Response of the ionospheric total electron content to stratospheric normal modes

P. Hoffmann, Ch. Jacobi

Summary

The response of stratospheric planetary wave (PW) activity over the higher middle latitudes (~52.5°N) in global gridded ionospheric data of the total electron content (TEC) are investigated to estimate the vertical coupling by PW. The monitoring of ionospheric variability is regularly operated by DLR Neustrelitz since 2002 producing TEC maps covering the northern hemisphere. This data base is considered for comparing simultaneous observations of wave activity in both stratosphere and ionosphere. The analysis technique of planetary wave type oscillations (PWTO) is carried out by separating waves into their zonal wavenumber, period and travelling direction.

A previous case study of autumn 2004 has shown that among other things the quasi 6-day wave (m2w) is visible in the mean spectrum of stratospheric geopotential height at 1 hPa pressure level and of ionospheric TEC data. The actual results give hints for a simultaneous occurrence of this wave type with a quasi 6-day oscillation in the mesopause region. But in comparison to the stratosphere, the wave signatures seem to be somewhat shifted and modified.

1. Introduction

Ionospheric variability with respect to planetary waves (PW) in a range of several days has already been studied by Altadill et al. (2003) by using measurements of the ionosonde critical plasma frequency (foF2). Wave signatures with periods of nearly 6 days were found in the spectral response of the ionosphere, obviously forced by a global scale wave,
having zonal wavenumber 2, in the mesosphere and lower thermosphere (MLT). The role of "meteorological influences" through PW from below seen in ionospheric variability has first been estimated by Forbes et al. (2000). They found that these exceed the effects induced by the solar photon flux or solar wind variability. As linking mechanisms between wave activity in the lower atmosphere and ionosphere vertical plasma drift, interaction of gravity waves (GW) with tides and PW modulation of the solar tides are proposed (Lastovicka, 2006). However, direct propagation of long-period PW into the upper thermosphere cannot happen due to strong wave dissipation at thermospheric heights (ion drag, molecular viscosity and thermal conduction).

The middle atmosphere shows a broad spectrum of travelling PW. They are generated, among others, by the quasi-stationary planetary waves (SPW), which modify the mean flow through depositing heat and momentum, as well as by irregular thermal or mechanical forcing in the lower atmosphere. SPW are induced by orography and diabatic heating in the troposphere. As extra-tropical Rossby normal modes for zonal wavenumber m=1, symmetric with respect to the equator, the 5-, 10- and 16-day wave were theoretically predicted by Salby (1984) and they are predominantly westward propagating. As normal symmetric modes for m=2, periods of 4-, 6,- and about 12-days can be obtained by calculating an averaged spectrum for westward propagating PW. The propagation of the first symmetric modes with zonal wavenumber 1 and 2 into the upper atmosphere was investigated by Fedulina et al. (2004). The 5-day wave propagates from the lower atmosphere with a delay of about one week during spring and autumn transition. Numerical simulations show that the 4-day wave is only capable of propagating into the upper stratosphere when the background wind is eastward and relatively weak (Pogoreltsev et al. 2002).

The mesosphere region is strongly influenced through upward propagating GW from the troposphere. Their wave amplitudes grow with height, break and lead to turbulences, small-scale mixing and dissipation in the upper mesosphere region. GW tend to drag the mean flow and influence the penetration of PW into the thermosphere. Due to relevance of GW, the behaviour of PW in the mesopause region differs from that in the stratosphere. Short periodic PW are generally strong in summer, while longer periodic PW dominate in winter. But similarities can be found during winter, when a direct wave propagation is possible.

The transmission of PW activity from the stratosphere into the ionosphere is interesting for understanding the atmospheric dynamics on the whole, as well as for understanding the origin of ionospheric oscillations. The variability in the ionospheric free electron density is mainly connected with the geomagnetic and solar activities. Thus, mainly during undisturbed conditions, signatures of upward propagating PW are expected in observations of the vertically integrated total electron content (TEC) values. The space-time analysis of global maps of TEC in comparison to stratospheric assimilated fields may give hints of wave activity from lower atmospheric layers.

