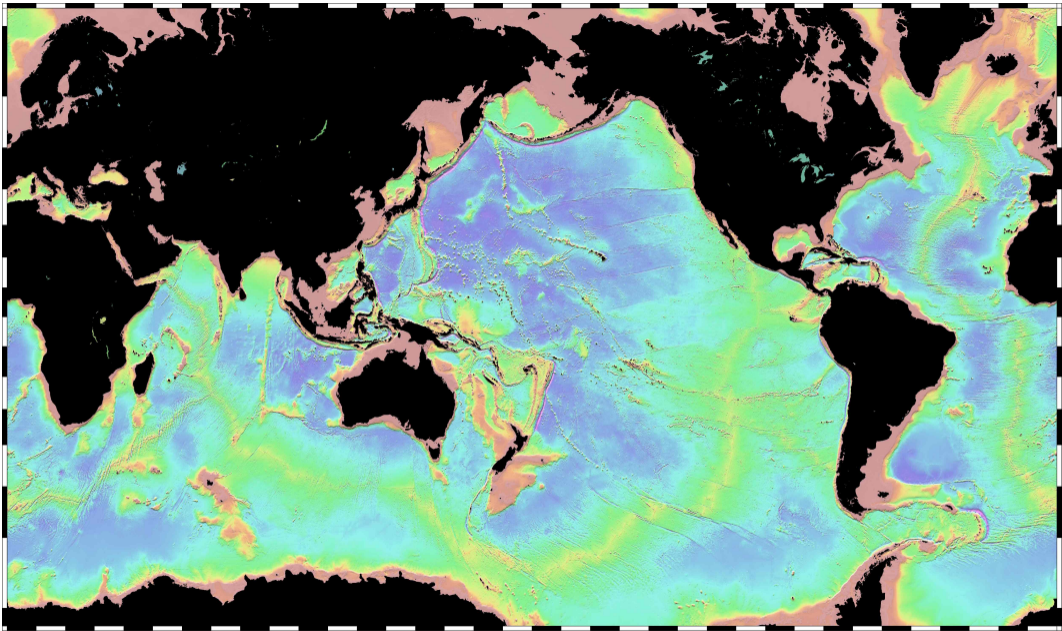
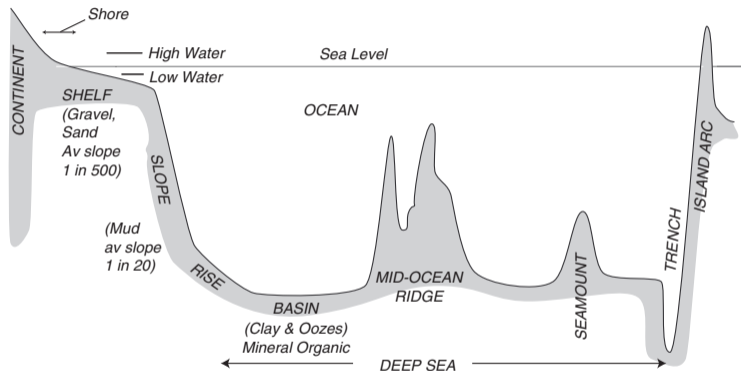


