Modul 04-006-1001:

WiSe 2024-2025

Formale Grundlagen (Logik)

Excercises 7

Excercise 1: Truth tables

- Construct truth tables for the following statements. Note whether any of them are logically equivalent.
- (1) a. $(p \lor (\neg q))$
 - b. $(\neg((\neg p) \land q))$
 - c. $((p \leftrightarrow q) \land p)$
 - d. $((p \rightarrow (q \lor (\neg r))) \land (p \rightarrow (q \lor (\neg r))))$
 - e. $(((p \rightarrow q) \rightarrow p) \rightarrow q)$

Excercise 2: Tautology, contradiction, contingency

- Let p, q, and r be atomic statements. Which of the following are tautologies, contradictions, or contingent statements?
- (2) a. $(p \lor (\neg p))$
 - b. $(p \lor q)$
 - c. $((p \land q) \rightarrow (p \lor r))$
 - d. $((\neg p) \land (\neg (p \rightarrow q)))$
 - e. $((p \lor r) \to (\neg p))$

Excercise 3: Definition of connectives

- Certain of the logical connectives can be defined in terms of others. Example: $(p \to q)$ can be defined as $((\neg p) \lor q)$ (i.e. \to is expressible in terms of \neg and \lor), since the two statements are logically equivalent.
- Define \rightarrow in terms of \land and \neg .
- Define \wedge in terms of \vee and \neg .
- Define \leftrightarrow in terms of \rightarrow and \land .
- Show how the five connectives can be reduced to \wedge and \neg .

Excercise 4: Laws of statement logic

- Prove the following equivalence: $((p \land q) \lor p) \Leftrightarrow p$.
- Use the laws of statement logic (and, possibly, the equivalence you proved previously) to reduce each of the following statements to the simplest equivalent statement.
- (3) a. $((\neg p) \lor (p \land q))$
 - b. $(((\neg p) \land q) \lor (\neg (p \lor q)))$
 - c. $((\neg p) \land ((p \land q) \lor (p \land r)))$
 - d. $(((\neg p) \land q) \leftrightarrow (p \lor q))$