

## Today's Lecture: Introduction

### Reference

Peixoto and Oort, Sec. 3.1, 3.2, 3.4, 3.5 (but skip the discussion of oceans until next week)

# Organization

**Lectures** Wednesdays 15:30–17:00 vor dem Hospitaltore

**Exercises** First session on April 20 in the CIP Pool  
Wednesdays 12:30–14:00

**Slide copies** On course web page: <http://home.uni-leipzig.de/jmuelmen>, hopefully with a link from the Sommersemester page soon

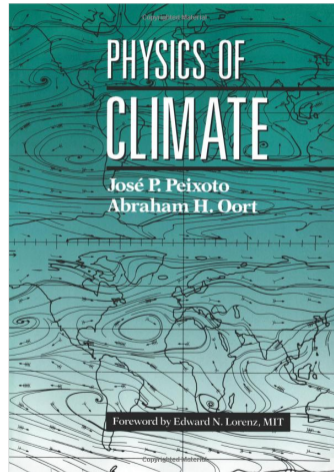
**Language** Input: de/en, output: en

**Miscellaneous** Please interrupt with questions! Comments welcome. Also by email:  
[johannes.muelmenstaedt@uni-leipzig.de](mailto:johannes.muelmenstaedt@uni-leipzig.de)

**Exams** July or August by appointment, 30 minutes

## Course materials

- ▶ Books available at the library
- ▶ Papers (occasionally) linked from course web page



# 1 – Introduction

## 1. Introduction

1.1 Overview of the climate system and its subsystems

1.2 Atmosphere

1.3 Ocean

1.4 Land and cryosphere

1.5 The climate system

1.6 Internal variability

1.7 Forcing and feedbacks

1.8 Anthropogenic climate change

## 1.1 – Overview of the climate system and its subsystems

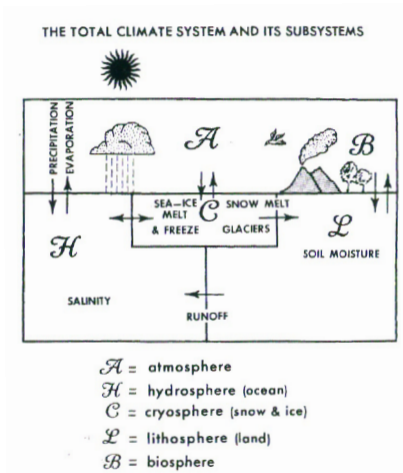
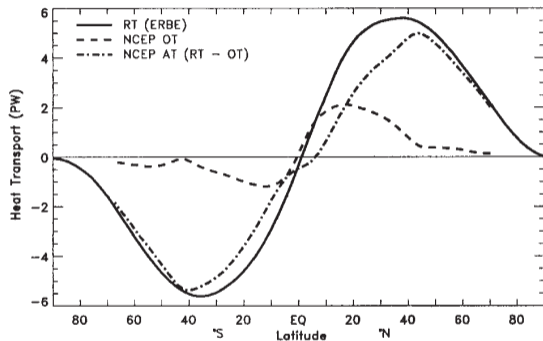


Figure: Peixoto and Oort

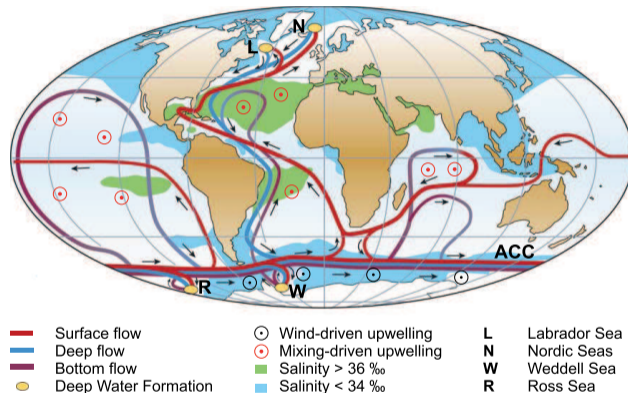
## 1.2 – Atmosphere

- ▶ Primitive equations
- ▶ The role of water
- ▶ The role of greenhouse gases
- ▶ The role of aerosols
- ▶ Atmospheric circulation
- ▶ Coupling to land and sea, perturbation response time scales
- ▶ What is the function of the atmosphere in the climate system?