2. Monitoring of ionospheric variability using TEC of GPS

The ionospheric conditions may be described by the electron density ($n_e$). The TEC map, as a two dimensional representation of the total electron content, see Equation 1, can be used to investigate PW type oscillations (PWTO) in the ionosphere. TEC is derived by ground based GPS measurements available from the International GPS Service (IGS). Because these TEC values are measured along slant rays (s), a transformation to vertical TEC values needs to be done. Further the normalised TEC values are mapped and blended into a TEC model, which was established by the DLR (Jakowski et al., 1996) especially
for this purpose. The result of this data assimilation is used to calculate the TEC value of each grid point of a fixed map grid covering upper middle and polar latitude with a longitudinal resolution of 7.5°. These TEC maps are processed by DLR Neustrelitz with a time resolution of ten minutes and an accuracy of 1 to 3·10^{16} \text{el/m}^2 (1 to 3 TECU):

\[ \text{TEC}_s = \int n_e \, ds. \] (1)

### 2.1 Tidal Waves

The strong solar tides observed in the ionospheric TEC data show the well known temporal and spatial structure. Due to the earth rotation the dominant tidal wave types of 24h, 12h and 8h with their zonal modes m=1, m=2 and m=3 are visible in the ionised component of the atmosphere (Fig.1). The diurnal tides is the most dominantly observed oscillation in the spectrum and represents the ionosation processes forced by sunrise and the recombination processes during sunset. Besides the tidal waves, a signature of the quasi 2-day wave (QTDW) is visible. The right panel in Figure 1 indicates the winter anomaly visible in the tidal amplitudes of TEC, representing the ionospheric F-region. Moreover, a secondary minimum every mid-winter is apparent in the time series of the diurnal tide. The reason for the strongest values around 2002 is the maximum of the 11-years solar circle.

![Figure 1: The amplitude spectrum of hourly TEC data over 52°N,15°E (left panel) during January 2002 shows the dominant tidal modes (24h, 12h, 8h) as well as the long period range of several days considering the planetary wave type oscillations. The time series of the monthly mean tidal wave amplitudes gives their seasonal variability from 2002 to 2005 and their solar circle dependency.](image)

### 2.2 Elimination of tides by generating daily prevailing TEC (PTEC) values

This transformation from hourly zonal distributed values to a daily mean representation of TEC along one latitudinal circle simplifies the analysis of PW with a period range of several days. A simple scheme in Figure 3 illustrates the principle. At every UT hour, there are 48 TEC values for one latitudinal circle discribing the zonal structur in degree or LT hour. For example, at 6 UT and 180°E the local time is 18 LT. To obtain the longitudinal structure for every LT hour the initial value is set UT=LT which is equal to the observation at diurnal migrating conditions.

The result of this procedure for one example depicts the left panel in Figure 3, where all 24 zonal maxima are brought in phase. The longitudinal variability during one day is clearly visible on the course of the two wrapped curves. The lower solid line is assumed as the prevailing TEC for one day on every zonal grid point, because the external impact seems to be weaker.
Figure 2: Scheme, how daily prevailing TEC (PTEC) values are generated. The universal time (UT) is shifted according to the contrast with local time (LT), which corresponds to the longitudinal (zonal) grid.

Figure 3: The lower wrapped thick solid line (left panel) represents the daily prevailing TEC (PTEC) values along one latitudinal circle for 2 January 2002 at 52.5°N. The hourly zonal distributions of TEC were shifted, so that all maxima lie in phase. The right panel gives the timeseries for the first three zonal wavenumber amplitudes of the differential PTEC values starting 2002 to 2005.

The mathematical formulation for such a phase shift is declared in Equation 2. TEC is dependent on the zonal coordinate \( \lambda \) (\( n\lambda = 48 \; [360°] \)) with a temporal resolution of one hour \( (nh=24) \). This means an angular speed of \( n\lambda/nh = \tau = 15°/h \):

\[
\text{TEC}(\lambda, h) \rightarrow \text{PTEC}(\lambda) = \text{TEC}_{\text{min}}(\lambda - h \cdot \tau, h),
\]

using periodic lateral boundary conditions.

