

Excercises 9

Excercise 1: Proofs

- Give a formal proof of the validity of each of the following statements.

- (1)
- $(p \vee p) \Rightarrow p$
  - $((\neg p) \rightarrow (\neg q)) \Rightarrow (q \rightarrow p)$
  - $(p \vee ((\neg p) \wedge q)) \Rightarrow (p \vee q)$
  - $(p \rightarrow (q \rightarrow r)) \Rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$
  - $(p \vee q) \Rightarrow (q \vee p)$
  - $(p \rightarrow q) \Rightarrow ((p \wedge r) \rightarrow (q \wedge r))$
  - $(\neg p) \Rightarrow (p \rightarrow q)$

Excercise 2: Conditional proofs

- Give a conditional proof of the validity of each of the following arguments. (Note: In the solutions to the previous exercise sheet, the first argument was proven without auxiliary assumption, the second argument was proven indirectly, namely by contradiction. The present task is to provide a direct conditional proof for both.)

- (2)
- $(p \rightarrow (\neg q))$
  - $(r \rightarrow q)$
  - $((\neg r) \rightarrow s)$
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- $\therefore (p \rightarrow s)$

- (3)
- $((\neg p) \rightarrow q)$
  - $(r \rightarrow (s \vee t))$
  - $(s \rightarrow (\neg r))$
  - $(p \rightarrow (\neg t))$
- 
- $\therefore (r \rightarrow q)$