## 1.3 – Ocean

- ▶ Primitive equations
- ▶ The role of salt
- ▶ “Thermohaline” (oceanic) circulation
- ▶ Coupling to atmosphere and cryosphere, perturbation response time scales
- ▶ What is the function of the ocean in the climate system?



## 1.4 – Land and cryosphere

### Land (lithosphere and biosphere)

- ▶ Primitive equations? – unknown
- ▶ Time scales from very short (energy cycle, diurnal) to very long (carbon cycle, geologic)

### Cryosphere

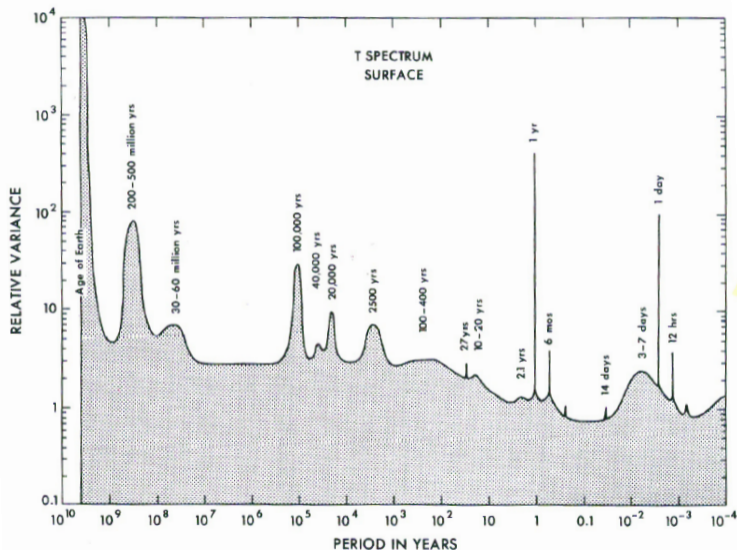
- ▶ Primitive equations? – unknown
- ▶ Coupling to land, sea, atmosphere
  - ▶ Albedo change
  - ▶ Sea-level rise
  - ▶ Release of permafrost methane
- ▶ Response to perturbation very slow, but can be irreversibly “locked in” far in advance — example of “committed climate change”



## 1.5 – The climate system

- ▶ Definition of “system”
- ▶ System types
  - Open Energy and mass exchanged with the environment
  - Closed Only energy exchanged with the environment
  - Isolated Neither energy nor mass exchanged with the environment
- ▶ The climate system as a meridional heat transport mechanism
- ▶ The climate system as a heat engine
- ▶ Radiative–convective equilibrium
- ▶ The general circulation – cycles of energy, momentum, angular momentum, entropy
- ▶ Hydrologic cycle

## 1.6 – Internal variability

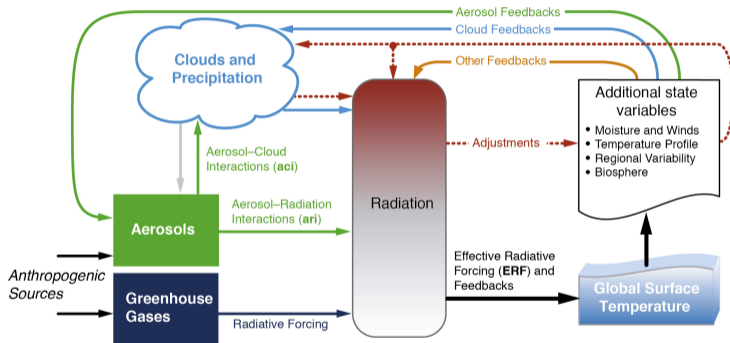


### A variety of time scales

- ▶ Mid-latitude storms
- ▶ Madden-Julian oscillation
- ▶ ENSO
- ▶ Teleconnections
- ▶ PDO/NAO/AO

