Modul 04-006-1001: **WiSe 2023-2024**
Formale Grundlagen (Logik)

Excercises 3

Excercise 1: Set operations and membership

- Given the sets in (1), what are the sets defined in (2)?
- Is a a member of $\{A, B\}$?
- Is a a member of $A \cup B$?

(1) a. $A = \{a, b, c\}$
b. $B = \{c, d\}$
c. $C = \{d, e, f\}$

(2) a. $A \cup B$
b. $A \cap B$
c. $A \cup (B \cap C)$
d. $C \cup A$
e. $B \cup \emptyset$
f. $A \cap (B \cap C)$
g. $A - B$

Excercise 2: Set theoretic equations

- Show by using the set-theoretic equalities that were introduced (idempotent laws, commutative laws, etc.) that the following holds for any sets A and B: $A \cap (B - A) = \emptyset$.

Excercise 3: Venn diagramms and distributive law

- Show by means of Venn diagramms that the equation in (3) holds (one of the distributive laws).

(3) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Excercise 4: Symmetric difference

- The symmetric difference between two sets A and B is defined as in (4-a).
- Draw the Venn diagramm for the symmetric difference of two sets.
- Show that (4-b) holds by making reference to set theoretic equalities. Verify that the Venn diagramm for $(A - B) \cup (B - A)$ is the same as the diagramm for $A + B$.
- Show that for all sets A and B: $A + B = B + A$.

(4) a. $A + B =_{def} (A \cup B) - (A \cap B)$
b. $A + B =_{def} (A - B) \cup (B - A)$

Excercise 5: More on symmetric difference

- Redefine the sets in (5), getting rid of the $+$-operator.
- Show that the statements in (6-a,b) are correct.

(5) a. $A + A$
b. $A + U$
c. $A + \emptyset$
d. $A + B$, where $A \subseteq B$
e. $A + B$, where $A \cap B = \emptyset$

(6) a. $((A - B) + (B - A)) = A + B$
b. $(A + B) \subseteq B$ iff $A \subseteq B$

Excercise 6: Carthesian products and relations

- Given are the sets $A = \{b, c\}$ and $B = \{2, 3\}$.
- Specify the sets in (7) by listing their members.

(7) a. $A \times B$
b. $B \times A$
c. $A \times A$
d. $(A \cup B) \times B$
e. $(A \cap B) \times B$
f. $(A - B) \times (B - A)$

- Consider now the following relation from A to $(A \cup B)$:
  $R = \{\langle b, b\rangle, \langle b, 2\rangle, \langle c, 2\rangle, \langle c, 3\rangle\}$
- Specify the domain and the range of R.
- Specify R' and R^{-1}.
- Is $(R')^{-1}$ equal to $(R^{-1})'$?