Figure: SIO

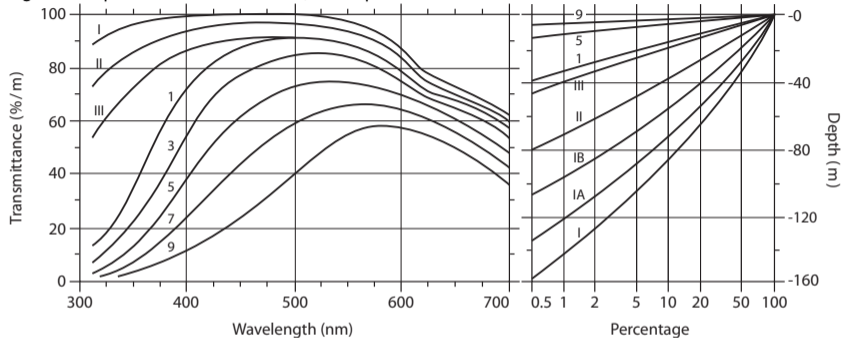
## 2.4 – Ocean



- ▶ Three oceans (Atlantic, Pacific, Indian) with marginal seas (but these are often called “ocean” as well, e.g., Southern Ocean, Arctic Ocean)
- ▶ Circumpolar in the Southern Ocean. Elsewhere, zonal flow is blocked by continents
- ▶ Subsurface orography: continental shelves, deep ocean, ridges, trenches; ridges separate oceanic basins, seamounts generate turbulence and vertical mixing
- ▶ Energy sources: wind, heat fluxes, tides

## Vertical structure

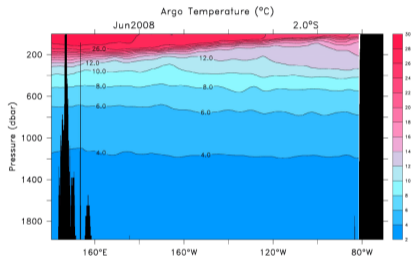
- ▶ Light absorption – albedo < 10%, absorption within the first 100 m:



- ▶ Heating at the top leads to stable stratification
- ▶ *Thermocline/pycnocline* forms the bottom of the *mixed layer* – seasonal and permanent
- ▶ Density profiles determine buoyancy and therefore stability –  $\rho = \rho(p, T, S)$ , and  $\rho - 1000 \text{ kg m}^{-3} \approx 27 \pm 2 \text{ kg m}^{-3}$ , so high precision is needed
- ▶  $\rho$  increases with  $S$ , decreases with  $T$  and (weakly) increases with  $p$

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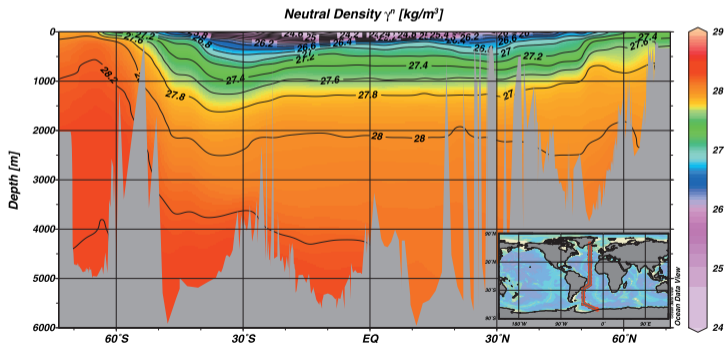


Figure: Kuhlbrodt et al 2008

# Surface circulation

## Primitive equations

Like the atmosphere, the ocean is a continuous medium and satisfies conservation of mass, momentum and energy. Unlike the atmosphere, the ocean is very nearly incompressible.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad \text{continuity equation, incompressible} \quad (2.144)$$

$$\frac{du}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + fv + F_x \approx 0 \quad \text{u-momentum equation, geostrophic + friction} \quad (2.145)$$

$$\frac{dv}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial y} - fu + F_y \approx 0 \quad \text{v-momentum equation, geostrophic + friction} \quad (2.146)$$

$$\frac{dw}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial z} - g \approx 0 \quad \text{z-momentum equation, geostrophic} \quad (2.147)$$

$z$  is (close to) zero at the surface and decreases downward.

## Scale analysis

Horizontal and vertical velocities both two orders of magnitude smaller than in the atmosphere. Geostrophy is a strong constraint in much of the ocean because  $Ro \sim 10^{-3} \ll 1$  (except very near the equator).