Figure: Peixoto and Oort

## 1.7 – Forcing and feedbacks



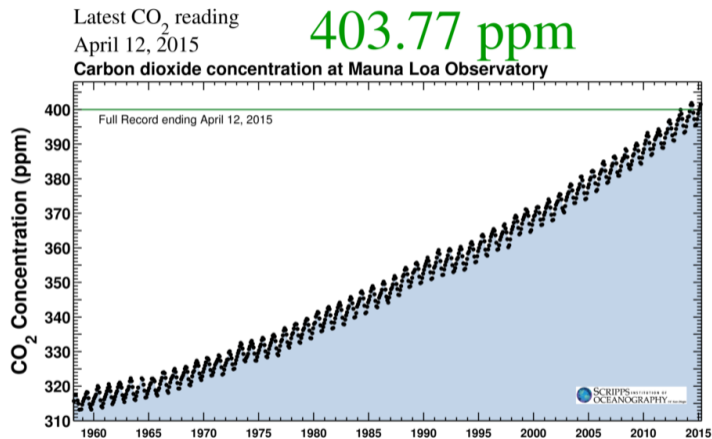
### Forcing

- ▶ Natural: solar cycles, orbital cycles, volcanic eruptions, geologic carbon cycle
- ▶ Anthropogenic: greenhouse gases, aerosols, land-use change

### Feedbacks

- ▶ “Planck” feedback
- ▶ Water vapor feedback
- ▶ Lapse rate feedback
- ▶ Cloud feedback
- ▶ Ice albedo feedback

## 1.8 – Anthropogenic climate change – the uncontrolled experiment



- ▶ History
- ▶ Projections and uncertainties
- ▶ Attribution
- ▶ Mitigation, adaptation, geoengineering
- ▶ The scientist/policy-maker dichotomy
- ▶ How to counter denialists?

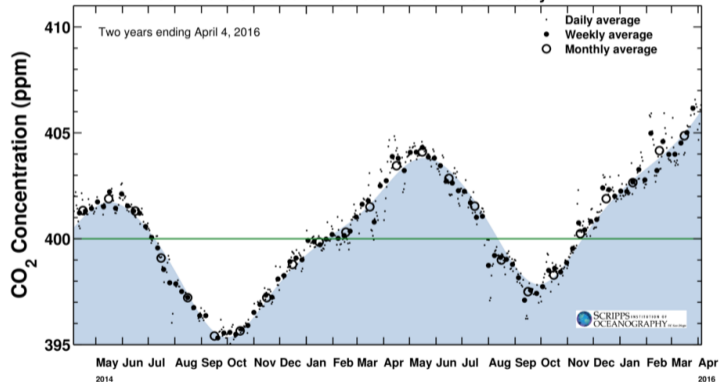
Figure: Scripps CO<sub>2</sub> program

## 1.8 – Anthropogenic climate change

Latest CO<sub>2</sub> reading  
April 04, 2016

406.31 ppm

Carbon dioxide concentration at Mauna Loa Observatory



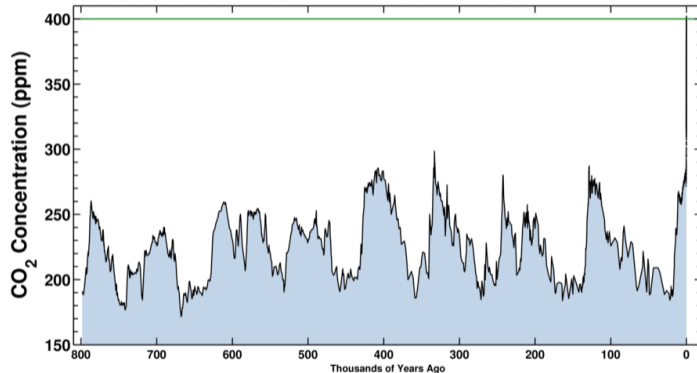
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## 1.8 – Anthropogenic climate change

Latest CO<sub>2</sub> reading  
April 12, 2015

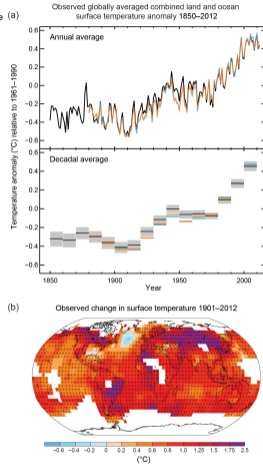
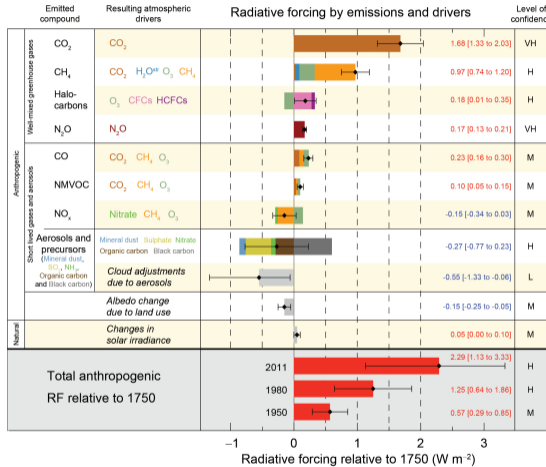
403.77 ppm

Ice-core data before 1958. Mauna Loa data after 1958.



