Modul 04-006-1001: **WiSe 2024-2025**
Formale Grundlagen (Logik)

Excercises 4

Excercise 1: Relations and functions
Let $A = \{a, b, c\}$ and $B = \{1, 2\}$.

- How many distinct relations are there from A to B?
- How many of these are total functions from A to B?
- How many of these total functions are onto (surjective)?
- How many of these total functions are one-to-one (injective)?
- Do any of these functions have inverses that are also total functions?
- Answer the same questions for all relations from B to A.

Excercise 2: Composition
Let R_1 and R_2 be the following two relations in $A = \{1, 2, 3, 4\}$:

(1) a. $R_1 = \{\langle 1, 1\rangle, \langle 2, 1\rangle, \langle 3, 4\rangle, \langle 2, 2\rangle, \langle 3, 3\rangle, \langle 4, 4\rangle, \langle 4, 1\rangle\}$
b. $R_2 = \{\langle 3, 4\rangle, \langle 1, 2\rangle, \langle 1, 4\rangle, \langle 2, 3\rangle, \langle 2, 4\rangle, \langle 1, 3\rangle\}$

- Form the composites $R_2 \circ R_1$ and $R_1 \circ R_2$. Are they equal?
- Show that $R_1^{-1} \circ R_1 \neq id_A$ and that $R_2^{-1} \circ R_2 \not\subseteq id_A$.

Excercise 3: Composition and the inverse

- Let F and G in (2-a,b) be functions from $A = \{a, b, c\}$ to $B = \{1, 2, 3, 4\}$ and from $C = \{1, 2, 3, 4\}$ to $D = \{p, q, r\}$, respectively. Show that $(G \circ F)^{-1} = F^{-1} \circ G^{-1}$.

(2) a. $F = \{\langle a, 1\rangle, \langle b, 3\rangle, \langle c, 3\rangle\}$
b. $G = \{\langle 1, p\rangle, \langle 2, q\rangle, \langle 3, q\rangle, \langle 4, r\rangle\}$

Excercise 4: Reflexivity and symmetry

- Give the status for the two relations “is a child of” and “is a brother of” (in the set of human beings) with respect to reflexivity and symmetry. Only mention the strongest property if the relation in question has more than one (e.g., a relation that is irreflexive is also non-reflexive, but not vice versa).

Excercise 5: More reflexivity and symmetry
Let $A = \{1, 2, 3, 4\}$.

- Describe the properties of each relation R_i in A below, of its inverse (R_i^{-1}), and of its complement (R_i') with respect to reflexivity and symmetry.

(3) a. $R_1 = \{\langle 1, 1\rangle, \langle 2, 1\rangle, \langle 3, 4\rangle, \langle 2, 2\rangle, \langle 3, 3\rangle, \langle 4, 4\rangle, \langle 4, 1\rangle\}$
b. $R_2 = \{\langle 3, 4\rangle, \langle 1, 2\rangle, \langle 1, 4\rangle, \langle 2, 3\rangle, \langle 2, 4\rangle, \langle 1, 3\rangle\}$
c. $R_3 = \{\langle 2, 4\rangle, \langle 3, 1\rangle, \langle 3, 4\rangle, \langle 2, 2\rangle, \langle 1, 3\rangle, \langle 4, 3\rangle, \langle 4, 2\rangle\}$
d. $R_4 = \{\langle 1, 1\rangle, \langle 2, 4\rangle, \langle 1, 3\rangle, \langle 2, 2\rangle, \langle 3, 1\rangle, \langle 4, 4\rangle, \langle 3, 3\rangle, \langle 4, 2\rangle\}$