- ▶ Forces are gravity (tides), buoyancy (due to density differences), wind stress
- ▶ Dominant terms in the equation of motion are pressure gradient, Coriolis and friction (wind stress)

## Surface circulation is mostly wind-driven

Phenomena that result from wind stress at the ocean surface:

- ▶ Inertial currents are the response of the ocean to a transient wind forcing
- ▶ Ekman transport describes horizontal and vertical motion in the mixed layer in response to a steady-state wind forcing
- ▶ Sverdrup transport describes the zonal interior ocean response to Ekman forcing at the surface
- ▶ Western boundary current intensification is a departure from symmetric gyres to form narrow, fast currents



# Summary of the surface currents

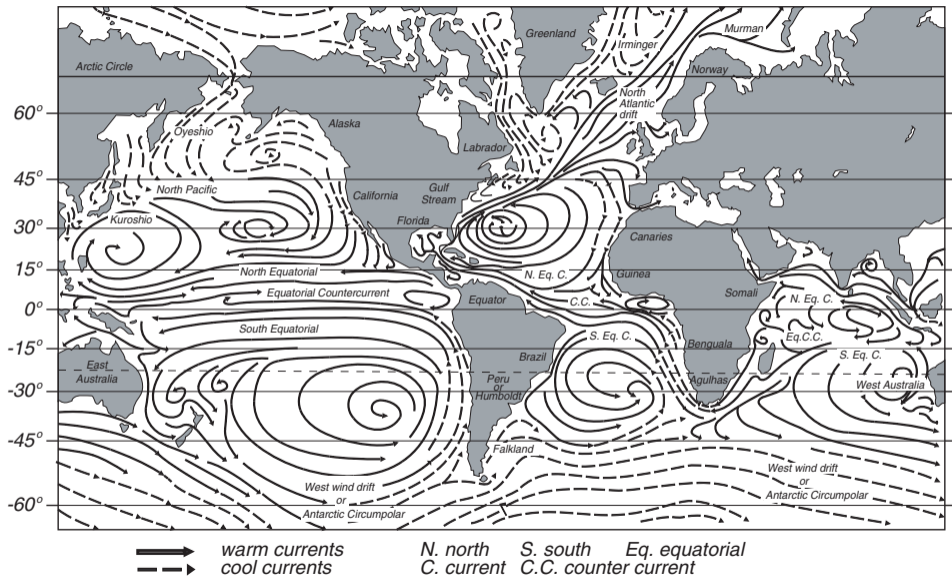
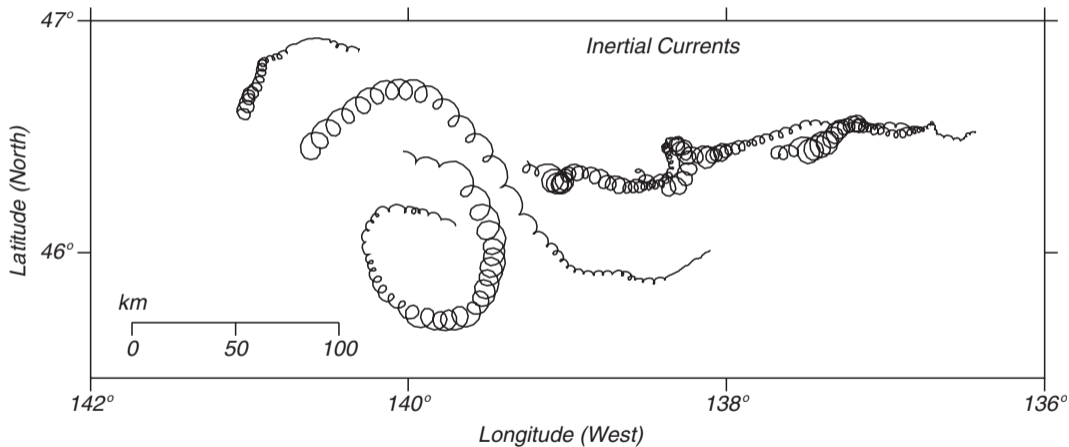


Figure: Stewart 2008

## Inertial currents

Inertial currents are excited by a transient wind stress. After the wind stress ceases, the entire mixed layer of the ocean rotates uniformly ("slab ocean") at the inertial frequency  $f$



# Ekman layer

How does the ocean surface respond to a non-transient wind? Instead of a fluid element at the ocean surface, think of the force balance on an iceberg floating on the ocean:

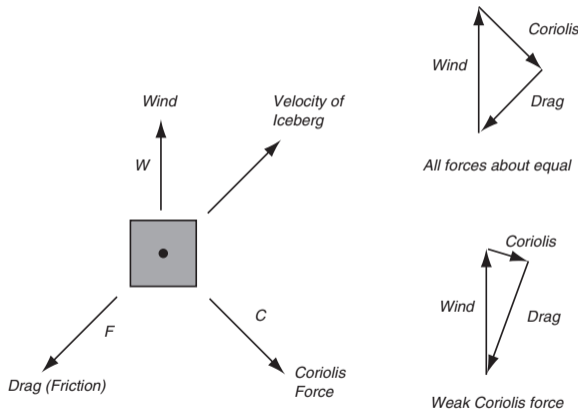


Figure: Stewart 2008

## Ekman spiral

Consider a steady-state homogeneous ocean:

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0 \quad (2.148)$$

Under these conditions, the equation of motion reduces to a balance between the Coriolis force and wind friction at the ocean surface (the stress acts *along* the wind direction):

$$fv = -F_x = -\frac{1}{\rho} \frac{\partial T_{xz}}{\partial z} = -A_z \frac{\partial^2 u}{\partial z^2} \quad \text{x-momentum equation, eddy viscosity friction} \quad (2.149)$$

$$-fu = -F_y = -\frac{1}{\rho} \frac{\partial T_{yz}}{\partial z} = -A_z \frac{\partial^2 v}{\partial z^2} \quad \text{y-momentum equation, eddy viscosity friction} \quad (2.150)$$

This system of equations has the *Ekman spiral* solution, here in the case of a southerly wind:

$$u_E = V_0 \exp(az) \cos\left(\frac{\pi}{4} + az\right) \quad (2.151)$$

$$v_E = V_0 \exp(az) \sin\left(\frac{\pi}{4} + az\right) \quad (2.152)$$

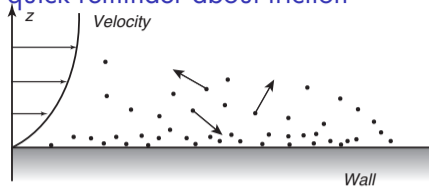
where

$$V_0 = \frac{T}{\sqrt{\rho^2 f A_z}} \approx \frac{7 \cdot 10^{-3} U_{10}}{\sqrt{\sin|\phi|}} \quad \text{current at sea surface} \quad (2.153)$$

$$a = \sqrt{\frac{f}{2A_z}} \quad \text{rotation of the current with depth, } \sim \text{reciprocal of Ekman layer depth} \quad (2.154)$$

The surface current is rotated  $\pi/4 = 45^\circ$  to the right (northern hemisphere) or left (southern hemisphere) of the wind.

## A quick reminder about friction



The *shear stress* in the fluid in the viscous layer is

$$T_{zx} = -\mu \frac{\partial u}{\partial z} \quad (2.155)$$

(and similarly for  $T_{zy}$  and  $v$ ), where the subscripts denote the stress is in the  $x$  direction in the  $z = \text{const}$  plane. The friction in the  $x$  direction is

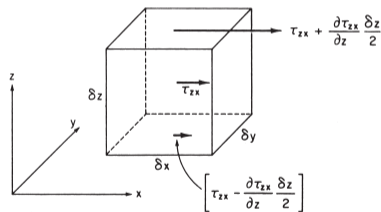
$$F_x = -\frac{1}{\rho} \left( \frac{\partial T_{xx}}{\partial x} + \frac{\partial T_{yx}}{\partial y} + \frac{\partial T_{zx}}{\partial z} \right) \quad (2.156)$$

$$\approx \frac{1}{\rho} \frac{\partial}{\partial z} \left( \mu \frac{\partial u}{\partial z} \right) = \nu \frac{\partial^2 u}{\partial z^2} \quad (2.157)$$

$\nu = \mathcal{O}(10^{-5} \text{ m}^2 \text{ s}^{-1})$ , so the depth of the viscous layer is  $\ll 1 \text{ m}$ .