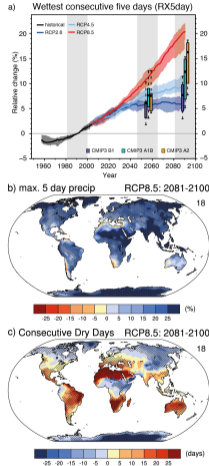
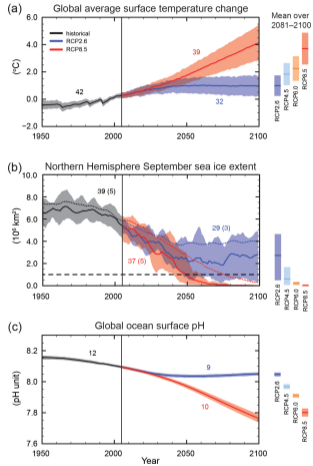
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- ▶ History
- ▶ Projections and uncertainties
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- ▶ How to counter denialists?



# What you should get out of this course

If you want to work in climate science: Context for your Master's thesis topic

If you want to work in any other area: A general introduction to the climate system

Either way: Respond knowledgeably when friends and family ask you about the climate or climate change

So please ask lots of questions!

## 2 – Atmosphere

### 1. Introduction

### 2. Atmosphere

2.1 Description of the state of the atmosphere

2.2 Equations of state

2.3 Conservation laws

2.4 Thermodynamics of the moist atmosphere

2.5 Exchange processes of the atmosphere with land and ocean

### 3. Ocean

### 4. Land, biosphere, cryosphere

### 5. The climate system

### 6. Internal variability

### 7. Forcing and feedbacks – guest lecture by Karo Block

### 8. Anthropogenic climate change

### Reference

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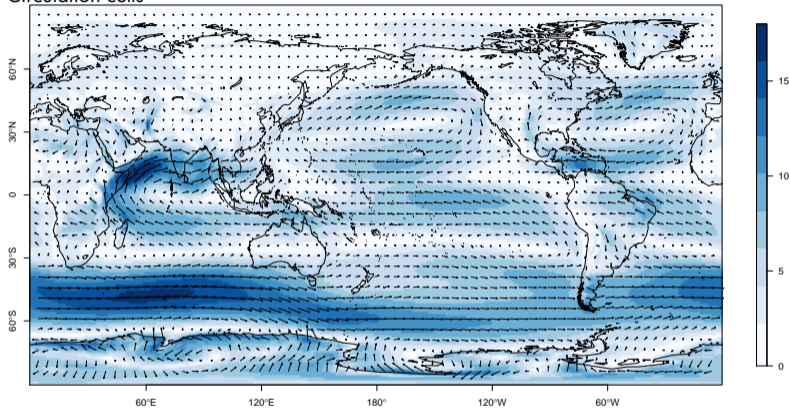
## 2.1 – Description of the state of the atmosphere

- ▶ Constituents – dry air, reactive gases, absorbing gases, aerosols, water vapor, liquid water, ice
- ▶ Thermodynamic state – pressure, temperature, mixing ratios
- ▶ Dynamic state – velocity field
- ▶ Circulation



