Modul 04-006-1001: **WiSe 2023-2024**
Formale Grundlagen (Logik)

Excercises 7

Excercise 1: Truth tables

- Construct truth tables for the following statements. Note whether any of them are logically equivalent.

(1) a. $(p \lor (\neg q))$
b. $(\neg((\neg p) \land q))$
c. $((p \leftrightarrow q) \land p)$
d. $((p \rightarrow (q \lor (\neg r))) \land (p \rightarrow (q \lor (\neg r))))$
e. $(((p \rightarrow q) \rightarrow p) \rightarrow q)$

Excercise 2: Tautology, contradiction, contingency

- Let p, q, and r be atomic statements. Which of the following are tautologies, contradictions, or contingent statements?

(2) a. $(p \lor (\neg p))$
b. $(p \lor q)$
c. $((p \land q) \rightarrow (p \lor r))$
d. $((\neg p) \land (\neg(p \rightarrow q)))$
e. $((p \lor r) \rightarrow (\neg p))$

Excercise 3: Definition of connectives

- Certain of the logical connectives can be defined in terms of others. Example: $(p \rightarrow q)$ can be defined as $((\neg p) \lor q)$ (i.e. $\rightarrow$ is expressible in terms of $\neg$ and $\lor$), since the two statements are logically equivalent.
- Define $\rightarrow$ in terms of $\land$ and $\neg$.
- Define $\land$ in terms of $\lor$ and $\neg$.
- Define $\leftrightarrow$ in terms of $\rightarrow$ and $\land$.
- Show how the five connectives can be reduced to $\land$ and $\neg$.

Excercise 4: Laws of statement logic

- Prove the following equivalence: $((p \land q) \lor p) \Leftrightarrow p$.
- Use the laws of statement logic (and, possibly, the equivalence you proved previously) to reduce each of the following statements to the simplest equivalent statement.

(3) a. $((\neg p) \lor (p \land q))$
b. $(((\neg p) \land q) \lor (\neg(p \lor q)))$
c. $((\neg p) \land ((p \land q) \lor (p \land r)))$
d. $(((\neg p) \land q) \leftrightarrow (p \lor q))$