**Turbulent eddies** can support deeper BL; eddy "viscosity" is similar to (2.157) but with larger coefficients, as in (2.149)–(2.150)

**Viscous** friction is due to molecular effects that resist shear in flow. At the surface, the wind velocity must be zero, while in the free stream, the wind velocity will have some arbitrary magnitude. The random motion of air molecules results in exchange of mass between the low-momentum layers and the high-momentum layers, resulting in the velocity profile shown on the left. The macroscopic friction is the result of the molecular-scale transport of momentum from the free stream into the boundary layer (BL).



## Ekman transport

How much mass is transported by the Ekman current? In what direction?

$$M_{Ex} = \int_{-d}^0 \rho v_E dz \quad (\text{integral from bottom of Ekman layer to surface}) \quad (2.158)$$

$$M_{Ey} = \int_{-d}^0 \rho u_E dz \quad (2.159)$$

From (2.149),

$$fM_{Ey} = f \int_{-d}^0 \rho v_E dz = - \int_{z=-d}^{z=0} dT_{xz} = -T_{x0} \quad (\text{the surface wind stress}) \quad (2.160)$$

$$fM_{Ex} = f \int_{-d}^0 \rho u_E dz = \int_{z=-d}^{z=0} dT_{yz} = T_{y0} \quad (2.161)$$

the *horizontal Ekman transport* is proportional to the surface wind stress (acting on the ocean) and at a right angle, to the right in the northern hemisphere.

## Ekman pumping

Because of conservation of mass, horizontally divergent surface Ekman transport must be balanced by vertical motion (upwelling or downwelling). From the (integral over the Ekman layer of) the continuity equation,

$$\rho \int_{-d}^0 \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) dz = 0 \quad (2.162)$$

so that

$$\frac{\partial}{\partial x} \int_{-d}^0 \rho u dz + \frac{\partial}{\partial y} \int_{-d}^0 \rho v dz = -\rho (w(0) - w(-d)) \quad (2.163)$$

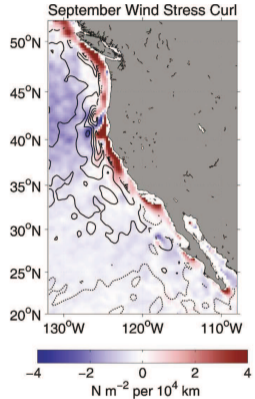
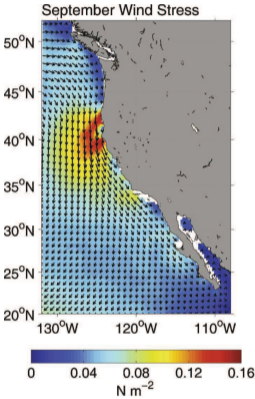
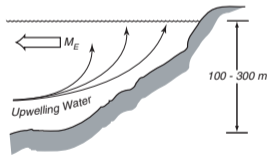
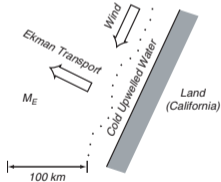
or

$$\nabla_h \cdot \vec{M}_E = \rho w(-d) = \rho w_E \quad (2.164)$$

Inserting (2.160), we find that Ekman pumping is proportional to the curl of the wind stress:

$$w_E = \frac{1}{\rho} \hat{k} \cdot \nabla \times \begin{pmatrix} \vec{\tau} \\ f \end{pmatrix} \quad (2.165)$$

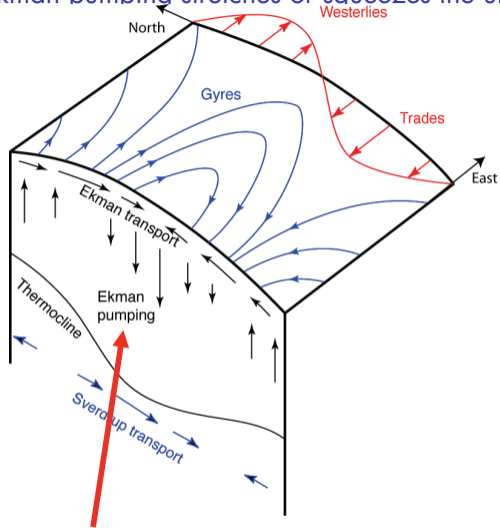
# Ekman pumping in an eastern boundary current



