Climate Dynamics
Summer Semester 2017

UNIVERSITÄT LEIPZIG

Homework 9
Due 21 June 2017

Problem 1 Time evolution of the surface temperature

Recall from lecture that the response of the surface temperature to climate forcing depends on the strength of the various feedbacks, and that we can write the following linearized equation:

$$\Delta R = \lambda \Delta T_S + F. \tag{1}$$

The radiative flux imbalance ΔR increases the internal energy of the climate system (on short timescales, mostly the upper ocean):

$$c\frac{\partial \Delta T_S}{\partial t} = \lambda \Delta T_S + F(t) \tag{2}$$

where c is the heat capacity of the upper ocean, λ is the climate feedback parameter, F is the forcing, and ΔT_S is the ocean surface temperature perturbation. The base-state temperature is taken to be the temperature at $t = 0$, so that $\Delta T_S(t = 0) = 0$.

(a) Solve (2) for F a general function of t. *Hint: To solve an inhomogeneous linear differential equation such as (2), try writing the solution as the product of two functions,*

$$\Delta T_S(t) = \Delta T_{hom}(t) \cdot f(t) \tag{3}$$

$\Delta T_{hom}(t)$ solves the homogeneous differential equation

$$c\frac{\partial \Delta T_{hom}}{\partial t} = \lambda \Delta T_{hom} \tag{4}$$

Applying the product rule to (3), inserting the result into (2), and using (4), you can solve for $f(t)$. As a check that you got the right result, your solution should have this form:

$$\Delta T_S = \exp(At) \int_0^t \exp(-At') BF(t')\, dt' \tag{5}$$

(b) Find $\Delta T_S(t)$ when the perturbation is a sudden step at $t = 0$:

$$F(t) = \begin{cases} 0 & (t \le 0) \\ Q_0 & (t > 0) \end{cases} \tag{6}$$

What is the initial behavior ($t \ll c/|\lambda|$)? What is the asymptotic behavior ($t \gg c/|\lambda|$)? (Note: we write $|\lambda|$ because $\lambda < 0$.)

(c) Find $\Delta T_S(t)$ when the perturbation is a linear increase,

$$F(t) = \begin{cases} 0 & (t \leq 0) \\ Q_t t & (t > 0) \end{cases} \tag{7}$$

What is the initial behavior ($t \ll c/|\lambda|$)? What is the asymptotic behavior ($t \gg c/|\lambda|$)?

Problem 2 A tale of two climate changes

The feedback parameter λ of the climate system is not well known. Let us explore the consequence of uncertainties on λ for the climate system. Consider two extreme cases: in one case the feedback parameter has a certain value $\lambda = \hat{\lambda}$, while in the other $\lambda = \hat{\lambda}/3$.

(a) Which climate system is more sensitive (greater equilibrium temperature change for the same forcing)?

(b) How does the temperature change in the early period of global warming ($t \ll c/|\lambda|$) depend on λ? You may use your solution from 1(b) or 1(c).

(c) How does the equilibrium temperature change ($t \gg c/|\lambda|$) depend on λ?

(d) What does this mean for using historical observations to predict equilibrium climate change?

Problem 3 The importance of using TOA forcings

When analyzing forcing mechanisms, it is important to consider their effect on the climate system as a whole, which is why we look at their effect on the TOA energy balance. Otherwise, things can get confusing quickly, as these two problems illustrate.

(a) "Absorbing aerosols prevent a fraction of the incident solar radiation from reaching the surface. Therefore, they exert a cooling effect on the surface temperature, which corresponds to a negative radiative forcing." This argument is incorrect in at least two ways. What are they?

(b) "Increasing the atmospheric GHG concentration causes the atmosphere to emit more downwelling thermal radiation to the surface at a given atmospheric temperature. For the climate system to return to equilibrium, the surface energy budget must come back into balance. This requires the climate system to cool, so the GHG forcing is negative." This argument has at least two flaws. What are they?