Modul 04-006-1001: **WiSe 2024-2025**
Formale Grundlagen (Logik)

Excercises 11

Excercise 1: Bound vs. free variables

- For each of the expressions below, state whether the statement is open (i.e.. contains unbound variables). Name the free variables (i.e. the variables that are unbound).

(1) a. $(\forall x)(P(x) \vee Q(x, y))$
b. $(\forall y)(Q(x) \rightarrow (\forall z)P(y, z))$
c. $(\forall x)(P(x) \rightarrow (\exists y)(Q(y) \rightarrow (\forall z)R(y, z)))$

Excercise 2: Translation from English into predicate logic

- Translate the following English sentences into predicate logic. Choose your own variables and predicate letters, giving the key.

(2) a. Susan will go jogging only if Bill doesn't fall sick.
b. Leipzig is in Sachsen.
c. Jill likes red shoes.
d. Some girls like red shoes.
e. All people detest cold houses, especially when they are sick!
f. When John saw his friend who had tricked him, he got very angry.
g. No one saw any red boots.

Excercise 3: Well-formed formulas

- For the following expressions, say whether they represent a well-formed formula (wff) in predicate logic or not. Explain your answer.

(3) a. $(\forall x)P(x)$
b. $(\forall x)P$
c. $(P(Q(x)) \rightarrow (\exists y)F(y, x))$
d. $(x \wedge (y \vee z))$
e. $(\forall x)(\exists x)(P(x, y) \vee Q(j, m))$
f. $((\forall y) \rightarrow (\exists z))$
g. $(\exists x)(P(x, z, j) \rightarrow (\forall y)(\neg K(f)))$
h. $(P(y) \leftrightarrow (J(k) \wedge (Q(p) \vee (\neg Y(y)))))$
i. $(\forall x)(\exists y)P(x \wedge y)$
j. $(P \wedge Q(x))$
k. $(\forall x)s(x)$
l. $(\exists y)K(m)$

Exercises 4:

- Given the equivalences in (4), prove the equivalence between (5-a,b). Give the names of the laws of logic that you make reference to in your proof.

(4) a. $(\neg(\forall x)(P(x))) \Leftrightarrow (\exists x)\neg(P(x))$
b. $(\neg(\exists x)(P(x))) \Leftrightarrow (\forall x)\neg(P(x))$

(5) a. Kein Kind fährt nicht nach Rom.
b. Alle Kinder fahren nach Rom.