Figures: Stewart 2008 and Risien and Chelton 2008



## Ekman pumping stretches or squeezes the underlying ocean



Convergent Ekman transport squeezes the interior ocean, leading to equatorward motion

## Vorticity conservation governs the response of the interior ocean

Recall that for a frictionless, barotropic fluid (the interior ocean), the *potential vorticity* of a fluid element of depth  $H$

$$\Pi = \frac{f + \zeta}{H} \quad (2.166)$$

is conserved. The fluid element's absolute vorticity  $f + \zeta$  can respond to vertical stretching of the element by increasing  $\zeta$  (cyclonic rotation) or by migrating poleward (increased  $f$ ).

## Sverdrup balance in response to Ekman pumping

Interior ocean is in geostrophic balance

$$f u = -\frac{1}{\rho} \frac{\partial p}{\partial y} \quad (2.167)$$

$$f v = \frac{1}{\rho} \frac{\partial p}{\partial x} \quad (2.168)$$

Cross-differentiate and add:  $\frac{\partial}{\partial x}(2.167) + \frac{\partial}{\partial y}(2.168)$

$$f \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + \frac{df}{dy} v = 0 \quad (2.169)$$

By the continuity equation, and with  $\beta = df/dy = 2\Omega \cos \phi/R$ :

$$\beta v = f \frac{\partial w}{\partial z} \quad (2.170)$$

Integrating this vertically through the interior ocean,

$$\beta M_y = \int \beta \rho v dz = f \rho w_E = f \hat{k} \cdot \nabla \times \left( \frac{\vec{T}}{f} \right) \approx \hat{k} \cdot \nabla \times \vec{T} \quad (2.171)$$

Over the oceans, the atmospheric flow is mostly zonal, so that

$$M_y \approx -\frac{1}{\beta} \frac{\partial T_x}{\partial y} \quad (2.172)$$

## Sverdrup transport

Integrated over the entire ocean depth, the continuity equation implies

$$\int \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) dz = - \int \frac{\partial w}{\partial z} dz = 0 \quad (2.173)$$

because  $w$  vanishes at the top and bottom limits of integration; therefore

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_y}{\partial y} = 0 \quad (2.174)$$

This implies

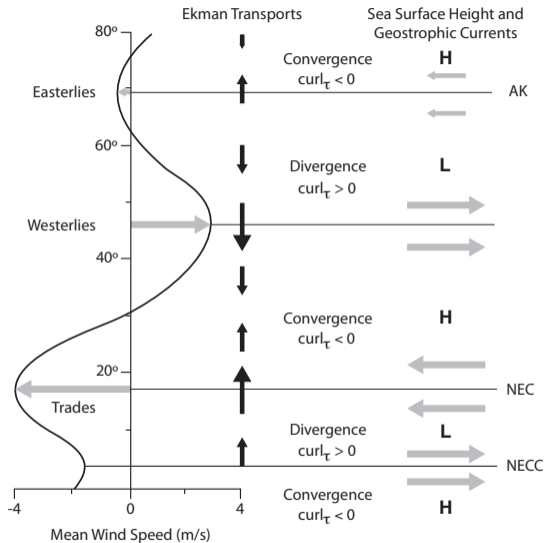
$$\frac{\partial M_x}{\partial x} = \frac{1}{2\Omega \cos \phi} \left( \frac{\partial T_x}{\partial y} \tan \phi + \frac{\partial^2 T_x}{\partial y^2} R \right) \quad (2.175)$$

Integrating from the eastern boundary of an idealized rectangular ocean, where  $x = 0$  and  $M_x = 0$ , with constant zonal wind stress,

$$M_x = - \frac{|\Delta x|}{2\Omega \cos \phi} \left( \frac{\partial T_x}{\partial y} \tan \phi + \frac{\partial^2 T_x}{\partial y^2} R \right) \quad (2.176)$$

Near the equator ( $\tan \phi \ll 1$ ), the sign of the zonal transport depends on the second derivative of the wind stress  $\rightarrow$  currents can flow upwind ("countercurrents").