# Some features of the atmospheric circulation

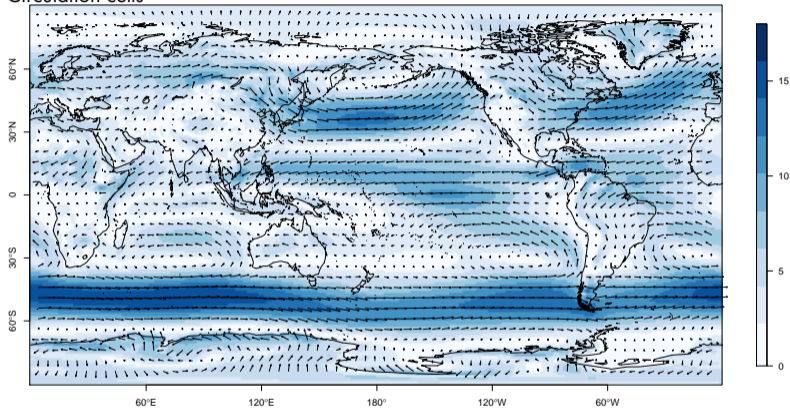
## ► Circulation cells



- Eddies
- Boundary layer
- Clouds

# Some features of the atmospheric circulation

## ► Circulation cells

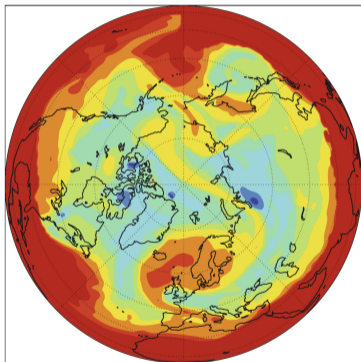


- Eddies
- Boundary layer
- Clouds

## Some features of the atmospheric circulation

- ▶ Circulation cells
- ▶ Eddies

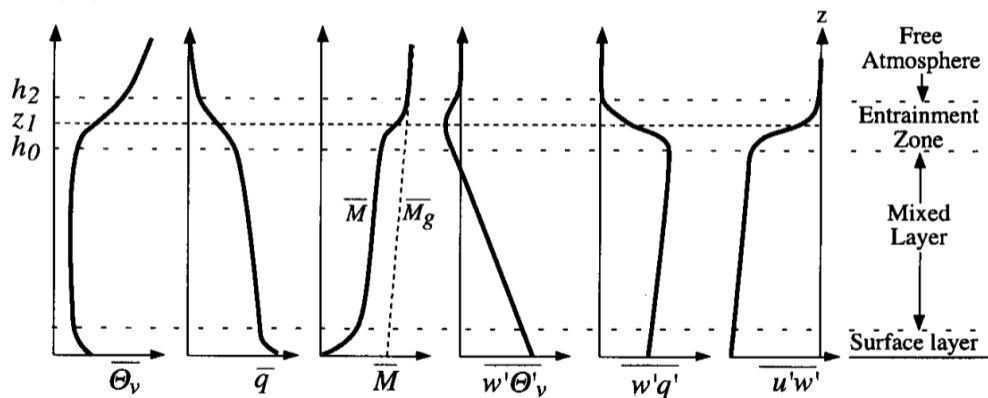
$\theta_2$ : Feb 14, 1994



- ▶ Boundary layer
- ▶ Clouds

## Some features of the atmospheric circulation

- ▶ Circulation cells
- ▶ Eddies
- ▶ Boundary layer



- ▶ Clouds



## Some features of the atmospheric circulation

- ▶ Circulation cells
- ▶ Eddies
- ▶ Boundary layer
- ▶ Clouds

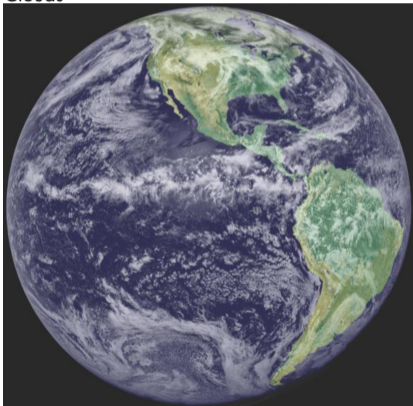


Figure: Bellon, 2013

## 2.2 – Equations of state

To good approximation, atmospheric gases are ideal

Dry air  $p_d = \rho_d R_d T$

Water vapor  $e = \rho_v R_v T$  ( $e$  is water vapor pressure)

Combination of dry air and water vapor  $p = \rho R_d T_v$  with *virtual temperature*  $T_v$ , derivation below:

The ratio of gas constants is reciprocal to the ratio of molecular weights of water vapor and dry air,

$$\frac{R_d}{R_v} = \frac{m_w}{m_d} = 0.622 \quad (2.1)$$

The total pressure and density are additive:

$$p = p_d + e \quad (2.2)$$

$$\rho = \rho_d + \rho_v = \frac{p - e}{R_d T} + \frac{e}{R_v T} = \frac{p - e}{R_d T} + 0.622 \frac{e}{R_d T} = \frac{p}{R_d T} \left( 1 - 0.378 \frac{e}{p} \right) \quad (2.3)$$