The dependency on the solar circle is eliminated by introducing a differential prevailing TEC (DPTEC) value, which is obtained from the relative deviation of a 30 days running median value at the center of this window and for every grid point (Eq. 3). The median is suitable for averaging, because the probability of a randomly selected number above and below this value is equal:


\[
DPTEC(\lambda, d) = \frac{\text{PTEC}(\lambda, d) - \text{Median}(\lambda, d)}{\text{Median}(\lambda, d)} \cdot 100\%.
\]

The right panel in Figure 3 shows the resulting first two zonal wavenumber amplitudes at mid-latitude (52.5°N) for DPTEC of the years 2002 to 2005. The solar circle effect seems to be suppressed and an almost continuous annual circle is obtained.

2.3 External Influences

As shown, the seasonal variability in the total electron content (TEC) is more visible during the solar maximum near 2002, see Figure 2. Near the solar minimum (2005) the annual circle is much weaker. Nonetheless, the total number of free electrons is greater in winter than in summer. This seasonal anomaly is first measured in the ionospheric absorption by Appleton (1937) and occurs due to meridional transport processes of neutral gas (NO) from the summer to the winter hemisphere.

Figure 4: The time series of ionospheric PTEC (middle panel) and DPTEC (lower panel) over 52°N,15°E as well as the solar radio flux (F10.7) and geomagnetic activity index (Ap), see (upper panel), show the forcing of short and long periodic variabilities from above.

From the US National Geophysical Data Center (NGDC) observed solar variability and its influence on the earth’s magnetic field are shown in the time series for the considered time interval of the solar flux and the ap-index in figure 4. The solar flux intensity (F10.7) decreases from 2002 to 2005 and is visibly modulated by the 27-days rotational period of the sun. This influences the winter maximum in TEC. On the other hand, the geomagnetic disturbances are primarily solar forced. Both correspond temporally but differ in absolute values. Several ionospheric storm events are observed, especially at the end of 2003 and two in the second half of 2004. The effect of these major external disturbances
becomes visible in the response of the ionospheric TEC. The lower panel in Figure 4 shows the temporal behaviour of DPTEC. The solar circle effect seems to be eliminated and the annual variability strengthened, as it is known from stratospheric wave activities.

### 3. Middle and Upper Atmospheric Data Base

The data sources for middle and upper atmospheric investigations can be divided into global gridded data taken from a background assimilation model (e.g. UKMO) and hemispheric TEC maps, as well as local wind measurements from LF radar at Collm observatory (51.3°N, 13.0°E) and foF2 observations from ionosonde at Juliusruh (54.6°N, 13.4°E) over one grid point.

For global ionospheric monitoring a system of polar distributed GPS receivers are the basis of the northern hemispheric TEC map. The zonal wind parameter represents the meteorological state of the neutral atmosphere up to the mesopause.

The global operational analysis of meteorological parameters combines observations and the simulation of the atmospheric dynamics. A global time-dependent image of the state of the atmosphere is generated covering both areas which are rich or rare in observations. The assimilated data base strongly varies horizontally (dense above Europe and less dense above oceans) and decreases with height. At lower pressure levels the model output is more determined by the background physics than by observational data. This fact has to be considered in later interpretations. Global assimilated stratospheric fields (Swinbank and O’Neill, 1994) are regularly produced by Met Office and provided by the British Atmospheric Data Center (BADC) since 1990 for the meteorological parameters geopotential height, temperature and the wind components. The resolution in time and space amounts to one day and 2.5° x 2.5° horizontally. Vertically, the model atmosphere is divided into 25 pressure levels up to 0.1 hPa.

### 4. PW Separation Method from Global Gridded Fields

The analysis of PW in space-time was first investigated by Hayashi (1971). The derived cross-spectral method isolates stationary and travelling wave parts which are generated by different mechanism. The later developed phase-difference technique by Pogoreltsev et al. (2002) uses the observations of longitudinal change of phase for separation. Those method is applied in this present study and is explained in the following two subsections.