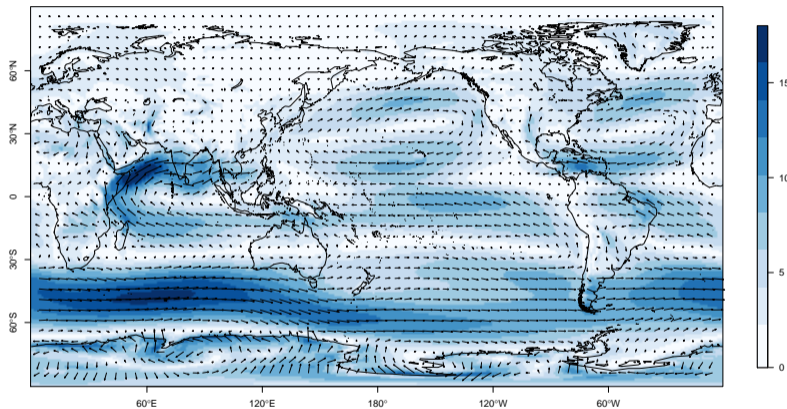
## Zonal currents in the oceans are the result of Sverdrup transport



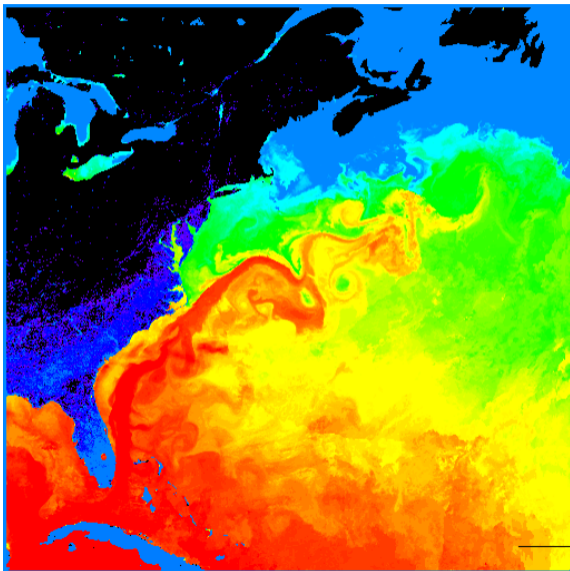
- ▶ In the midlatitudes,  $\partial^2 T_x / \partial y^2 < 0 \Rightarrow M_x > 0$
- ▶ Farther equatorward,  $\partial^2 T_x / \partial y^2 > 0 \Rightarrow M_x < 0$  (the north equatorial current)
- ▶ Near the equator,  $\partial^2 T_x / \partial y^2 < 0 \Rightarrow M_x > 0$  (the equatorial countercurrent)

Figure: Stewart 2008

(Recall that there are two trade wind maxima — northern and southern)



## Western boundary currents

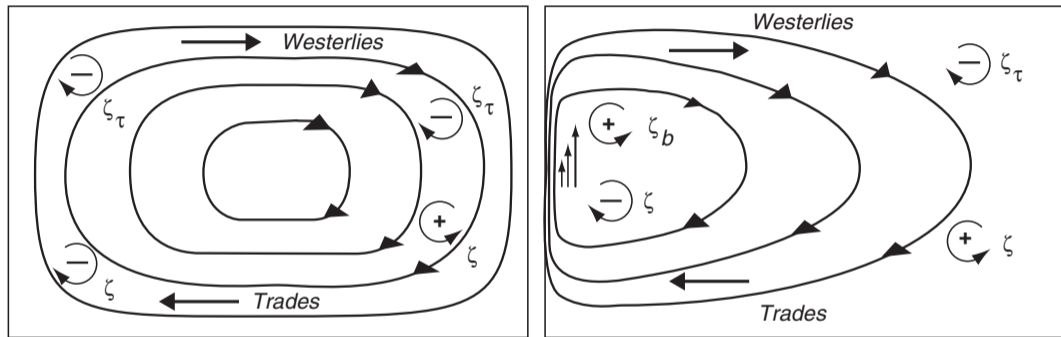


At the western boundaries of oceans basins, the currents are narrower and stronger than in the open ocean gyres – this is called *western boundary current intensification*. Examples are the Gulf Stream or the Kuroshio. They are warm currents because they originate near the equator.

