Climate Dynamics, Summer 2016
Tom Goren / Johannes Quaas

UNIVERSITÄT LEIPZIG

Homework 7
Due 15 June 2016

Problem 1 The surface temperature response to forcing

Recall from lecture that the response of the surface temperature to climate forcing depends on the strength of the various feedbacks, and that we can write the following linearized equation:

$$\Delta R = \lambda \Delta T + F. \tag{1}$$

The radiative flux imbalance ΔR increases the internal energy of the climate system (on short timescales, mostly the upper ocean):

$$c\frac{\partial \Delta T}{\partial t} = \lambda \Delta T + F(t) \tag{2}$$

where c is the heat capacity of the upper ocean, λ is the climate feedback parameter, F is the forcing, and ΔT is the ocean surface temperature perturbation. The base-state temperature is taken to be the temperature at $t = 0$, so that $\Delta T(t = 0) = 0$.

(a) Solve (2) for F a general function of t. *Hint: To solve an inhomogeneous linear differential equation such as (2), try writing the solution as the product of two functions,*

$$\Delta T(t) = \Delta T_{hom}(t) \cdot f(t) \tag{3}$$

$\Delta T_{hom}(t)$ *solves the homogeneous differential equation*

$$c\frac{\partial \Delta T_{hom}}{\partial t} = \lambda \Delta T_{hom} \tag{4}$$

Applying the product rule to (3), inserting the result into (2), and using (4), you can solve for $f(t)$. *As a check that you got the right result, your solution should have this form:*

$$\Delta T = \exp(At) \int_0^t \exp(-At') B F(t')\, dt' \tag{5}$$

(b) Find $\Delta T(t)$ when the perturbation is a sudden step at $t = 0$:

$$F(t) = \begin{cases} 0 & (t \leq 0) \\ Q_0 & (t > 0) \end{cases} \tag{6}$$

What is the initial behavior ($t \ll c/|\lambda|$)? What is the asymptotic behavior ($t \gg c/|\lambda|$)? (Note: we write $|\lambda|$ because $\lambda < 0$.)

(c) Find $\Delta T(t)$ when the perturbation is a linear increase,

$$F(t) = \begin{cases} 0 & (t \leq 0) \\ Q_t t & (t > 0) \end{cases} \tag{7}$$

What is the initial behavior ($t \ll c/|\lambda|$)? What is the asymptotic behavior ($t \gg c/|\lambda|$)?

Problem 2 A tale of two climate changes

The feedback parameter λ of the climate system is not well known. Let us explore the consequence of uncertainties on λ for the climate system. Consider two extreme cases: in one case the feedback parameter has a certain value $\lambda = \hat{\lambda}$, while in the other $\lambda = \hat{\lambda}/3$.

(a) Which climate system is more sensitive (greater equilibrium temperature change for the same forcing)?

(b) How does the temperature change in the early period of global warming ($t \ll c/|\lambda|$) depend on λ? You may use your solution from 1(b) or 1(c).

(c) How does the equilibrium temperature change ($t \gg c/|\lambda|$) depend on λ?

(d) What does this mean for using historical observations to predict equilibrium climate change?