Modul 04-006-1001: **WiSe 2023-2024**
Formale Grundlagen (Logik)

Solutions 1

Excercise 1: Identifying sets
Which of the following are valid sets, and which are not?

(1) a. {♗, ♕, ♔, ♙, ♖, ♘}
b. {Obama, 'Obama', {Obama}}
c. {x | x is a multiple of 17}
d. {x, sofa, prime, Pizza, 0, Φ, ♘, x}
e. {{{{{Φ}}}}}
f. {} (also written as ∅)

All of (1-a-f) are valid sets: (1-a) is the set of (white) chess pieces. (1-b) is the set containing the person Barack Obama, the name 'Obama', and the set containing the person Barack Obama. (1-c) is an infinite set, and therefore it is impossible to list all its members (but one may resort to the dot/etc.-notation {17, 34, 51, 68, . . .}), (1-d) is a random collection, (1-e) is the set containing (as its only member) the set {{{{Φ}}}}; finally, (1-f) is the empty set (the set that does not contain anything).

Excercise 2: Identifying set members and sets

- What are the members of the following sets?

(2) a. {x | x is a multiple of 12 below 60}
b. {x | x is the set containing all multiples of 12 below 60}
c. {vowel, yellow, 'gelb', {vowel, yellow, 'gelb'}, Φ}
d. {{{{{Φ}}}}}
e. {Bart}
f. {Bart, 'Bart', {Bart}}

The members of (2-a) are 12, 24, 36, 48. The only member of (2-b) is the set {12, 24, 36, 48}. The members of (2-c) are the concepts vowel and yellow, the name 'gelb', the set containing these three elements, and the Greek letter Φ. The only member of (2-d) is the set {{{{Φ}}}}. The only member of (2-e) is a person named Bart (possibly the Simpson character). And (2-f) contains this person plus the name 'Bart' plus the set containing the person Bart.

- For the set OS = {Olaf Scholz}, which of the following is True or False?

(3) a. Olaf Scholz is a member of OS.
b. {Olaf Scholz} is a member of OS.
c. The current Bundeskanzlerin of Germany is a member of OS.

(3-a) is obviously true, (3-b) is false because OS does not have a set as its member. (3-c) is true again because the description given refers to Olaf Scholz (which is the only member of the set).

- What is the set whose only member is {S}?

Obviously, the answer is {{S}}.

Excercise 3: List to predicate notation

- Convert each of the sets in list notation below to predicate notation.

(4) a. {2, 4, 6, 8, 10}
b. {Bart, Lisa, Homer, Marge, Maggie}
c. {Leipzig}
d. {'Bart', 'Lisa', 'Homer', 'Marge', 'Maggie'}
e. {}

Note that often one can come up with various different descriptions, all defining the same set. Possible answers are the following. (4-a): {x | x is an even natural number smaller than 12 (exluding 0)}. (Just one alternative: {x | x is a non-negative even number smaller than 11}.) (4-b): {x | x is a character of the (fictional) Simpson family}. (4-c): {x | x is the city where Leibniz was born}. (4-d): {x | x is the name of a character of the (fictional) Simpson family}. (4-e): {x | x is an even prime number that does not equal 2}.

Excercise 4: Predicate to list notation

- Convert each of the sets in predicate notation below to list notation.

(5) a. {x | x is an odd integer smaller than 12}
b. {z | z is the name of the first president of the USA}
c. {a | a is a triangular circle}

(5-a): {11, 9, 7, 5, 3, 1, –1, –3, –5, . . .} (again, listing all elements is impossible because the set is infinite); (5-b): {'George Washington'}; (5-c): {}.

Excercise 5: Subset superset relations

- List the proper subset or proper superset relations that hold between the following sets.

(6) a. A = { x | x is a Beatles song written by Paul McCartney}
b. C = { x | x is a Beatles song written by Ringo Starr}
c. D = {Let it be, Maxwell's Silver Hammer, Hey Jude}
d. F = {}

$D \subset A$, $A \supset D$ (D exlusively contains songs written by Paul McCartney)
$F \subset A$, $A \supset F$ (the empty set is subset of every set)
$F \subset C$, $C \supset F$ (Ringo Starr actually wrote songs, hence $F \neq C$)
$F \subset D$, $D \supset F$ (the empty set is subset of every set)