The Gulf Stream, a western boundary current, is narrow, fast

## Western boundary current intensification

Why do the currents look like the picture on the right (western boundary intensification) and not like the picture on the left?



This is the result of vorticity conservation. As the fluid element completes one full revolution around the gyre, its absolute vorticity must be conserved (otherwise the gyre will accelerate). There is an anticyclonic contribution due to anticyclonic wind stress that must be balanced. The balance comes from positive (cyclonic) shear friction vorticity at the western boundary, which requires the bulk of the flow to be close to shore.

## Antarctic circumpolar current

North of Antarctica there is a circumpolar gap in the continents that coincides with the maximum intensity of the southern polar jetstream; this results in a nearly zonal current (*Antarctic circumpolar current*). Because of its importance for the overturning circulation of the ocean, we will discuss it later (in connection with the energy cycle in the climate system).



# Summary of the surface currents

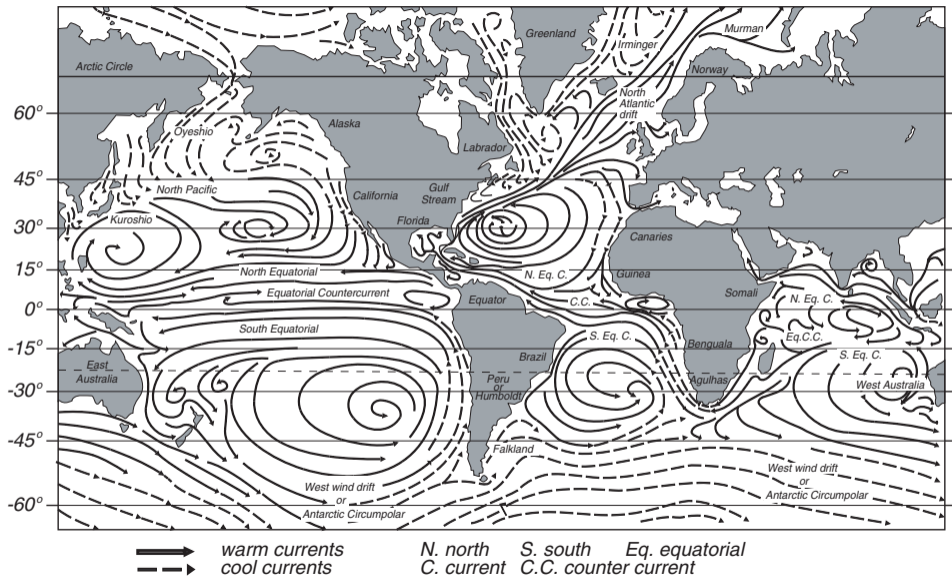


Figure: Stewart 2008

# Exchange processes of the ocean with atmosphere, land and cryosphere

## Annual average heat fluxes:

$F_{SW}$  depends on insolation and reflectivity of the atmosphere (mostly clouds); deep ocean has  $\alpha < 10\%$ ;  
 $30 < F_{SW} < 250 \text{ W m}^{-2}$

$F_{LW}$  depends on emission (SST) and back radiation by the atmosphere ( $T$ ,  $q$ , clouds);  
 $-60 < F_{LW} < -30 \text{ W m}^{-2}$

$F_{LH}$  depends on wind speed (turbulent mixing) and relative humidity of air (evaporation);  
 $-130 < F_{LH} < -10 \text{ W m}^{-2}$

$F_{SH}$  depends on wind speed (turbulent mixing) and temperature difference between air and ocean;  
 $-40 < F_{SH} < -2 \text{ W m}^{-2}$  (because water vapor is a more efficient heat transport)

Mixed layer of the ocean is a deeper heat reservoir than land