#### 4.1 The zonal harmonic decomposition using SVD

The transformation of the spatial variation into the spectral (wavenumber) domain can be calculated by solving the following set of equation (Eq.4). It is supposed that a vector $\lambda_i$ can be expressed by spectral coefficients $s_j$ of sine basis functions $A_{ij}$:

$$\lambda_i = s_j \cdot A_{ij} = s_j \cdot \sin \left\{ i \cdot \frac{2\pi}{n_i} \cdot n \right\}.$$  \hspace{1cm} (4)

To receive the vector elements of $s_j$ the matrix $A_{ij}$ has to be inverted. The singular value decomposition (SVD) is known as a powerful technique for solving sets of equation or matrix, based on the following theorem of the linear algebra (Eq.5):

$$A = V \cdot D \cdot U^T.$$  \hspace{1cm} (5)

It means that the matrix $A$ can be written as the product of an orthogonal matrix $V$, \ldots
a diagonal matrix $D$ and the transpose of an orthogonal matrix $U^T$. The orthogonal matrices $V$ and $U$ are symmetric containing the normalized eigenvectors of $A \cdot A^T$ and $A^T \cdot A$. The diagonal elements of $D$ are the nonzero eigenvalues of $A$ called singular values. The inverse matrix $A^{-1}$ can be obtained by calculating the reciprocals of the diagonal elements of the matrix $D$.

If this procedure is repeated for the cosine basis functions, resulting in the coefficients $c_i$, the amplitude $A_m$ and phase $\varphi_m$ of the first three zonal harmonics ($m=1-3$) can be obtained.

### 4.2 Phase difference method

The time series resulting from the zonal harmonic decomposition of the spectral information along one latitudinal circle, two waves can be generated representing the behaviour of the field at longitudes $0^\circ W$ and $90^\circ W$ ($m=1$) and $0^\circ W$ and $45^\circ W$ ($m=2$), equal to the phase shift between the sine (imaginary) and cosine (real) function. An applied multiple regression analysis estimates the Fourier coefficients and the stationary part for both time series during a sliding time segment of 48 days. A component is regarded as stationary if its amplitude remains constant at one time interval. The coefficients $A_{wc}$, $A_{ws}$, $A_{ec}$, $A_{es}$ in the set of Equation 6 for the east- and westward travelling waves are calculated using the mentioned phase-difference method (Pogoreltsev et al., 2002) from the Fourier amplitudes $A_r$, $A_i$ and phases $\varphi_r$, $\varphi_i$ of the cosine ($r$) and sine ($i$) wave:

$$
\begin{align*}
A_{wc} &= +0.5[A_r \cos\{\varphi_r\} + A_i \cos\{\frac{\pi}{2} - \varphi_i\}] \\
A_{ws} &= +0.5[A_r \cos\{\frac{\pi}{2} - \varphi_r\} - A_i \cos\{\varphi_i\}] \\
A_{ec} &= +0.5[A_r \cos\{\varphi_r\} - A_i \cos\{\frac{\pi}{2} - \varphi_i\}] \\
A_{es} &= -0.5[A_r \cos\{\frac{\pi}{2} - \varphi_r\} + A_i \cos\{\varphi_i\}]
\end{align*}
$$

(6)

To filter waves, the spectral information of periods between 3 and 20 days are combined and transformed back into the time domain. For this resulting time series a wavelet amplitude spectrum is calculated for each separated mode.

### 5. Results

The comparison study of PW type oscillations in the stratosphere-mesosphere-ionosphere system is performed at a mid-latitude region ($52.5^\circ N$) during 2002 to 2005. For a local measurement validation around ($15.0^\circ E$) the ionospheric parameters foF2 and TEC are compared with respect to their tidal waves and short-periodic waves of quasi 2- and 6-days by calculating a 30d-running spectrum shifted by one day. The external influence on such oscillations is considered in this context.

The separation of PWTO from global stratospheric and ionospheric fields is applied for comparing wave signatures. The strong solar circle dependency of TEC was eliminated by introducing a relative TEC (DPTEC) value. The hourly TEC data base was reduced to daily prevailing TEC fields. This simplifies the analysis of PW and the comparison to stratospheric reanalysis data.

Simultaneous observation of several PW types in the stratosphere and ionosphere is distinguished from zonal wavenumber ($m=1,2$), travelling/stationary parts and quasi period bands (3-7d, 7-12d, 12-20d, 3-20d). This technique allows to investigate the response to characteristic wave types of stratospheric origin in the ionosphere (e.g. quasi 6-day wave, $m2w$).

For validation of local wind measurements near mesopause region and for proving the
external impact on ionospheric oscillations using the ap-index, a wavelet spectrum is calculated for these both timeseries to compare the results from the global stratospheric and ionospheric space-time analysis.

