Modul 04-006-1001: **WiSe 2023-2024**
Formale Grundlagen (Logik)

Excercises 5

Excercise 1: Transitivity and connectedness
Let $A = \{1, 2, 3, 4\}$.

- Describe the properties of each relation R_i in A below, of its inverse (R_i^{-1}), and of its complement (R_i') with respect to transitivity and connectedness.

(1)
a. $R_1 = \{\langle 1, 1\rangle, \langle 2, 1\rangle, \langle 3, 4\rangle, \langle 2, 2\rangle, \langle 3, 3\rangle, \langle 4, 4\rangle, \langle 4, 1\rangle\}$
b. $R_2 = \{\langle 3, 4\rangle, \langle 1, 2\rangle, \langle 1, 4\rangle, \langle 2, 3\rangle, \langle 2, 4\rangle, \langle 1, 3\rangle\}$
c. $R_3 = \{\langle 2, 4\rangle, \langle 3, 1\rangle, \langle 3, 4\rangle, \langle 2, 2\rangle, \langle 1, 3\rangle, \langle 4, 3\rangle, \langle 4, 2\rangle\}$
d. $R_4 = \{\langle 1, 1\rangle, \langle 2, 4\rangle, \langle 1, 3\rangle, \langle 2, 2\rangle, \langle 3, 1\rangle, \langle 4, 4\rangle, \langle 3, 3\rangle, \langle 4, 2\rangle\}$

Excercise 2: Partitions

- Is any of the R_i in exercise 1 an equivalence relation (see excercise 5 on sheet 4 for reflexivity and symmetry)? If so, then give the partition that is induced on A.
- Give the equivalence relation that induces the following partition on A:
  $P = \{\{1\}, \{2, 3\}, \{4\}\}$.
- How many different partitions on A are possible?

Excercise 3: Orders
Let $A = \{1, 2, 3, 5, 6, 10, 15, 30\}$ and let R be the relation in A defined as $R = \{\langle x, y\rangle \mid x$ divides y without remainder$\}$

- List the members of R and determine whether it forms an order (and if so, whether the order is weak or strong).
- Do the same for the set $\wp(B)$, where $B = \{a, b, c\}$, and the relation “is a subset of”.