

Excercises 8

Excercise 1: Proofs

- Give formal proofs of the validity of each of the following argument forms. If a direct proof seems impossible, try a proof by contradiction (*reduction ad absurdum*).

$$(1) \quad \begin{array}{l} 1. (p \rightarrow q) \\ 2. (q \rightarrow r) \\ 3. (\neg r) \\ \hline \therefore (\neg p) \end{array}$$

$$(2) \quad \begin{array}{l} 1. p \\ 2. (\neg r) \\ 3. ((p \wedge (\neg r)) \rightarrow q) \\ \hline \therefore q \end{array}$$

$$(3) \quad \begin{array}{l} 1. (p \vee q) \\ 2. (\neg q) \\ 3. (r \rightarrow (\neg p)) \\ \hline \therefore (\neg r) \end{array}$$

$$(4) \quad \begin{array}{l} 1. (p \rightarrow (\neg q)) \\ 2. (r \rightarrow q) \\ 3. ((\neg r) \rightarrow s) \\ \hline \therefore (p \rightarrow s) \end{array}$$

$$(5) \quad \begin{array}{l} 1. ((\neg p) \vee q) \\ 2. ((\neg q) \wedge r) \\ 3. ((\neg(p \vee q)) \rightarrow s) \\ \hline \therefore (r \wedge s) \end{array}$$

$$(6) \quad \begin{array}{l} 1. (p \vee (q \wedge r)) \\ 2. (\neg t) \\ 3. ((p \vee q) \rightarrow (s \vee t)) \\ 4. (\neg p) \\ \hline \therefore (r \wedge s) \end{array}$$

$$(7) \quad \begin{array}{l} 1. (p \rightarrow q) \\ 2. (r \rightarrow s) \\ 3. ((\neg q) \vee (\neg s)) \\ 4. p \\ 5. ((t \wedge u) \rightarrow r) \\ \hline \therefore ((\neg t) \vee (\neg u)) \end{array}$$

$$(8) \quad \begin{array}{l} 1. ((p \wedge q) \rightarrow (p \rightarrow (r \wedge s))) \\ 2. ((p \wedge q) \wedge u) \\ \hline \therefore (r \wedge s) \end{array}$$

$$(9) \quad \begin{array}{l} 1. (p \leftrightarrow q) \\ 2. (\neg p) \\ 3. ((q \wedge (\neg r)) \vee t) \\ 4. ((s \vee t) \rightarrow r) \\ \hline \therefore (r \wedge (\neg q)) \end{array}$$

$$(10) \quad \begin{array}{l} 1. ((\neg p) \rightarrow q) \\ 2. (r \rightarrow (s \vee t)) \\ 3. (s \rightarrow (\neg r)) \\ 4. (p \rightarrow (\neg t)) \\ \hline \therefore (r \rightarrow q) \end{array}$$

$$(11) \quad \begin{array}{l} 1. (p \rightarrow (q \wedge r)) \\ 2. (q \rightarrow s) \\ 3. (r \rightarrow t) \\ 4. ((s \wedge t) \rightarrow (\neg u)) \\ 5. u \\ \hline \therefore (\neg p) \end{array}$$

$$(12) \quad \begin{array}{l} 1. p \\ 2. ((p \wedge q) \vee (p \wedge r)) \\ 3. ((p \vee q) \rightarrow (\neg r)) \\ \hline \therefore (p \leftrightarrow q) \end{array}$$