(2.3) has the form of an ideal-gas equation of state with

$$T_v = \frac{T}{1 - 0.378 \frac{e}{p}} \approx T \left( 1 + 0.378 \frac{e}{p} \right) = T(1 + 0.61q), \quad \text{where} \quad (2.4)$$

$$q = \frac{M_v}{M} = \frac{\rho_v}{\rho} = 0.622 \frac{e}{(p - 0.378e)} \approx 0.622 \frac{e}{p} \quad (\text{the water vapor mixing ratio}) \quad (2.5)$$

## 2.3 – Conservation laws

### Physical principles

Conservation of mass applies to each constituent (dry air, water) individually → continuity equation

Conservation of momentum → equation of motion

Conservation of energy → thermodynamic heat equation

Conservation of angular momentum → vorticity equation

## Coordinate systems

All the coordinate systems we use in this course are non-inertial (co-moving with the Earth)

Rectangular *local* coordinates:  $x$  eastward,  $y$  northward,  $z$  upward

Pressure coordinates: replace  $z$  (upward-pointing) with  $p$  (downward-pointing); any other vertical coordinate that is locally monotonic also works;  $\delta p = -\rho g \delta z$  (hydrostatic equilibrium)

Spherical coordinates:  $\lambda$  (longitude),  $\phi$  (latitude),  $p$ ;  $\delta x = R_E \cos \phi \delta \lambda$ ,  $\delta y = R_E \delta \phi$

## Associated differential operators

Total derivative  $d/dt$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{c} \cdot \nabla, \quad \text{with } \vec{c} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

Gradient  $\nabla$

$$\nabla = \begin{pmatrix} \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \end{pmatrix}, \quad [\nabla] = \text{m}^{-1}$$

"Horizontal"  $\nabla_p$

$$\nabla_p = \begin{pmatrix} \partial/\partial x \\ \partial/\partial y \\ 0 \end{pmatrix}_p, \quad (p \text{ means on isobars})$$

## Continuity equation

Conservation of mass (of a dry air parcel):

$$\frac{1}{\delta m} \frac{d(\delta m)}{dt} = \frac{1}{\rho \delta V} \frac{d(\rho \delta V)}{dt} = \frac{1}{\rho} \frac{d\rho}{dt} + \frac{1}{\delta V} \frac{d(\delta V)}{dt} = 0 \quad (2.6)$$

Recognizing the relative rate of expansion of the air parcel as the divergence of the wind field,

$$-\frac{1}{\rho} \frac{d\rho}{dt} = \nabla \cdot \vec{c} \quad (2.7)$$

In pressure coordinates  $\delta m = \delta x \delta y \rho \delta z = \delta x \delta y \delta p/g$  (in hydrostatic equilibrium), and thus the wind field is non-divergent (meaning we can treat the atmosphere as incompressible):

$$\frac{1}{\delta m} \frac{d(\delta m)}{dt} = \frac{1}{\delta x \delta y} \frac{d(\delta x \delta y)}{dt} + \frac{1}{\delta p} \frac{d(\delta p)}{dt} = \nabla_p \cdot \vec{v} + \frac{\partial \omega}{\partial p} = 0, \quad \text{with } \vec{v} = \begin{pmatrix} u \\ v \end{pmatrix}, \quad \omega = \frac{dp}{dt} \quad (2.8)$$

In spherical coordinates:

$$\frac{1}{R_E \cos \phi} \frac{\partial u}{\partial \lambda} + \frac{1}{R_E \cos \phi} \frac{\partial (v \cos \phi)}{\partial \phi} + \frac{\partial \omega}{\partial p} = 0 \quad (2.9)$$

## Equation of motion

In an inertial frame (designated by  $A$ ), the forces acting on an air parcel are the pressure gradient force  $-\frac{1}{\rho}\nabla p$ , gravitational acceleration  $-\nabla\Phi_N$  and friction  $\vec{F}$ :

$$\frac{d_A \vec{c}_A}{dt} = -\frac{1}{\rho} \nabla p - \nabla \Phi_N + \vec{F} \quad (2.10)$$