5.1 External disturbances in local ionospheric parameters TEC and foF2

Local observations of the ionospheric F-region (∼ 300km) are regularly measured by ionosondes represented by the critical plasma frequency (foF2). Hourly measurements at Juliusruh and local TEC values taken from global gridded maps at 52.5°N and 15.0°E are windowed analysed, resulting in the time dependency of tidal amplitudes and signatures of PWTO shorter than 10-days.

![Ionospheric Wave Amplitudes](image)

**Figure 5:** The temporal course for the tidal wave amplitudes (diurnal-, semidiurnal- and terdiurnal tides) as well as amplitudes for short-periodic PW of quasi 2- and 6-days are analysed for the ionospheric parameters TEC (solid lines) and foF2 (dotted lines) during the time interval around winter 2004/05.

The result of this comparison between different wave signatures and their response to ionospheric disturbances shows Figure 5. Some differences in wave activity can be observed between TEC and foF2. Tidal wave types in the F-region do maximize in winter. This is clearly visible in the semidiurnal and terdiurnal tide for both. But the diurnal oscillation in TEC indicates a local minimum during mid-winter, while the course in foF2 remains unchanged. This deviation could be the consequence of the integrated electron density over all ionospheric layers. Thereby, the behaviour of the E-region may affect the total electron content in spite of the maximum in the F-region. A correspondence to
stationary waves in the stratosphere seems to be obvious (see Figure 6 of the following subsection). The signature of the quasi 2-day oscillation fits here and there quite well. During summer 2005 the ionosphere is more disturbed through geomagnetic activities. Its effect is stronger in foF2 parameter than in TEC and can be observed in the spectral response of the ionosphere. A seasonal variability in occurrence of the quasi 2- or 6-day wave is not really visible in this data in comparison to the behaviour of PW in the mesopause region.

5.2 Correspondence of stationary wave type observations

Based on the global space-time analysis separating stationary wave parts SPW1 and SPW2 for the stratosphere and ionosphere (Fig. 5), a correspondence in the temporal behaviour of SPW2 during the years 2002 to 2005 is visible. The seasonal effect in analysed SPW1 of the ionosphere seems somewhat reduced probably through the differential PTEC introduction. The local minima every winter in SPW1 and SPW2 may be a consequence of stratospheric warming events, while a minimum in SPW1 corresponds to a maximum in SPW2. At the end of 2002 there is a strong peak in the stratospheric and ionospheric SPW2 signature.

The accumulated phase difference on the right panels shows the phase propagation of the SPW with time. In case of the stratospheric SPW1, the phase remains almost constant during summer and winter. Such an oscillation is named vasillation. In transition, the phase is changing while a meridional circulation constitutes and changes the middle atmosphere conditions. The SPW play an essential role in such dynamic processes. In some cases the SPW may propagate in opposite direction, see the behaviour of SPW1 in 2004. The accumulated phase difference in the ionosphere shows only a slight temporal course. Nevertheless, there are some samples for SPW2, which corresponds quit well to the stratosphere, especially the phase course around 2003.

5.3 Signatures of westward travelling PW

The observation of simultaneous PW activities in the stratospheric zonal wind at 1hPa and its ionospheric response in differential PTEC is presented in Figure 7. The waves are westward propagating with zonal mode m=1 (left panel) and m=2 (right
panel) and the signatures are compared for several quasi-period bands. Short-periodic waves are more variable than longer ones. The seasonal variability of wave activities in the stratosphere and ionosphere fits well for longer periodic oscillations (m=1), but single maxima are shifted to one another. Short ionospheric waves occur nearly over the whole year, favoured during spring and autumn transition. Their seasonal circle is weak. The comparison of waves with zonal wavenumber 2 shows only a few simultaneous signatures for oscillations lower 12 days, predominately in winter (e.g. 2003/04).

A comparison with waves in MLT region, based on local LF radar measurements, for these bandpass filtered period intervals (without figure), shows a better agreement for periods lower 12-days than in comparison to the stratosphere. In the following subsection the wave analysis of MLT wind is involved for interpretation.

