Modul 04-006-1001: **WiSe 2024-2025**
Formale Grundlagen (Logik)

Excercises 12

Excercise 1: Predicate logic; scope

- Classify all the wffs in this exercise as: statement, open statement, or having vacuous quantification. If the wff involves a quantifier or quantifiers, indicate the scope of each quantifier by means of underlining.

(1) a. $(\forall x)(P(x))$
b. $(\forall x)(\exists x)(P(x,y) \vee Q(j,m))$
c. $((\exists x)P(x,z,j) \rightarrow (\forall y)(\neg K(f)))$
d. $(P(y) \leftrightarrow (J(k) \wedge (Q(p) \vee (\forall y)(Y(y)))))$
e. $(\forall x)(\exists y)(P(x \wedge y))$
f. $(\exists y)(K(m))$

Excercise 2: Predicate logic; models and quantifiers

- Evaluate the truth-values of the expressions in (3-a-c) based on the model M given in (2):

(2) $M = \langle D, I \rangle$, where:
a. $D = \{$*Sokrates, Aristotle, Plato, Michelangelo, Bach, Tarski*$\}$
b. $I =$

term	value	predicate	value
s	*Sokrates*	H	$\{$*Sokrates, Aristotle, Plato*$\}$
m	*Michelangelo*	M	$\{$*Sokrates, Aristotle, Plato,*
b	*Bach*		*Michelangelo, Bach, Tarski*$\}$
t	*Tarski*	L	$\{\langle$*Sokrates, Sokrates*$\rangle, \langle$*Sokrates, Aristotle*$\rangle,$
a	*Aristotle*		$\langle$*Michelangelo, Bach*$\rangle, \langle$*Bach, Michelangelo*$\rangle,$
p	Plato		$\langle$*Tarski, Plato*$\rangle, \langle$*Plato, Michelangelo*$\rangle,$
			$\langle$*Aristotle, Tarski*$\rangle\}$

(3) a. $[\![(\exists y)(\forall x)L(x,y)]\!]^M =$
b. $[\![(\forall x)\neg(\exists y)L(x,y)]\!]^M =$
c. $[\![((\exists z)M(z) \wedge (\forall y)(H(y) \rightarrow L(y,b)))]\!]^M =$

Excercise 3: Predicate logic; multiple quantifiers

- The statement in (4) is ambiguous, i.e. has two different meanings. Show how the two meanings can be expressed via predicate logic. Discuss how the ambiguity is the result of (quantifier) scope ambiguity.

(4) Everyone is jealous of someone.