Modul 04-006-1001: **WiSe 2023-2024**
Formale Grundlagen (Logik)

Excercises 2

Excercise 1:

- Besides the list notation and the predicate notation there is a third way to define sets: by recursive rules. An example for such a definition is given in (1-b), which defines the set of all even natural numbers greater than or equal to 4 (cf. (1-a)) by list notation :

(1) a. $E = \{4, 6, 8, 10, \ldots\}$
   b. (i) $4 \in E$,
      (ii) if $x \in E$, then $x + 2 \in E$.

- (1-b) contains two clauses: the base clause (i), which defines one concrete element as being a member of E (namely 4), and the recursive clause (ii), which allows to generate all other members of E on the basis of 4.
- This is called a recursive definition because the clause (ii), which is supposed to define the members of E, makes reference to (one property of) E already. Note that the definition is not circular because of the existence of the base clause (i).
- Give two definitions for each of the following sets, one in terms of predication and one in terms of recursive rules.

(2) a. $A = \{5, 10, 15, 20, \ldots\}$
   b. $B = \{7, 17, 27, 37, \ldots\}$
   c. $C = \{300, 301, 302, \ldots, 399, 400\}$
   d. $D = \{3, 4, 7, 8, 11, 12, 15, 16, 19, 20, \ldots\}$
   e. $E = \{0, 2, -2, 4, -4, 6, -6, \ldots\}$
   f. $F = \{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots\}$

Excercise 2:

- Given the sets in (3), answer the questions in (4).

(3) a. $S_1 = \{\emptyset, \{A\}, A\}$
   b. $S_2 = A$
   c. $S_3 = \{A\}$
   d. $S_4 = \{\{A\}\}$
   e. $S_5 = \{\{A\}, A\}$
   f. $S_6 = \emptyset$
   g. $S_7 = \{\emptyset\}$
   h. $S_8 = \{\{\emptyset\}\}$
   i. $S_9 = \{\emptyset, \{\emptyset\}\}$

(4) Of the sets S_1–S_9, ...
   a. ... which are members of S_1?
   b. ... which are subsets of S_1?
   c. ... which are members of S_9?
   d. ... which are subsets of S_9?
   e. ... which are members of S_4?
   f. ... which are subsets of S_4?

... to be continued on next page ↪

Excercise 3:

- Specify each of the sets in (5) by listing its members ($\wp(A)$ = power set of A):

(5)
a. $\wp(\{a,b,c\})$
b. $\wp(\{a\})$
c. $\wp(\emptyset)$
d. $\wp(\{\emptyset\})$
e. $\wp(\wp(\{a,b\}))$

Excercise 4:

- Given the sets in (6), list the members of the sets in (7).

(6)
a. $A = \{a,b,c,2,3,4\}$
b. $B = \{a,b\}$
c. $C = \{c,2\}$
d. $D = \{b,c\}$
e. $E = \{a,b,\{c\}\}$
f. $F = \{\}$
g. $G = \{\{a,b\},\{c,2\}\}$

(7)
a. $B \cup C$
b. $A \cup B$
c. $D \cup E$
d. $B \cup G$
e. $D \cup F$
f. $A \cap B$
g. $A \cap E$
h. $C \cap D$
i. $B \cap F$
j. $C \cap E$
k. $A - B$
l. $B - A$
m. $C - D$
n. $E - F$
o. $F - A$

Excercise 5:

- Given the sets in (6), and assuming that the universe of discourse U is defined as $\bigcup\{A,B,C,D,E,F,G\}$, list the members of the following sets:

(8)
a. $(A \cap B) \cup C$
b. $A \cap (B \cup C)$
c. $(B \cup C) - (C \cup D)$
d. $A \cap (C - D)$
e. $(A \cap C) - (A \cap D)$
f. G'
g. $(D \cup E)'$
h. $D' \cap E'$
i. $F \cap (A - B)$
j. $(A \cap B) \cup U$
k. $(C \cup D) \cap U$
l. $C \cap D'$
m. $G \cup F'$