5.4 Signals of the quasi 6-day wave (m2w) observed in the Ionosphere

The last Figure 8 represents a time-period image for a selected time interval around winter 2004/05. Several parameters are analysed to obtain a whole picture of simultaneous wave activity in the middle and upper atmosphere. Two quasi 7-day wave signatures with zonal wavenumber m=1 westward propagating (m1w) were detected in the stratospheric zonal wind at 1hPa in December 2004 and January 2005 (lower panel). The westward propagating waves with m=2 show no signatures of such period during this considered time interval.

From local wind measurement near the mesopause region over Collm, an observation of two quasi 6-day wave maxima (third panel) with a time delay of about two weeks are visible. The vertical propagation of PW from stratosphere to the mesopause region can take several days, while a wave modification may occur by interaction with other wave types (e.g. GW).
The response of the analysed ionospheric fields for (m2w) represented through DPTEC (second panel) gives a good agreement to the quasi 6-day wave signature in MLT. A wave signature of quasi 10-days may also be found in both spectra. This could correspond with (m2e) in the stratosphere (not shown). The external impact (upper panel) of the period range shorter 10-days can be neglected.

6. Conclusions and Outlook

The investigation of vertical coupling processes in the stratosphere-ionosphere system through upward propagating PW is a current focus for understanding the upper atmosphere dynamics with regard to the interaction between ionospheric plasma and neutral wind.

Since 2002, the DLR Neustrelitz regularly produces hemispheric maps of the total electron content (TEC) derived from GPS measurements. Continuous monitoring of the ionospheric variability enables to analyse such data in space and time in comparison to meteorological fields taken from a global stratospheric reanalysis model (e.g. UKMO). The results of the present study gives the following cognition with respect to the PWTO observed in the stratosphere and ionosphere. Due to the winter anomaly of the F-region, the long periodic wave in the ionosphere do maximize in winter and show a similar annual circle as it is known from the stratosphere. Nevertheless, local minima in winter occur in the diurnal tide of TEC and in the stationary part of the stratospheric and ionospheric fields, while the magnitude of the diurnal oscillation in foF2 show no such winter phenomenon. The external influence on ionospheric oscillations may affect the whole spectrum of PW. Especially the solar and geomagnetic impact on short-periodic waves in a range of 2- and 6-days is hardly estimated.

Through the introduction of a daily prevailing TEC (PTEC) and its relative deviation from a sliding 30-day median value, the tidal and solar circle dependency could be eliminated. Thereby, a strong seasonal variability is also seen during the solar minimum around 2005. The investigation of simultaneous occurrence of PWTO in the middle atmosphere and ionosphere for quasi periods of 5-,10-,16-days westward propagating and for the stationary part offers some correspondence between both. Predominantly longer-periodic waves, which are as far as possible consistent during winter, show some simultaneous activities for zonal mode (m=1), while the stationary part of the ionosphere seems somewhat noised. The analysed SPW2 for both shows a good agreement in the temporal behaviour of the amplitude and phase.

In contrast to that, the shorter-periodic oscillations are transient and difficult to interpret. In case of the PWTO of quasi 6-day, a comparison of wave activity is applied for local and global data sets. Two signatures of such a wave type with zonal mode (m=1) westward travelling were detected in the stratosphere around December 2004 and January 2005. These are somewhat shifted to the local observation near the mesopause. The response of the ionosphere to wave signatures of around 6-days shows simultaneous PWTO with zonal wavenumber (m=2), while the geomagnetic variability seems to have no influence on such oscillation. A connection between neutral atmospheric waves and oscillations observed in the ionospheric plasma is obvious.

The investigations of global TEC maps with respect to PW from below appears quite complex without a conception of vertical coupling mechanisms between the middle atmosphere and the ionospheric plasma. From the numeric modelling of the middle and upper atmosphere it is known that tidal waves can penetrate the thermosphere, while PW are blocked. The meteorological influence of the ionosphere can only be transferred indirectly.
through tidal modulation or their interaction with gravity waves (GW). More efforts have to be made to achieve a global picture of PWTO in the ionospheric TEC and other upper atmospheric data sets (e.g. SABER) should be used for comparison. Also the analysis technique has to be carefully proved and a cross-spectral analysis can be applied to find the coherence between wave activity in the neutral atmosphere and the ionosphere.

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