

Excercises 10

Excercise 1: Translation into English

- Translate the following English sentences into predicate logic. Choose your own variables and predicate letters, giving the key.

- (1)
- Everything is black or white.
 - A dog is a quadruped.
 - Everybody loves somebody.
 - There is someone whom everyone loves.
 - No one loves himself, unless it is John.
 - People who live in Leipzig love it.
 - If someone does not love Leipzig, he does not know it.
 - Give him a finger, and he takes the whole hand.
 - If someone is noisy, he annoys everybody.
 - Although no one made noise, John was annoyed.
 - Only drunk drivers under 18 cause bad accidents.

Excercise 2: Bound vs. free variables

- For each of the expressions below, state whether the statement is open (i.e.. contains unbound variables). Name the free variables (i.e. the variables that are unbound).

- (2)
- $(\forall x)(P(x) \vee Q(x, y))$
 - $(\forall y)(Q(x) \rightarrow (\forall z)P(y, z))$
 - $(\forall x)(P(x) \rightarrow (\exists y)(Q(y) \rightarrow (\forall z)R(y, z)))$

Excercise 3: Predicate logic; models and quantifiers

- Evaluate the truth-values of the expressions in (4-a-c) based on the model M given in (3):

- (3) $M = \langle D, I \rangle$, where:

- a. $D = \{Sokrates, Aristotle, Plato, Michelangelo, Bach, Tarski\}$

term	value	predicate	value
s	<i>Sokrates</i>	H	$\{Sokrates, Aristotle, Plato\}$
m	<i>Michelangelo</i>	M	$\{Sokrates, Aristotle, Plato, Michelangelo, Bach, Tarski\}$
b	<i>Bach</i>		
t	<i>Tarski</i>	L	$\{\langle Sokrates, Sokrates \rangle, \langle Sokrates, Aristotle \rangle, \langle Michelangelo, Bach \rangle, \langle Bach, Michelangelo \rangle, \langle Tarski, Plato \rangle, \langle Plato, Michelangelo \rangle, \langle Aristotle, Tarski \rangle\}$
a	<i>Aristotle</i>		
p	<i>Plato</i>		

- (4)
- $\llbracket (\exists y)(\forall x)L(x, y) \rrbracket^M =$
 - $\llbracket (\forall x)\neg(\exists y)L(x, y) \rrbracket^M =$
 - $\llbracket ((\exists z)M(z) \wedge (\forall y)(H(y) \rightarrow L(y, b))) \rrbracket^M =$ continues on next page \leftrightarrow

Exercises 4:

- Given the equivalences in (5), prove the equivalence between (6-a,b). Give the names of the laws of logic that you make reference to in your proof.

- (5) a. $(\neg(\forall x)(P(x))) \Leftrightarrow (\exists x)\neg(P(x))$
b. $(\neg(\exists x)(P(x))) \Leftrightarrow (\forall x)\neg(P(x))$
- (6) a. Kein Kind fährt nicht nach Rom.
b. Alle Kinder fahren nach Rom.