Modul 04-006-1001: **WiSe 2023-2024**
Formale Grundlagen (Logik)

Solutions 2

Excercise 1:

- Give two definitions for each of the following sets, one in terms of predication and one in terms of recursive rules.

(1) a. $A = \{5, 10, 15, 20, \ldots\}$
$\{x | x$ is a multiple of $5\}$
(where proper is supposed to exclude the case 0×5)
(i) $5 \in A$,
(ii) if $x \in A$, then $x + 5 \in A$.

b. $B = \{7, 17, 27, 37, \ldots\}$
$\{x | x$ is a multiple of 10 plus $7\}$
(i) $7 \in B$,
(ii) if $x \in B$, then $x + 10 \in B$.

c. $C = \{300, 301, 302, \ldots, 399, 400\}$
$\{x | 300 \leq x \leq 400$, with $x \in \mathbb{N}\}$
(i) $300 \in C$,
(ii) if $x \in C$ and if $x \leq 399$, then $x + 1 \in C$.

d. $D = \{3, 4, 7, 8, 11, 12, 15, 16, 19, 20, \ldots\}$
$\{x | x = 3 + 4y$ or $x = 4 + 4y$, with $y \in \mathbb{N}_0\}$
(i) $3 \in D$,
(ii) if $x \in D$ and if x is odd, then $x + 1 \in D$,
(iii) if $x \in D$ and if x is even, then $x + 3 \in D$.

e. $E = \{0, 2, -2, 4, -4, 6, -6, \ldots\}$
$\{x | x \in \mathbb{Z}$ and x is even$\}$
(i) $0 \in E$,
(ii) if $x \in E$, then $x \pm 2 \in E$.

f. $F = \{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots\}$
$\{x | x = \frac{1}{2^y}$, with $y \in \mathbb{N}_0\}$
(i) $1 \in F$,
(ii) if $x \in F$, then $\frac{x}{2} \in F$.

Excercise 2:

- Given the sets in (2), answer the questions in (3).

(2) a. $S_1 = \{\emptyset, \{A\}, A\}$
b. $S_2 = A$
c. $S_3 = \{A\}$
d. $S_4 = \{\{A\}\}$
e. $S_5 = \{\{A\}, A\}$

f. $S_6 = \emptyset$
g. $S_7 = \{\emptyset\}$
h. $S_8 = \{\{\emptyset\}\}$
i. $S_9 = \{\emptyset, \{\emptyset\}\}$

(3) Of the sets S_1–S_9, ...

a. ... which are members of S_1?
$\rightarrow S_2, S_3, S_6$

b. ... which are subsets of S_1?
$\rightarrow S_1, S_3, S_4, S_5, S_6, S_7$

c. ... which are members of S_9?
$\rightarrow S_6, S_7$

d. ... which are subsets of S_9?
$\rightarrow S_6, S_7, S_8, S_9$

e. ... which are members of S_4?
$\rightarrow S_3$

f. ... which are subsets of S_4?
$\rightarrow S_4, S_6$

Excercise 3:

- Specify each of the sets in (4) by listing its members:

(4)
a. $\wp(\{a, b, c\}) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$
b. $\wp(\{a\}) = \{\emptyset, \{a\}\}$
c. $\wp(\emptyset) = \{\emptyset\}$
d. $\wp(\{\emptyset\}) = \{\emptyset, \{\emptyset\}\}$
e. $\wp(\wp(\{a, b\})) =$
$\wp(\{\emptyset, \{a\}, \{b\}, \{a, b\}\}) =$
$\{\emptyset, \{\emptyset\}, \{\{a\}\}, \{\{b\}\}, \{\{a, b\}\},$
$\{\emptyset, \{a\}\}, \{\emptyset, \{b\}\}, \{\emptyset, \{a, b\}\}, \{\emptyset, \{a\}, \{b\}\},$
$\{\emptyset, \{a\}, \{a, b\}\}, \{\emptyset, \{b\}, \{a, b\}\}, \{\emptyset, \{a\}, \{b\}, \{a, b\}\},$
$\{\{a\}, \{b\}\}, \{\{a\}, \{a, b\}\}, \{b, \{a, b\}\}, \{\{a\}, \{b\}, \{a, b\}\}\}$

Excercise 4:

- Given the sets in (5), list the members of the sets in (6) (or give a shorthand).

(5)
a. $A = \{a, b, c, 2, 3, 4\}$
b. $B = \{a, b\}$
c. $C = \{c, 2\}$
d. $D = \{b, c\}$
e. $E = \{a, b, \{c\}\}$
f. $F = \{\}$
g. $G = \{\{a, b\}, \{c, 2\}\}$

(6)
a. $B \cup C = \{a, b, c, 2\}$
b. $A \cup B = A$
c. $D \cup E = \{a, b, c, \{c\}\}$
d. $B \cup G = \{a, b, \{a, b\}, \{c, 2\}\}$
e. $D \cup F = D$
f. $A \cap B = \{a, b\}$
g. $A \cap E = \{a, b\}$
h. $C \cap D = \{c\}$
i. $B \cap F = \emptyset$
j. $C \cap E = \emptyset$
k. $A - B = \{c, 2, 3, 4\}$
l. $B - A = \emptyset$
m. $C - D = \{2\}$
n. $E - F = E$
o. $F - A = F$

Excercise 5:

- Given the sets in (5), and assuming that the universe of discourse (i.e. the universal set U) is defined as $\bigcup\{A, B, C, D, E, F, G\}$, list the members of the following sets (or give a shorthand):

(7)

a. $(A \cap B) \cup C = \{a, b, c, 2\}$

b. $A \cap (B \cup C) = \{a, b, c, 2\}$

c. $(B \cup C) - (C \cup D) = \{a\}$

d. $A \cap (C - D) = \{2\}$

e. $(A \cap C) - (A \cap D) = \{2\}$

f. $G' = \{a, b, c, 2, 3, 4, \{c\}\}$

g. $(D \cup E)' = \{2, 3, 4, \{a, b\}, \{c, 2\}\}$

h. $D' \cap E' = \{2, 3, 4, \{a, b\}, \{c, 2\}\}$

i. $F \cap (A - B) = \emptyset$

j. $(A \cap B) \cup U = U$

k. $(C \cup D) \cap U = \{b, c, 2\}$

l. $C \cap D' = \{2\}$

m. $G \cup F' = U$