

Homework 4
Due 16 May 2018

Problem 1 Saturation water vapor pressure

Consider the Clausius–Clapeyron equation, (2.65):

$$\frac{d \ln e_s}{dT} = \frac{L}{R_v T^2} \quad (1)$$

- (a) Show that e_s increases by approximately $7\% \text{ K}^{-1}$ for typical values of the near-surface air temperature.
- (b) Approximately how large of a temperature increase is required for e_s to double?

Problem 2 Diabatic heating

Starting with the thermodynamic energy equation (2.46),

$$c_p \frac{dT}{dt} = T \frac{ds}{dt} + \alpha \frac{dp}{dt} = Q + \alpha \omega, \quad (2)$$

show that an equivalent formulation (involving the static stability $\partial\theta/\partial p$) is

$$c_{pd} \frac{T}{\theta} \left(\frac{\partial\theta}{\partial t} + \vec{v} \cdot \nabla_p \theta + \omega \frac{\partial\theta}{\partial p} \right) = Q \quad (3)$$

Problem 3 Thermodynamic structure of the atmosphere

ERA Interim reanalysis air temperature data is available in the file

`/home_local/quaas/data/ERA__Interim__T_GDS0_ISBL_123__1.5x1.5xL37__198901-200712.nc`.

Calculate zonal and temporal means of the following quantities and plot them as contour lines in the ϕ - p plane:

- (a) T
- (b) θ
- (c) q_s ; you may use the empirical approximation

$$e_s(T) = 6.1094 \text{ hPa} \times \exp \left(\frac{17.625T}{T + 243.04} \right) \quad (4)$$

for T in Celsius.

- (d) θ_e

Problem 4 Averaging operators

The temporal mean of an arbitrary field A over a time period τ is defined as

$$\bar{A} = \bar{A}(\lambda, \phi, p) = \frac{1}{\tau} \int_{-\tau/2}^{\tau/2} A(\lambda, \phi, p, t) dt \quad (5)$$

The zonal mean (over all longitudes) is defined as

$$[A] = [A](\phi, p, t) = \frac{1}{2\pi} \int_0^{2\pi} A(\lambda, \phi, p, t) d\lambda \quad (6)$$

The instantaneous value of A is given by

$$A = \bar{A} + A' \quad (7)$$

where A' is called the *fluctuating* component of A . Likewise

$$A = [A] + A^* \quad (8)$$

where A^* is the departure from the zonal mean.

Show that for arbitrary fields A and B

(a) $A = [\bar{A}] + [A'] + \bar{A}^* + A'^*$

(b) $[\overline{AB}] = [\bar{A}] [\bar{B}] + [\bar{A}^* \bar{B}^*] + [\overline{A'B'}]$