Consider a point with position vector  $\vec{r}_r$  in a rotating frame; the transformation to position in the inertial frame includes motion of the point due to rotation of the frame:

$$\frac{d_A \vec{r}_A}{dt} = \frac{d\vec{r}_r}{dt} + \vec{\Omega} \times \vec{r}_r \quad \text{or} \quad \frac{d_A}{dt} = \frac{d}{dt} + \vec{\Omega} \times \quad (2.11)$$

or

$$\vec{c}_A = \vec{c} + \vec{\Omega} \times \vec{r}_r. \quad (2.12)$$

Applying the transformation (2.11) to  $\vec{c}_A$  gives the acceleration in the inertial reference frame:

$$\frac{d_A \vec{c}_A}{dt} = \frac{d\vec{c}_A}{dt} + \vec{\Omega} \times \vec{c}_A = \frac{d}{dt}(\vec{c} + \vec{\Omega} \times \vec{r}_r) + \vec{\Omega} \times (\vec{c} + \vec{\Omega} \times \vec{r}_r) = \frac{d\vec{c}}{dt} + 2\vec{\Omega} \times \vec{c} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}_r) \quad (2.13)$$

By a vector identity, the last term can be written as

$$\vec{\Omega} \times (\vec{\Omega} \times \vec{r}_r) = \frac{1}{2} \nabla (\vec{\Omega} \times \vec{r}_r)^2 = \frac{1}{2} \nabla (\Omega R_E \cos \phi)^2 \quad (2.14)$$

In the rotating reference frame, the equation of motion then becomes

$$\frac{d\vec{c}}{dt} = -2\vec{\Omega} \times \vec{c} - \frac{1}{\rho} \nabla p - \nabla \Phi + \vec{F}, \quad (2.15)$$

where the term  $\nabla \Phi$  has subsumed the centrifugal term  $\vec{\Omega} \times (\vec{\Omega} \times \vec{r}_r)$  by (2.14) into the “apparent geopotential”

$$\Phi = \Phi_N - \frac{1}{2} \Omega^2 R_E^2 \cos^2 \phi. \quad (2.16)$$

In component form, the equation of motion is

$$\frac{du}{dt} - \frac{uv \tan \phi}{R_E} + \frac{uw}{R_E} = -\frac{1}{\rho} \frac{1}{R_E \cos \phi} \frac{\partial p}{\partial \lambda} + 2\Omega v \sin \phi - 2\Omega w \cos \phi + F_{r\lambda} \quad (2.17)$$

$$\frac{dv}{dt} + \frac{u^2 \tan \phi}{R_E} + \frac{vw}{R_E} = -\frac{1}{\rho} \frac{1}{R_E} \frac{\partial p}{\partial \phi} - 2\Omega u \sin \phi + F_{r\phi} \quad (2.18)$$

$$\frac{dw}{dt} - \frac{u^2 + v^2}{R_E} = -\frac{1}{\rho} \frac{\partial p}{\partial z} - g + 2\Omega u \cos \phi + F_{rz} \quad (2.19)$$

## Simplifying approximations

### Hydrostatic approximation

Neglecting all but the highest-order terms in the vertical equation of motion yields a diagnostic relationship between the vertical pressure gradient and density:

$$\frac{\partial p}{\partial z} = -\rho g \quad (2.20)$$

The equation of motion in pressure coordinates then simplifies with the following substitutions (note that there is no vertical acceleration under the hydrostatic approximation):

$$\omega = \frac{dp}{dt} = \frac{\partial p}{\partial t} + \vec{v} \cdot \nabla p + w \frac{\partial p}{\partial z} \approx w \frac{\partial p}{\partial z} \approx -\rho g w \quad (2.21)$$

$$\left. \frac{\partial p}{\partial \lambda} \right|_z = \left. \frac{\partial p}{\partial z} \right|_\lambda \left. \frac{\partial z}{\partial \lambda} \right|_p \approx -\rho g \left. \frac{\partial \Phi}{\partial \lambda} \right|_p \quad (2.22)$$

### Geostrophic approximation

Proceeding similarly in the horizontal yields a diagnostic relationship between the horizontal wind and pressure gradients (in the height coordinate system) or geopotential gradients (in the pressure coordinate system):

$$f v_g = \frac{1}{\rho} \frac{1}{R_E \cos \phi} \frac{\partial p}{\partial \lambda} = \frac{1}{R_E \cos \phi} \frac{\partial \Phi}{\partial \lambda} \quad \text{and} \quad f u_g = -\frac{1}{\rho} \frac{1}{R_E} \frac{\partial p}{\partial \phi} = -\frac{1}{R_E} \frac{\partial \Phi}{\partial \phi} \quad (2.23)$$

The quality of the geostrophic approximation depends on  $Ro = U/fl \ll 1$ ; not the case in the tropics ( $f \ll 1$ ) or for small-scale phenomena ( $fl \ll U$ ).  $f = 2\Omega \sin \phi$  is the *Coriolis parameter*.



## Thermodynamic energy equation

Combined first and second laws of thermodynamics for a reversible process (all variables are *state functions*):

$$dU = T dS - p dV \quad (2.24)$$

Since  $U$  is a state function, so is the sum (or difference) of  $U$  and other state functions:

$$H = U + pV \quad \text{enthalpy} \quad (2.25)$$

$$dH = T dS + V dp \quad (2.26)$$

$$G = H - TS \quad \text{Gibbs function} \quad (2.27)$$

$$dG = -S dT + V dp \quad (2.28)$$

Which differential is most convenient depends on the process under consideration:

**isentropic**  $S = \text{const} \Rightarrow T dS = 0$

**isobaric**  $p = \text{const} \Rightarrow V dp = 0$

**isothermal**  $T = \text{const} \Rightarrow S dT = 0$

For our (atmospheric) purposes, the extensive variables are replaced by their intensive counterparts. For example, (2.24) becomes

$$du = T ds - p d\alpha \quad (2.29)$$

## Heat capacities

For an ideal gas,  $U$  is a function only of  $T$ . The same is true of  $H = U + pV$ , since  $pV$  can be related to  $T$  by the equation of state. At constant volume, any heat added to the system leads directly to an increase in internal energy, since the second term in (2.24) vanishes; we therefore define the *heat capacity at constant volume*

$$C_V = \left. \frac{dU}{dT} \right|_V \quad \text{or} \quad c_V = \left. \frac{du}{dT} \right|_V \quad (2.30)$$

At constant pressure, any heat added to the system leads directly to an increase in enthalpy, since the second term in (2.26) vanishes; we therefore define the *heat capacity at constant pressure*

$$C_P = \left. \frac{dH}{dT} \right|_P \quad \text{or} \quad c_P = \left. \frac{dh}{dT} \right|_P \quad (2.31)$$

(Note that the notation  $\left. \frac{d}{dT} \right|_V$  and  $\left. \frac{d}{dT} \right|_P$  is not necessary since the differentials are total differentials.)

Differentiating the definition of (specific) enthalpy with respect to temperature yields

$$c_P = \frac{dh}{dT} = \frac{d}{dT}(u + p\alpha) = \frac{d}{dT}(u + R_d T) = c_V + R_d \quad (2.32)$$

Furthermore, for dry air at atmospheric temperatures,  $u \approx \frac{5}{2}R_d T$  (see Section 2.6), so

$$c_V = \frac{5}{2}R_d \quad \text{and} \quad c_P = \frac{7}{2}R_d \quad (2.33)$$

## Thermodynamic energy equation for atmospheric dynamics

The most convenient expression of the thermodynamic energy balance for atmospheric dynamics is derived as follows. Recall the differential form of the first and second laws of thermodynamics in enthalpy form:

$$dh = T ds + \alpha dp \quad (2.34)$$

Taking the time derivative and substituting from (2.31), we find

$$c_p \frac{dT}{dt} = T \frac{ds}{dt} + \alpha \frac{dp}{dt} = Q + \alpha \omega, \quad (2.35)$$

where  $Q$  is the diabatic heating rate by clouds and radiation, and  $\alpha \omega$  is the heating due to adiabatic compression.

### Potential temperature

For an adiabatic process, divide (2.35) by  $T$  and substitute for  $\alpha/T$  from the equation of state:

$$c_p \frac{d \ln T}{dt} = R_d \frac{d \ln p}{dt} \quad (2.36)$$

which implies conservation of the *potential temperature*  $\theta$ ,

$$\theta = T \left( \frac{p_0}{p} \right)^\kappa, \quad \kappa = \frac{R_d}{c_p} \approx \frac{2}{7} \quad (2.37)$$

From (2.35) and (2.37) it can be shown that  $\theta$  is related to entropy (up to additive constants) by

$$s = c_p \ln \theta \quad (2.38)$$